

Optimal economic lot schedules with lost demand

Waldemar KACZMARCZYK 

This paper presents a new mathematical programming model for the *economic lot scheduling problem* (ELSP) with a *common cycle* for all products. The proposed model takes into account lost demand when capacity is insufficient or when set-up and inventory holding costs are too high. Two cases are modeled. In the first case, demand is fully satisfied only during a fraction of the production cycle. In the second case, a constant fraction of demand is met throughout the entire cycle. The general non-linear model of the considered problem is reformulated as a *mixed-integer quadratically constrained programming* (MIQCP) model, enabling a general-purpose solver to find an optimal solution in a short time. This paper aims to encourage researchers and engineers to tackle non-convex non-linear optimization problems using modern general-purpose solvers.

Key words: inventory control, production, scheduling, mixed-integer quadratically constrained programming

1. Introduction

The *economic lot scheduling problem* (ELSP) is one of the most important operations research models applied in production and inventory control. The task is to determine the optimal lot sizes and a schedule for several items produced on a single machine with limited capacity. It assumes that demand and production rates are constant, the planning horizon is infinite, and therefore, the solution is a cyclic schedule. The goal is to minimize the average per unit of time of set-up costs, inventory holding costs, and the costs of lost demand. In practice, the ELSP is applied when parameters are stable over a time interval during which the production cycle is expected to be executed several times. It may be used at both tactical and operational levels of planning.

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W. Kaczmarczyk (e-mail: [waldek@agh.edu.pl](mailto:walde@agh.edu.pl)) is with Faculty of Management, AGH University, al. Mickiewicza 30, 30-059, Kraków, Poland

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Several papers review the applications of the ELSP, among others, in the automotive, metal, food, chemical, and plastic industries [2,6,17,22]. Some recent publications describe adaptations of the ELSP to power energy management [3,9], batch shipments [1], deteriorating items and backlog costs [17]. A broad review of literature on the ELSP present [2,6,22].

The general ELSP model is very challenging to solve; therefore, most authors employ simplifying assumptions to make it easier to solve. Hanssmann [14] proposed the *common cycle approach*, which assumes that all products have the same cycle length. It is often adopted due to its simplicity, computational convenience, and satisfactory quality of solutions for problem instances with minor parameter differences across various products [7,22].

In cases with larger parameter variability, the *extended basic period approach* [7] may be more suitable, as it allows for different production cycles. This is possible when the production cycle is divided into several periods, during which each product is produced at most once, and the production cycles are integer multiples of the base period length. The scheduling subproblem of this approach, which involves assignment of lots to periods and sequencing, can be optimized using a *mixed-integer programming* (MIP) model [16].

The cyclic ELSP schedules are intended to be applied over several months or longer. The ELSP, therefore, may be seen as a method of mid-term planning, similarly to the *sales and operations planning* (SOP) [23]. In such a case, one of the most important decisions considered should be the choice of products to produce, the fraction of demand to meet, and the fraction that may be lost.

Taking this into account, the literature on the ELSP accounting for lost demand is limited. Its detailed review is presented in the next section; a summary of it is provided below. First of all, researchers consider only the common cycle approach, and there is no optimal method for solving this type of problem with non-zero set-up costs and times. Next, all proposed heuristics ignore either set-up costs or times, which are often substantial in production systems with steady demand due to high production scale, and therefore have a significant impact on optimal solutions. Moreover, all the known models and methods assume that the entire demand for some products may be unmet, which can be difficult for managers to accept, as it means that the company completely hands over this part of the market to competing companies. This may be contrary to the company's long-term goals. Therefore, managers may need to set an upper limit for the lost demand separately for each product.

In 1962, Hanssmann [14] presented an exact algorithm for the basic case of the ELSP with a common cycle. Then, for more than sixty years, researchers have developed numerous specialized algorithms for more complex ELSP cases, such as the extended basic period approach [7,16] or cases with shortages [2,22]. To

date, application of mathematical programming has been limited to specific ELSP subproblems with a linear goal function [16, 22].

During this time, enormous progress has been made in the development of optimization methods for mixed-integer non-convex non-linear problems. General-purpose solvers ensuring global optimality use the *spatial branch-and-bound* algorithm, piecewise relaxation techniques, bilinear programming techniques, and several other techniques, and are built on top of *mixed-integer (linear) programming* (MIP) solvers [5, 13]. The GAMS documentation [11] lists fifteen general-purpose solvers capable of optimizing general *mixed-integer non-linear programming* (MINLP) models and eighteen for *mixed-integer quadratically constrained programming* (MIQCP) models; however, not all ensure globally optimal solutions. Surprisingly, researchers working on inventory control problems ignore this development.

This paper presents a mathematical programming model for the ELSP with non-zero set-up costs and times, following the common cycle approach, which allows for insufficient capacity and lost demand. This model can be easily solved using standard methods of a general-purpose solver. To achieve this, a straightforward MINLP model is reformulated as a *mixed-integer quadratically constrained programming* model (MIQCP). Such reformulation is purposeful, because MIQCP solvers are significantly faster than solvers for general MINLP problems, and are already a component of several commercial optimization packages [11, 13].

For decades, a similar approach has been a standard for so-called *dynamic lot-sizing problems*, which assume variable demand during a finite planning horizon divided into periods. To address such problems, researchers use the *mixed-integer (linear) programming* (MIP), seek tight model formulations, employ various model reformulations, incorporate so-called valid inequalities, and introduce symmetry-breaking constraints. A comprehensive overview of such techniques is presented in the book by Pochet and Wolsey [20], and a specific example can also be found in [15]. This paper attempts to apply a similar approach to the ELSP problem using MINLP models and methods.

This article is organized as follows. Section 2 contains a review of the literature on the ELSP with lost demand. Section 3 presents mathematical programming models for the ELSP following the common cycle approach. Section 4 presents the proposed MIQCP model for two types of models with insufficient capacity and lost demand. Section 5 contains results for several problem instances. Section 6 gives the summary.

2. Literature

Most authors consider lost demand only in cases with *insufficient capacity*, when production rates are too low to meet the entire demand for all products.

The lost demand and inventory levels are modeled in two different ways. In models with the *ready rate*, the demand for a product is fully met during a time interval which is only a fraction of the total production cycle. In models with the *fill rate*, only a constant fraction of demand is satisfied throughout the entire production cycle. The names *ready rate* and *fill rate* are used here following Silver et al. [23] (p. 249), who employ them with a very similar meaning to describe service measures for stochastic inventory replenishment policies. The fill rate may be used when a company outsources a constant fraction of its demand to subcontractors; the ready rate, when all demand is met until the stock runs out.

Goyal and Gopalakrishnan [12] consider a single-product problem, in which the goal is to find a favorable trade-off between minimizing set-up costs, inventory holding costs, and lost demand (maximizing ready rate) due to capacity waste during set-up activities. The authors did not explain why it is necessary to make many set-ups of the machine producing only one product, which fully utilizes machine capacity.

Gallego and Moon [10] consider a problem similar to the ELSP, but with a fixed planning horizon divided into periods and a fixed total number of lots, aiming to minimize total stock-out duration and lost demand.

Bollapragada and Rao [4] consider the common cycle ELSP with insufficient capacity, fill rate, and parallel machines. They propose a straightforward MINLP model, but solve the problem using specialized heuristics developed only for the case with negligible set-up times, based on decomposition into a single machine subproblems. Pesenti and Ukovich [19] consider the same problem and provide another heuristic approach that generates feasible schedules.

Faaland et al. [8] consider a similar problem for a single machine and a large number of products, assuming sufficient capacity to meet the entire demand; however, for some products, it may be unprofitable due to high inventory holding costs. They present a straightforward MINLP model but solve the problem using a heuristic algorithm, which has two flaws. First, it ignores set-up cost. Second, it is based on an objective function that incorrectly models the inventory holding costs, underestimating inventories when part of the demand is lost.

Khoury et al. [18] developed an optimal algorithm for the common cycle ELSP with insufficient capacity and ready rate. Their method ignores, however, setup times and requires that production of some products is completely abandoned.

3. Models for common cycle approach

The basic parameters of the ELSP problem are shown in the list below. Several products j with constant and deterministic demand rate D_j are produced on a single machine at rate R_j . Before changing the product being made, the machine

must be set up, which takes some time A_j and cost S_j , both independent from the sequence of products. Inventory holding cost is proportional to the inventory level and unit holding cost h_j .

Basic parameters:

$\mathcal{N} = \{1, \dots, n\}$ – set of products, where n is their number;

D_j – demand of product j [product units per day];

R_j – production rate of product j [product units per day];

A_j – set-up cost of product j [monetary value];

h_j – unit holding cost of product j [monetary value per unit of product and per day];

S_j – set-up time of product j [days].

Derived parameters:

$\rho_j = D_j/R_j$ – rate ratio of product j ;

$\widehat{D}_j = D_j(1 - \rho_j)$ – adjusted demand of product j .

Variables:

T – cycle length.

Ratio ρ_j is the fraction of time (production cycle, capacity) that must be devoted to producing product j to meet the demand. Adjusted demand $\widehat{D}_j$ simplifies notation. For given T the lot sizes equal $Q_j = D_j T$.

3.1. General MINLP model

The economic lot-scheduling problem (ELSP) following the common cycle approach describes the following mixed-integer non-linear programming model (MINLP):

minimize

$$\sum_{j \in \mathcal{N}} \left(\frac{A_j}{T} + h_j \frac{\widehat{D}_j}{2} T \right) \quad (1a)$$

subject to

$$\sum_{j \in \mathcal{N}} S_j + \left(1 - \sum_{j \in \mathcal{N}} \rho_j \right) T \leq T, \quad (1b)$$

$$\underline{T} \leq T \leq \overline{T}. \quad (1c)$$

The objective function (1a) describes the average total set-up and inventory holding cost per unit of time. Inequality (1b) ensures that workload never exceeds the cycle length. Constraints (1c) define the domain of the decision variable T .

Setting bounds on variables is often very important in the non-linear programming [13]; therefore, in the above model, the cycle length has some lower $\underline{T}$ and upper $\bar{T}$ bounds. They may be arbitrarily set by the decision maker. For example, the lower bound may be equivalent to a single day or the sum of the set-up times of all products, and the upper bound may result from market features not reflected in the ELSP model. The bounds may also be obtained using the so-called *independent solution* [7] — optimal cycle times T_j^* determined for each product j separately using the *economic production quantity* (EPQ) formula [24]:

$$T_j^* = \sqrt{\frac{2A_j}{h_j \widehat{D}_j}}. \quad (2)$$

Optimal ELSP cycle length must be a compromise solution for all products; therefore, its lower bound may be estimated as the shortest independent cycle (2) obtained among all products, and the upper bound as the longest.

For the classic formulation of the common cycle ELSP, an optimal solution may be easily obtained without the help of mathematical programming. Hanssmann [14] presented following approach:

$$T_c = \sqrt{\frac{2 \sum_{j \in N} A_j}{\sum_{j \in N} h_j \widehat{D}_j}}, \quad (3a)$$

$$T_w = \sum_{j \in N} S_j / \left(1 - \sum_{j \in N} \rho_j\right), \quad (3b)$$

$$T^* = \max(T_w, T_c). \quad (3c)$$

T_c is the cost-optimal cycle length derived from (1a). T_w is the minimal feasible cycle length due to the workload and set-up time derived from (1b). The optimal cycle length T^* is the higher of these two values.

For more complex ELSP cases, obtaining an optimal solution is significantly more challenging. After more than sixty years, there is no method to determine an optimal solution for many seemingly simple cases, such as the one considered in this paper, involving lost demand, with non-zero setup times and costs. The goal of this paper is to fill this gap using mathematical programming.

3.2. MIQCP model

Conversion of the straightforward MINLP model of the classic common cycle ELSP to a quadratically constrained model (MIQCP) requires to eliminate division by

cycle length T in the objective function. This is possible through the introduction of additional variable:

$T^R = 1/T$ – reverse of cycle length.

With its help the general MINLP model (1) may be converted into following MIQCP model:

minimize

$$\sum_{j \in \mathcal{N}} \left(A_j T^R + h_j \frac{\widehat{D}_j}{2} T \right) \quad (4a)$$

subject to

$$\sum_{j \in \mathcal{N}} S_j + \sum_{j \in \mathcal{N}} \rho_j T \leq T, \quad (4b)$$

$$T T^R = 1, \quad (4c)$$

$$\underline{T} \leq T \leq \overline{T}, \quad (4d)$$

$$1/\overline{T} \leq T^R \leq 1/\underline{T}. \quad (4e)$$

In the objective function (4a), the division by cycle length T is replaced by multiplication by the new variable T^R representing the inverse of T . The bilinear constraint (4c) defines the relationship between cycle length and its inverse. A function $f(x, y)$ is bilinear if it is linear in x when y is fixed, and linear in y when x is fixed. Inequalities (4e) define the domain of the new variable. Constraints describing the workload (4b) and cycle domain (4d) remain the same as in model (1).

4. Models with lost demand

To describe problems with lost demand, the following additional parameters and variables are necessary:

Additional parameters:

l_j – lost demand unit cost of product j per unit of product (e.g., lost profit, loss of goodwill).

Additional variable:

$x_j = [0, 1]$ – fraction of demand of product j that is satisfied;

$y_j = [0, 1]$ – fraction of demand of product j that is lost, remains unsatisfied;

$\delta_j = 1$, if machine is set up to produce product j ; 0 otherwise.

The two types of models with planned lost demand, with the fill rate and ready rate, are illustrated in Figure 1, which shows the changes in single-product

inventory. The single cycle begins at point A and ends at point D, and has a length of T . Production of product j occurs within the time interval AB. The order Q_j satisfies only a fraction x_j of the demand over the entire cycle.

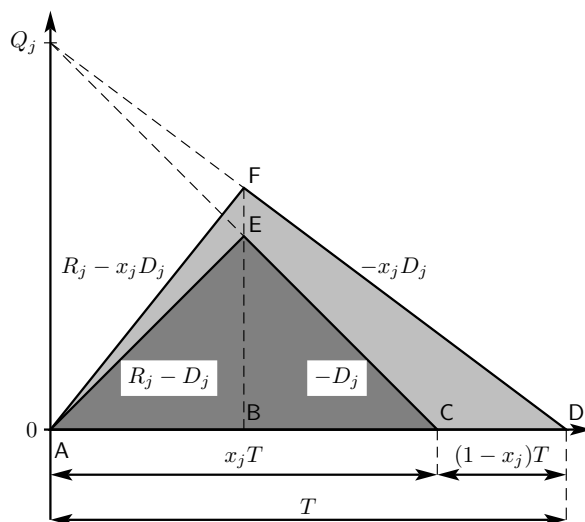


Figure 1: Changes in a single product inventory whose demand is only partially met.

The inventory in the ready rate model represents the dark-gray triangle ACE. The demand is fully met in the interval AC. In the rest of the cycle, in the interval CD, there is no inventory and demand is not met at all. During the production interval AB, the inventory rises at a rate $R_j - D_j$, and then decreases at a rate D_j .

The light-Grey triangle ADF shows inventory in the fill rate model. The inventory is positive over the entire cycle. First, the inventory level rises faster and higher than in the ready rate model, at a rate of $R_j - x_j D_j$, and then decreases more slowly, at a rate of $x_j D_j$.

4.1. Model with limited ready rate

The average total cost with limited *ready rate* ($x_j \leq 1$) describes the function below [18]:

$$\sum_{j \in \mathcal{N}} \left(\frac{A_j}{T} + h_j \frac{\widehat{D}_j}{2} x_j^2 T + l_j D_j y_j \right). \quad (5)$$

In the function (5) non-linear and non-quadratic is the product of three variables $x_j^2 T$, which may, however, be easily replaced by two types of quadratic constraints. The MIQCP model below describes the ELSP following common cycle approach with ready rate, denoted here ELSP/xT:

minimize

$$c^S + c^H + c^L \quad (6a)$$

subject to

$$\sum_{j \in \mathcal{N}} A_j \delta_j = \alpha, \quad (6b)$$

$$\alpha T^R = c^S, \quad (6c)$$

$$\sum_{j \in \mathcal{N}} h_j \frac{\widehat{D}_j}{2} x_j^2 = \beta, \quad (6d)$$

$$\beta T = c^H, \quad (6e)$$

$$\sum_{j \in \mathcal{N}} l_j D_j y_j = c^L, \quad j \in \mathcal{N}, \quad (6f)$$

$$x_j \leq \delta_j, \quad j \in \mathcal{N}, \quad (6g)$$

$$x_j + y_j = 1, \quad j \in \mathcal{N}, \quad (6h)$$

$$\sum_{j \in \mathcal{N}} \rho_j x_j = \gamma, \quad (6i)$$

$$\sum_{j \in \mathcal{N}} S_j \delta_j + \gamma T \leq T, \quad (6j)$$

$$T T^R = 1, \quad (6k)$$

$$\underline{T} \leq T \leq \bar{T}, \quad (6l)$$

$$1/\bar{T} \leq T^R \leq 1/\underline{T}, \quad (6m)$$

$$\alpha, \beta, \gamma, c^S, c^H, c^L \geq 0, \quad (6n)$$

$$0 \leq x_j \leq 1, \quad j \in \mathcal{N}, \quad (6o)$$

$$0 \leq y_j \leq \bar{y}_j, \quad j \in \mathcal{N}, \quad (6p)$$

$$\delta_j \in \{0, 1\}, \quad j \in \mathcal{N}. \quad (6q)$$

The objective function (6a) is linear and describes the average total cost per unit of time. Equation (6b) defines auxiliary variable α as a linear expression, which aggregates binary variables. Equation (6c) determines the average set-up cost, where variable α prevents multiplying the inverse of cycle length T^R by each binary variable δ_j separately. This way the set-up cost is determined using only one bilinear expression.

The quadratic equation (6d) defines auxiliary variable β , which aggregates variables x_j . Equation (6e) determines the average inventory holding cost and

thanks to β contains only one bilinear expression. The linear equation (6f) determines the cost of lost demand.

The big M constraint (6g) (with $M = 1$) allows non-zero production of product j only if machine is set up to produce this product. Equality (6h) couples variables x_j and y_j . Fundamentally, the variable y_j is not needed, could be replaced with $1 - x_j$, but it makes the model easier to build and read, and the pre-solver can easily optimize such structures. Equation (6i) aggregates variables x_j into one auxiliary variable γ to have only one bilinear expression in the next constraint. Constraint (6j) ensures that the workload does not exceed the cycle length. Equation (6k) ensures that T^R is the inverse of couple T . Constraints (6l)–(6q) define the domain of all variables.

The lower and upper bounds of cycle length, $\underline{T}$ and $\bar{T}$, are hard to set due to the lost demand. The individual optimal cycles of all products are no longer helpful, as they depend on the satisfied demands, which may be even zero. Therefore, cycle bounds should be set by managers based on their requirements and expertise. The upper bound of lost demand $\bar{y}_j$ is by definition equal to one or some smaller value desired by managers due to long-term plans of the company.

Model (6) with n products has n binary variables, $n + 3$ quadratic terms, 3 of which are bilinear. The square of one variable is a purely quadratic term, not a bilinear term. That is, not all quadratic terms are bilinear, but all bilinear terms are quadratic.

Khoury et al. [18], in their ready rate optimal method, use decision variables $T_j = x_j T$, which describe the product-individual time interval during which his demand is met, instead of x_j . For the five problem instances presented in [18], the above model (6) ensures the same solutions as the Khoury et al. algorithm.

4.2. Model with limited fill rate

The average total cost with limited *fill rate* ($x_j \leq 1$) describes the function below [4, 19]:

$$\sum_{j \in N} \left(\frac{A_j}{T} + h_j \frac{x_j D_j}{2} \left(1 - \frac{x_j D_j}{R_j} \right) T + l_j D_j y_j \right). \quad (7)$$

It differs from (5) only by the middle element describing the inventory holding cost.

The MIQCP model with the fill rate (8) presented below, denoted by ELSP/xD, is the same as the ELSP/xT model described by the set of equations (6a)–(6q), except for the constraint (6d), which the following constraint (8d) must replace:

minimize

$$c^S + c^H + c^L \quad (8a)$$

subject to

$$\begin{aligned} & \vdots \\ & \sum_{j \in \mathcal{N}} h_j \frac{x_j D_j}{2} \left(1 - \frac{x_j D_j}{R_j} \right) = \beta, \end{aligned} \quad (8d)$$

$$\begin{aligned} & \vdots \\ & \delta_j \in \{0, 1\}, \quad j \in \mathcal{N}. \end{aligned} \quad (8q)$$

Both constraints (6d) and (8d) contain the same number of quadratic terms.

5. Results

To test the scalability of the MIQCP models ELSP/xT (6) and ELSP/xD (8), five problem instances were created for $n = 10, 50$, or 100 products, and two data generation patterns [21], in a total of 30 problem instances. The second data pattern has higher parameter variability. Most authors consider only ELSP problems with $n \leq 15$ per machine [4, 18, 19, 21].

First, parameters were drawn from a uniform distribution using the value ranges presented in Table 1. Next, the demand for all products was proportionally adjusted so that the workload equals $\sum_{j \in \mathcal{N}} D_j / R_j = 1.2$, meaning the total demand is 20% higher than the capacity. The set-up time was set to $S_j = S'_j \times 10/n$ to maintain the total set-up time independent of the number of products. The unit inventory holding cost per day was determined as $h_j = h'_j / 250$, where h'_j is the cost per year and 250 is the number of workdays per year. All cases have been solved with the lost demand upper bound $\bar{y}_j = 1.0$ or 0.4 , the same for all products, which is equivalent to a minimal ready or fill rate of $x_j = 0.0$ or 0.6 . The cycle length bounds have been set fully arbitrarily: $\underline{T} = 1$ day and $\bar{T} = 365$ days.

Table 1: Value ranges used to draw parameters of problem instances [21]

	Data set 1	Data set 2
D_j	[1000, 2000]	[500, 2000]
R_j	[4000, 20 000]	[1500, 30 000]
h'_j	[1, 5]	[0.005, 5]
A_j	[50, 100]	[10, 350]
S'_j	[1, 4]	[1, 8]

The unit cost of lost demand was randomly generated using the following approach. First, the product value v_j was estimated as $v_j = h'_j/0.1$, where 0.1 is the annual interest rate. Next, the profit margin p_j was randomly drawn from a uniform distribution using the range $[0.05, 0.25]$. Finally, the unit cost of lost demand was set as $l_j = v_j p_j$. The obtained ratio l_j/h_j was in the range $[125, 625]$.

All instances were solved with the help of the GUROBI 11.0.3 solver on an Intel Core i9-7900X processor with 3.3 GHz clock speed, 16 GB RAM, 10 physical cores, and 20 logical processors, utilizing up to 20 threads, with a time limit of 60 seconds.

In all solutions obtained, machine utilization was 100%, which is not surprising given that workload exceeds the capacity.

The solver only twice reached the time limit, for the ELSP/xT model, $n = 100$ products, and $\bar{y}_j = 0.4$. In both cases, the final MIP gap was about 0.03%, which means the solutions were very close to optimality or optimal. For all other considered cases, the final MIP gap was 0%, indicating that the solver had completed the search and proven the optimality of the solutions.

Table 2 presents the average optimization effort, described by the number of nodes, iterations, and optimization time in seconds. These results prove that solving the ELSP using standard MIQCP methods is now quite easy. In most cases the average optimization time is smaller than 3 seconds. For the ELSP/xT model

Table 2: Average optimization effort

MIQCP model	Data set	n	$\bar{y}_j = 1.0$			$\bar{y}_j = 0.4$		
			Nodes [10 ³]	Iterations [10 ³]	Time [sec]	Nodes [10 ³]	Iterations [10 ³]	Time [sec]
xT	1	10	0.20	0.76	0.15	0.20	0.62	0.04
		50	4.90	50.06	1.21	6.02	15.54	0.29
		100	2.53	38.81	1.44	375.92	807.98	14.47
	2	10	0.10	0.66	0.11	0.76	1.84	0.05
		50	5.59	68.62	1.00	84.76	191.06	2.84
		100	2.92	51.38	1.66	601.54	1373.91	23.06
xD	1	10	0.32	0.93	0.10	0.04	0.16	0.03
		50	4.27	21.29	0.77	0.05	0.57	0.06
		100	3.07	18.83	0.90	0.03	0.46	0.11
	2	10	0.12	0.63	0.12	0.03	0.12	0.03
		50	5.26	40.07	0.68	0.05	0.45	0.11
		100	2.63	22.58	1.03	0.02	0.47	0.32

and $n = 100$ products, it is higher but still very short, less than 24 seconds on average.

For cases without any limits on lost demand ($\bar{y}_j = 1.0$), the number of nodes and iterations is higher for $n = 50$ than for 100, which is surprising and difficult to explain.

For problem instances with an upper bound on lost demand $\bar{y}_j = 0.4$, ensuring that at least 60% of the demand of each product will be satisfied, all binary variables must take the value one ($\delta_j = 1$), and all nodes in the search algorithm are created due to branching of continuous variables to improve approximation of the non-convex constraints.

For the ELSP/xT model with $\bar{y}_j = 0.4$, the optimization time is significantly longer than for $\bar{y}_j = 1.0$ and increases rapidly with the number of products. For the ELSP/xD model, the opposite is true; the optimization time is much shorter for $\bar{y}_j = 0.4$ than for $\bar{y}_j = 1.0$ and rises only moderately with the number of products. Such a significant difference between two very similar models is surprising.

Table 3 presents the average number of products with a given value of ready or fill rate x_j . The solutions obtained for the two models are pretty similar. For $\bar{y}_j = 1.0$, products with the minimal fill/ready rate $x_j = 0.0$ are about two times less frequent than the products with the maximal fill/ready rate $x_j = 1.0$. For $\bar{y}_j = 0.4$, the minimal rate equals $x_j = 0.6$ and is about two times more frequent than the maximal rate $x_j = 1.0$.

Table 3: Average number of products with given value of ready or fill rate x_j

Model	n	$\bar{y}_j = 1.0$			$\bar{y}_j = 0.4$		
		0.0	other	1.0	0.6	other	1.0
xT	10	2.9	0.4	6.7	6.0	1.6	2.4
	50	15.2	0.2	34.6	32.8	4.8	12.4
	100	31.5	0.6	67.9	65.0	8.0	27.0
xD	10	3.0	0.2	6.8	6.6	0.5	2.9
	50	15.3	0.0	34.0	34.6	0.3	15.1
	100	31.7	0.0	68.3	68.4	0.5	31.1

What is more interesting, the optimal solutions of the ELSP/xD model have at most one product with the rate x_j different from the minimal or maximal feasible value. For the ELSP/xT model, the number of such rate values is always greater than or equal to one, and for $\bar{y}_j = 0.4$ increases with the number of products. For $n = 100$ products, the average was 8, but the maximum was even 15 products, what may be the explanation for the significantly higher optimization time.

6. Summary

This paper presents a *mixed-integer quadratically constrained programming* (MIQCP) model for the *economic lot scheduling problem* (ELSP) with a *common cycle* for all products, insufficient capacity and lost demand. The proposed model considers the ready rate, which is the percentage of time that a product's on-hand stock is greater than zero; and the fill rate, which is the constant percentage of demand met throughout the entire production cycle. Moreover, this model allows managers to set the maximum lost demand for each product individually. Finally, optimizing the proposed model is the only known method to find optimal solutions to the ELSP with lost demand and non-zero setup cost and time.

Computational experiments demonstrated that all versions of the model can be solved efficiently using a general-purpose solver, ensuring the global optimality of the solutions through the use of the spatial branch-and-bound algorithm. Optimization time of less than a minute is very short for a production schedule intended to be applied cyclically over several months. In all solutions of the model with the fill rate, all products but one have either the minimal or maximal fill rate, which may be utilized to develop a better model or optimization approach in the future.

The obtained results demonstrate that some lot-sizing and scheduling problems with stable demand can be addressed using standard mathematical programming methods and model reformulation, similarly to those with dynamic demand. It should encourage researchers and engineers to tackle non-convex non-linear optimization problems using modern general-purpose solvers.

Future research should adapt the proposed model to the extended basic approach for problems with higher data variability and extend it to consider several special cases, such as production backlogs or variable production rates.

Data availability statement

The dataset, including all cases, models, and solutions, is available from the repository RODBUK of the AGH University (agh.rodbuk.pl) at DOI: [10.58032/AGH/TZARZF](https://doi.org/10.58032/AGH/TZARZF).

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