

A new 3-D chaotic system with a curve equilibrium: Bifurcation analysis, multistability, circuit design and complete synchronization design

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In this study, a new three-dimensional chaotic system has been derived by adding two nonlinearities to Sambas chaotic system (2019) with a curve equilibrium. It is shown that the new chaotic system also features the same curve equilibrium as the Sambas chaotic system (2019), namely a pair of circles in the (x, y) -plane with centres in the x -axis and intersecting at the origin. It is established that the new chaotic system with a curve equilibrium has more complexity than the Sambas chaotic system (2019) with a curve equilibrium. Next, a detailed bifurcation analysis is presented for the new chaotic system with a curve equilibrium. Multistability analysis for the new chaotic system with a curve equilibrium is exhibited by the coexistence of two chaotic attractors for the same parameters as in the chaotic case but different values of the initial states. Using Multisim, an electronic circuit design for the proposed chaotic system has been exhibited

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which will be useful for practical implementations. Using integral sliding mode control (ISMC), complete synchronization of a pair of new chaotic systems taken as the master and slave systems has been achieved. Simulations using MATLAB R2024b have been shown to illustrate the main results of this paper. The proposed chaotic system with a curve equilibrium will be useful for applications in science and engineering.

Key words: chaos, chaotic systems, dynamic analysis, bifurcation, multistability, circuit design, MultiSim, integral sliding mode control.

1. Introduction

Chaotic dynamical systems are nonlinear dynamical systems that are characterized by the presence of at least one positive Lyapunov exponent, which exhibit extreme sensitivity to their initial conditions, causing small variations at the start to amplify exponentially over time [1]. Examples of chaotic dynamics appear in various natural, scientific and engineering areas, such as weather systems [2, 3], chemical reactions [4, 5], lasers [6–8], memristors [9, 10], etc.

Many research works in the field of chaos theory have investigated chaotic systems characterized by special curves of equilibrium points [11]. Vaidyanathan *et al.* proposed a new chaotic system with an axe-shaped equilibrium curve [12]. Vaidyanathan *et al.* proposed a new conservative chaotic system with a lemniscate-shaped equilibrium curve [13]. Mobayen *et al.* proposed a novel chaotic system with a boomerang-shaped equilibrium curve [14]. Mohamed *et al.* designed a speech cryptosystem using a new chaotic system with a capsule shaped equilibrium curve [15]. Sambas *et al.* proposed a new 3-D chaotic system with a curve equilibrium consisting of two circles in the (x, y) -plane of $\mathbb{R}^3$ [16].

In this research, a new three-dimensional chaotic system has been derived by adding two nonlinearities to Sambas chaotic system with a curve equilibrium [16]. It is shown that the new chaotic system also features the same curve equilibrium as the Sambas chaotic system [16], namely a pair of circles in the (x, y) -plane with centres in the x -axis and intersecting at the origin. It is established that the new chaotic system with a curve equilibrium has more complexity than the Sambas chaotic system with a curve equilibrium [16].

Bifurcation analysis is a valuable method for exploring chaotic systems, as it uncovers how small changes in system parameters can lead to major shifts in behavior [17]. Bifurcation analysis shows the equilibrium points where the system's dynamics suddenly change, giving rise to new states such as stable fixed points, periodic orbits, or chaotic attractors [18]. We have presented a detailed bifurcation analysis for the new 3-D chaotic system with a curve equilibrium.

Multistability in chaotic dynamical systems manifests as the simultaneous presence of multiple long-term stable configurations, known as attractors, within a single framework under identical parameter values [19]. The existence of mul-

tistability in chaotic regimes stems from the diverse nature of these coexisting attractors amplifying the system's exquisite sensitivity to perturbations and onset conditions [20]. We have presented a detailed multistability analysis for the new 3-D chaotic system with a curve equilibrium.

Electronic simulations of chaotic systems using software or hardware are crucial for understanding, analyzing, and applying chaotic dynamics in a controlled and flexible manner [21]. Electronic simulations of chaotic systems deliver efficient, and cost-effective means for technological innovations across cryptography, communications, complex system modeling, etc [22]. Using Multisim, we have presented a detailed multistability analysis for the new 3-D chaotic system with a curve equilibrium.

Synchronization of chaotic systems enables secure communication by letting a transmitter and receiver generate identical chaotic signals even if they start from slightly different conditions [23]. Synchronization of chaotic systems has many applications in secure communication devices [25–27]. In this research work, we use Integral Sliding Mode Control (ISMC) to achieve chaos synchronization between a pair of new chaotic systems taken as the transmitter and receiver systems. ISMC excels in control applications like stabilization and synchronization involving chaotic systems by ensuring the system trajectory remains on the sliding surface from the initial instant, eliminating the reaching phase of classical SMC and delivering robustness across the entire state space against matched uncertainties and disturbances [28–30].

2. A new 3-D chaotic system with two circles of equilibrium points

In 2019, Sambas *et al.* [16] obtained a new 3-D chaotic system with two circles of equilibrium points. Sambas chaotic system [16] is given by the following 3-D dynamics:

$$\begin{cases} \dot{x} = z, \\ \dot{y} = (-ay - by^2 - cxz)z, \\ \dot{z} = x^2 + y^2 - |x|. \end{cases} \quad (1)$$

In the system (1), a, b, c are positive constants and $X = (x, y, z)$ is the state vector.

Sambas *et al.* [16] showed that the system (1) is chaotic when $a = 4$, $b = 4.5$ and $c = 1$. The Lyapunov characteristic exponents (LCE) for the Sambas system (1) were determined in MATLAB R2024b for the initial state $X(0) = (0.02, 0.01, 0.02)$ and $(a, b, c) = (4, 4.5, 1)$ for $T = 1 \times 10^4$ seconds as $l_1 = 0.0213$, $l_2 = 0$ and $l_3 = -0.0531$. Thus, the Maximum Lyapunov Exponent (MLE) for the Sambas system (1) is easily seen as $l_{\max} = 0.0213$.

The equilibrium points of the Sambas system (1) are obtained by solving the following three equations.

$$z = 0, \quad (2a)$$

$$(-ay - by^2 - cxz)z = 0, \quad (2b)$$

$$x^2 + y^2 - |x| = 0. \quad (2c)$$

We note that $z = 0$ satisfies both equations (2a) and (2b). Thus, the Sambas system (1) consists of a curve of equilibrium points in the (x, y) plane given by the following set:

$$S = \{X = (x, y, z) : z = 0, x^2 + y^2 - |x| = 0\}. \quad (3)$$

The equilibrium curve S is symmetric with respect to both x -axis and y -axis in the (x, y) coordinate plane. Geometrically, S represents a pair of circles with the centres in the x -axis and passing through the origin..

In this research work, we obtain a new 3-D chaotic system with a curve equilibrium by modifying the dynamics of the system (1) by means of adding a quadratic nonlinearity and a cubic nonlinearity in the second differential equation in (1). Thus, we propose the following 3-D dynamics:

$$\begin{cases} \dot{x} = z, \\ \dot{y} = (qx - ay - px^2 - by^2 - cxz)z, \\ \dot{z} = x^2 + y^2 - |x|. \end{cases} \quad (4)$$

We denote the state of the new dynamics (4) as $X = (x, y, z)$. Moreover, a, b, c, p, q are constant parameters which take positive values. The divergence of the flow is

$$\frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -z(a + 2by), \quad (5)$$

which is negative for bounded flows, and divergence-free for equality.

The equilibrium points of the system (4) are obtained by solving the following three equations.

$$z = 0, \quad (6a)$$

$$(qx - ay - px^2 - by^2 - cxz)z = 0, \quad (6b)$$

$$x^2 + y^2 - |x| = 0. \quad (6c)$$

We note that $z = 0$ satisfies both equations (6a) and (6b). Thus, the new system (4) consists of a curve of equilibrium points in the (x, y) plane given by

the following set:

$$S = \{X = (x, y, z) : z = 0, x^2 + y^2 - |x| = 0\}. \quad (7)$$

The equilibrium curve S is symmetric with respect to both x axis and y axis in the (x, y) coordinate plane. Geometrically, S represents a pair of circles with the centres in the x -axis and passing through the origin as shown in Figure 1.

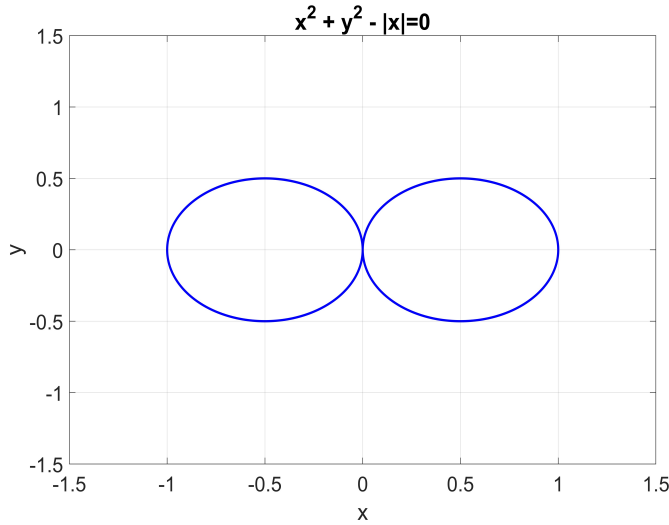


Figure 1: Two circles of equilibrium points for the new chaotic system (4)

We shall show that the new 3-D system dynamics (4) exhibits a chaotic attractor for the constant values

$$a = 4.1, \quad b = 5, \quad c = 1.3, \quad p = 0.9, \quad q = 0.3. \quad (8)$$

For the initial state $X(0) = (0.02, 0.01, 0.02)$ and the constant values as in (8), the Lyapunov characteristic exponents (LCE) of the new 3-D dynamics (4) were determined for $T = 1 \times 10^4$ seconds as follows:

$$l_1 = 0.0545, \quad l_2 = 0, \quad l_3 = -0.1784. \quad (9)$$

Since the MLE of the new chaotic system (4) ($l_{\max} = 0.0545$) is greater than the MLE of the SCS (1) ($l_{\max} = 0.0213$), we have shown that the new chaotic system (4) exhibits more complexity than the SCS (1) and both chaotic systems have the same curve of equilibrium points given by S in Eq. (3).

Figures 2 to 4 show the phase plots of the new chaotic system (4) with a curve of equilibrium points for the initial condition $X(0) = (0.02, 0.1, 0.02)$ and the parameter values as in (8).

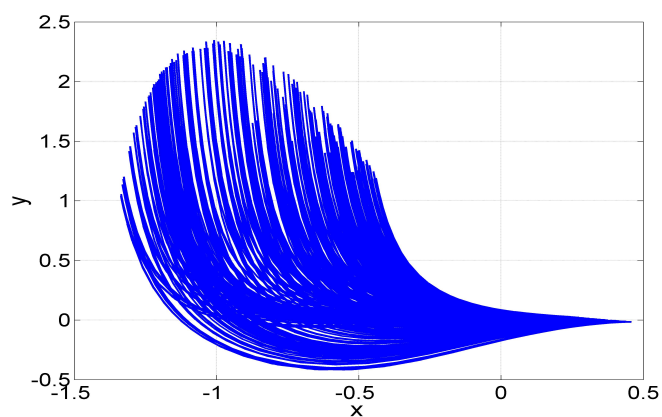


Figure 2: MATLAB (x, y) plot of the new 3-D dynamics (4)

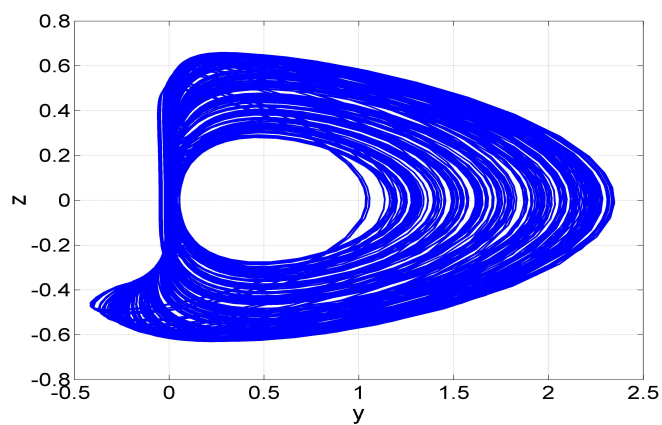


Figure 3: MATLAB (y, z) plot of the new 3-D dynamics (4)

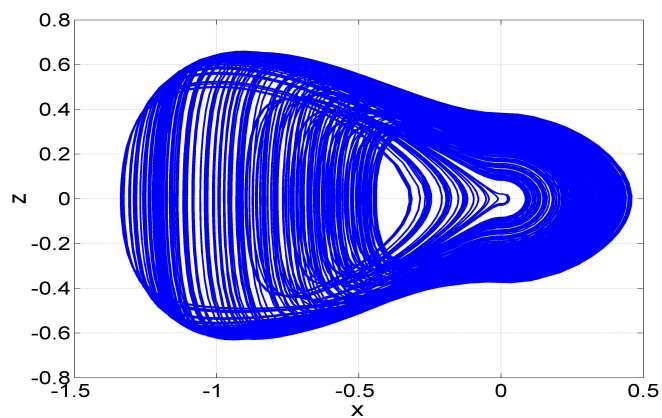


Figure 4: MATLAB (x, z) plot of the new 3-D dynamics (4)

3. Dynamic analysis of the new chaotic system with two circles of equilibrium points

From the results of Section 2, we find that the equilibrium state of the new 3-D dynamics (4) has $z = 0$. Computing the Jacobian matrix of the new 3-D dynamics (4) evaluated at this value gives

$$J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & M \\ 2x - s & 2y & 0 \end{bmatrix}, \quad (10)$$

where $s = \pm 1$, reflecting the modulus in (4) and

$$M = -(ay + by^2) - (px^2 - qx). \quad (11)$$

A simple calculation shows that

$$M = -p \left(x - \frac{q}{2p} \right)^2 - b \left(y + \frac{a}{2b} \right)^2 + p \left(\frac{q}{2p} \right)^2 + b \left(\frac{a}{2b} \right)^2. \quad (12)$$

The characteristic equation for the Jacobian matrix J is a cubic characteristic equation for λ , in which one of the eigenvalues is always $\lambda = 0$ (a degeneracy because of the curve of equilibria), together with a quadratic equation:

$$\lambda^2 + 2y\lambda - (2yM + 2x - s) = 0. \quad (13)$$

Here x and y are given by solutions to the curve of equilibrium states $(x - s/2)^2 + y^2 = 1/4$.

For example, there is a second zero eigenvalue when $2(yM + x) - s = 0$, with $y \neq 0$.

If $y = 0$, the constant term in the quadratic equation is then $-(2x - s)$.

From the curve of equilibrium states, we therefore have $x = 0$ or $x = \pm 1$.

When $y = 0$ and $x = \pm 1$, the constant term, $-(2x - s)$, gives a saddle for $(x, s) = (1, 1)$, and pure imaginary eigenvalues for $(x, s) = (-1, -1)$.

We focus on producing bifurcation transition diagrams in terms of the maximum values of x as the parameter a increases (or decreases), keeping the other parameters fixed. Firstly we demonstrate the differences in fixing the initial data, as opposed to varying it at each value of a . Figure 5 shows these plots for a for the Sambas chaotic system (SCS) (1) when $(b, c) = (4.5, 1)$. Figure 5a is produced, keeping the initial conditions the same at $(0.01, 0.02, 0.01)$ for each integration step. We used the NAG library routines, so that the details in the plots may differ from Figure 7a of the SCS (1). Figure 5b is produced by taking the final condition for one integration as the initial condition for the next integration. The chaotic dynamics has been lost for $a \approx 5.72$ as evolves to a steady state of $x \approx -0.10363$

at $a \approx 5.88$. SCS (1) remains chaotic for $a = 4$ in both cases. There is therefore a wide region of multistability. We have also extended the range of integration down to $a = 2.4$ in both cases.

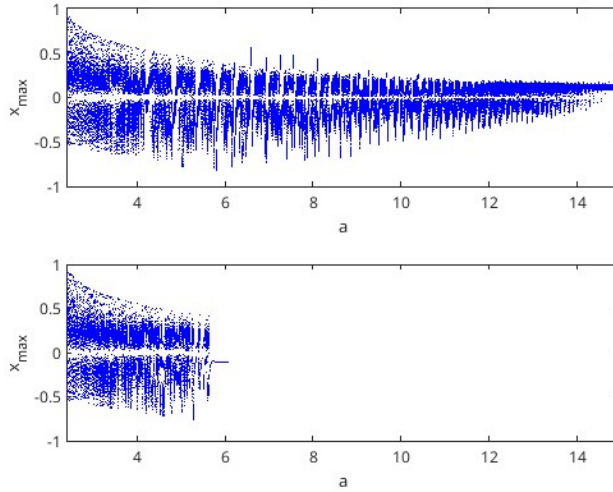


Figure 5: The bifurcation transition plot of $x_{\max}$ as a varies for $4 < a < 15$ for the SCS (1) with $(b, c) = (4.5, 1)$. In (a) the same initial data of $(0.01, 0.02, 0.01)$ is used for each integration step; in (b) the final data for one integration is used as the next initial data for the next increment of a

Figure 6 shows the corresponding plots for the new system (4) as a varies, but with the same initial conditions and parameter values as for the SCS (1) i.e. $(b, c) = (4.5, 1)$, but with the two additional terms and $(p, q) = (0.9, 0.3)$. These plots show the effect of the two additional terms in the $\dot{y}$ equation. One difference is that for both fixed and variable initial data, period-halving bifurcations replace the chaos found in $4 > a > 2.4$.

Figure 7 shows these two plots for the new system (4) parameter values of $(b, c, p, q) = (5, 1.3, 0.9, 0.3)$ and initial data of $(0.02, 0.01, 0.02)$. For variable initial data, there is again a much shorter region of chaotic dynamics as a increases; when a decreases, the chaos is again lost in a period-halving transition. There is the onset of a period-four cycle at $a \approx 3.65$, to period-two cycle for $a \approx 3.55$, and to a periodic cycle at $a \approx 3.02$. For fixed initial conditions, the loss of stability to a period-two orbit occurs at $a \approx 3.56$ and to a period-one orbit at $a \approx 2.98$.

Figure 8 shows the effects of integrating with fixed initial conditions, but changing the sign of $x(0)$. We used the new system (4) parameter values with two sets of initial conditions: $(0.3, 0.01, 0.02)$ (blue); $(-0.3, 0.01, 0.02)$ (red). This is the equivalent plot to SCS (1)'s Figure 4.

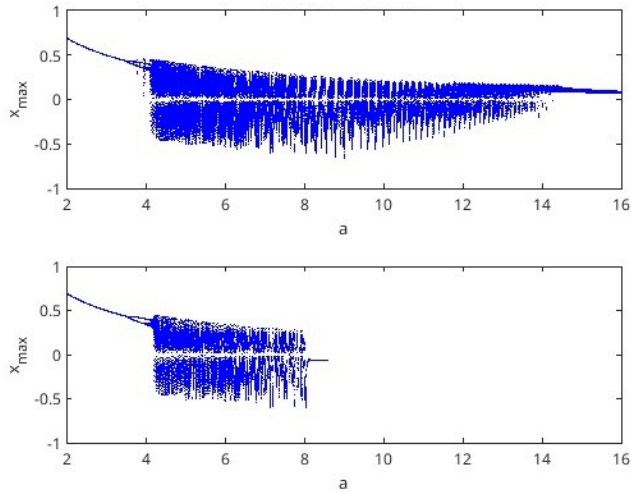


Figure 6: The bifurcation transition plot of $x_{\max}$ as a varies for $2 < a < 16$ for the SCS (1) and parameter values $(b, c) = (4.5, 1.0)$, but with the additional two terms of $(p, q) = (0.9, 0.3)$ in the new system (4). In (a) the same initial data of $(0.01, 0.02, 0.01)$ is used for each integration step; in (b) the final data for one integration is used as the next initial data for the next increment of a

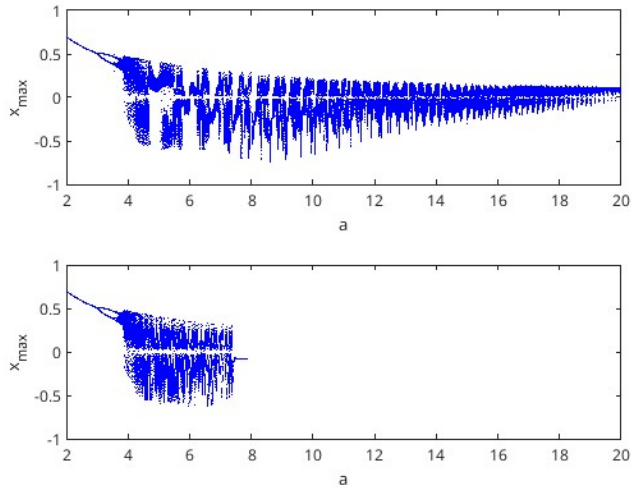


Figure 7: The bifurcation transition plot of $x_{\max}$ as a varies in $2 < a < 20$ for the new system (4). In the upper panel initial conditions are fixed at $(0.02, 0.01, 0.02)$, while in the lower panel initial conditions vary at each value for a . We again took $(b, c, p, q) = (5, 1.3, 0.9, 0.3)$

To produce these two bifurcation transition diagrams, we integrated the non-linear system for 2000s at each parameter value, omitting transients from each integration.

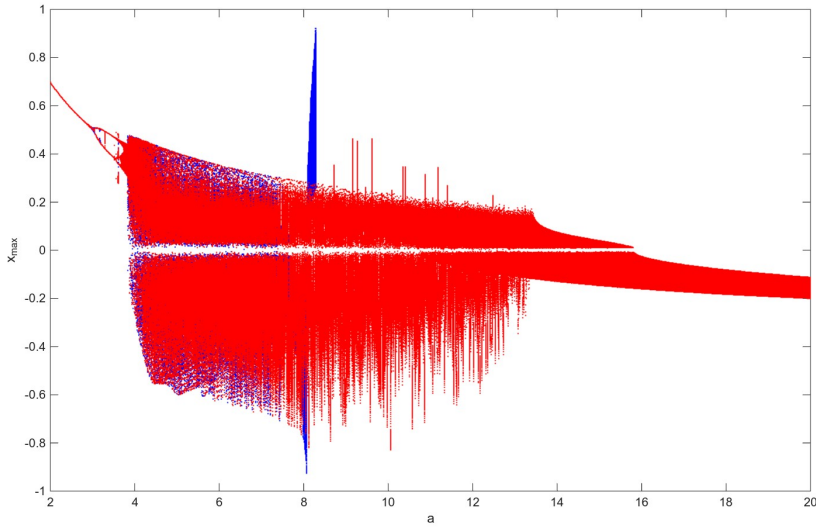


Figure 8: The bifurcation transition plot of $x_{\max}$ as a varies in $2 < a < 20$ for the new system (4) with $(b, c, p, q) = (5, 1.3, 0.9, 0.3)$. In blue is the plot with initial data $(0.3, 0.01, 0.02)$, while in red, the initial data is $(-0.3, 0.01, 0.02)$. Each iteration step is for $2000s$. The integration for initial data $(0.3, 0.01, 0.02)$ terminates after $a = 8.29$

For the initial conditions of $(0.3, 0.01, 0.02)$ (blue), we found that for $a = 8.29$, $x \rightarrow 2.9021844$, a constant, while $(y, z) \rightarrow (-3.5865274e + 07, 3.9898424e + 03)$; x solutions remained constant, and y solutions remained unbounded for $a > 8.29$. For $(-0.3, 0.01, 0.02)$ (red), all solutions remained bounded throughout.

We clearly see multistability, this time when initial conditions are fixed for each increment of a , but with a different value for $x(0)$.

In all three cases, there are wide regions of multistable states.

Figure 9 shows extended time integrations for both the SCS (1) and new system (4) for $a = 10$, with the remaining parameter values and initial data at either their SCS (1) or new system (4) values, apart from the choice of $x(0) = \pm 0.3$. Red shows the cases $x(0) = -0.3$ and blue shows the cases $x(0) = 0.3$. Figures 8 and 9 should be compared to those of SCS (1).

One notable feature is that in both SCS (1) and new system (4), the $x(0) = -0.3$ case remains bounded for $0 \leq t \leq 1500$, while in the $x(0) = 0.3$ case, x approaches a constant value of $x = 3.2127936e + 00$ and $(y, z) \rightarrow (-1.9907184e + 07, 2.9742798e + 03)$ at $t \approx 1.4367236e + 03$ for SCS (1). For the new system (4), $x \rightarrow 2.4496555$ at $t \approx 1.2026022e + 03$, where $(y, z) \rightarrow (-1.1753431e + 07, 2.1685306e + 03)$.

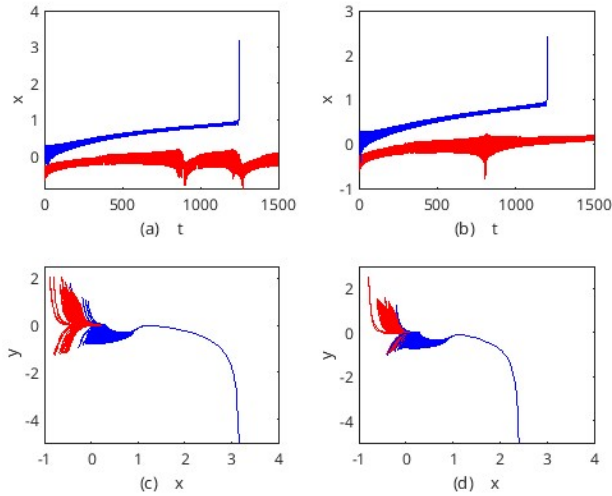


Figure 9: Comparison of the time series of $x(t)$, and the (x, y) phase portraits for the SCS (1) and new system (4) when $a = 10$ with their parameter values. For (a) and (c) the SCS (1) initial data is $(\pm 0.3, 0.02, 0.01)$ (red for -0.3 and blue for 0.3). For (b) and (d) the new system (4) initial data is $(\pm 0.3, 0.01, 0.02)$ with the same colours

4. Bifurcation analysis of the new chaotic system with two circles of equilibrium points

We numerically investigate the dynamical behaviors of the new chaotic system (4) in this section, utilizing the Lyapunov exponents (LE) spectrum and bifurcation diagrams.

Our numerical simulations demonstrate that the system exhibits either chaotic or periodic behaviors within certain ranges of its parameters. Specifically, these behaviors are observed when the parameters fall within the following intervals: $a \in [2, 6]$, $b \in [4, 5.5]$, $c \in [1, 2]$, $p \in [0, 4]$ and $q \in [0, 2]$.

4.1. Varying the parameter a

Figure 10 shows the Lyapunov exponents (LE) spectrum and the bifurcation diagram of the new 3-D chaotic system (4) with respect to parameter a . We fix the values the parameters: b, c, p, q as: $(b, c, p, q) = (5, 1.3, 0.9, 0.3)$. We can identify the dynamic behavior of the system (4) when the parameter a varies in the range $[2, 6]$ as follows: we define the two intervals A and B as $A = [2, 3.66]$ and $B = (3.66, 6]$.

When $a \in A$, we can see from Figure 10 that the system (4) exhibits periodic behavior where the maximal LE is zero, i.e. $l_1 = 0$, $l_2 < 0$ and $l_3 < 0$.

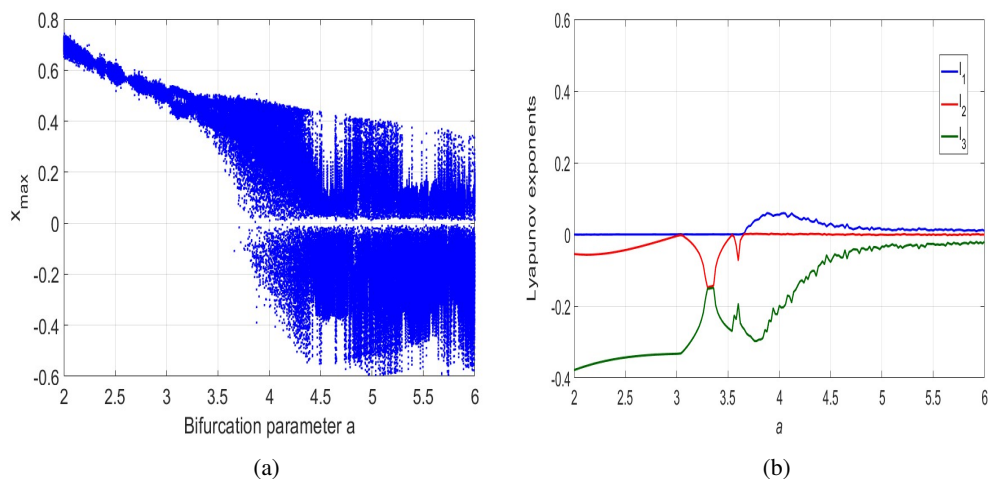


Figure 10: (a) bifurcation diagram and (b) LE spectrum of (4) for $a \in [2, 6]$

When $a \in B$, we can see from Figure 10 that the system (4) exhibits chaotic behavior where the maximal LE is positive, i.e. $l_1 > 0$, $l_2 = 0$ and $l_3 < 0$.

Dynamic behaviors and Lyapunov exponents of the new 3-D system(4) for different values of the parameter a are summarized in Table 1.

Table 1: Summary Table for the dynamic behavior of the new system (4) for various values of the parameter a , where $(b, c, p, q) = (5, 1.3, 0.9, 0.3)$

Dynamics	Parameter a	LEs		
		l_1	l_2	l_3
Periodic	2	0	-0.05447	-0.3781
	2.5	0	-0.04338	-0.3430
	3	0	-0.004325	-0.3326
	3.4	0	-0.06885	-0.2177
	3.6	0	-0.0730	-0.1924
Chaotic	3.86	0.05327	0	-0.2660
	4	0.05495	0	-0.1987
	4.36	0.04268	0	-0.116
	4.5	0.03064	0	-0.07701
	5	0.01326	0	-0.03322
	5.74	0.01320	0	-0.02861

4.2. Varying the parameter b

Figure 11 shows the Lyapunov exponents (LE) spectrum and the bifurcation diagram of the new (4) with respect to parameter b . We fix the values of the other parameters as $(a, c, p, q) = (4.1, 1.3, 0.9, 0.3)$.

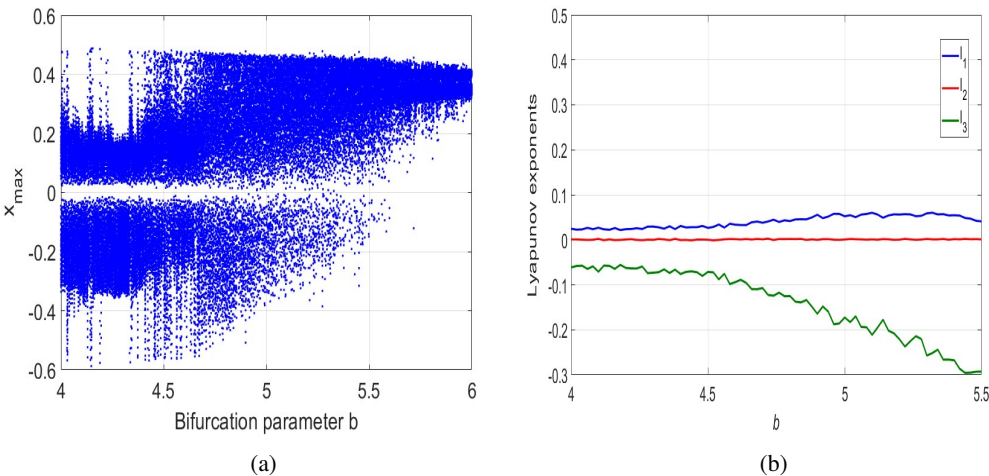


Figure 11: (a) bifurcation diagram and (b) LE spectrum of (4) for $b \in [4, 5.5]$

When we vary the parameter b in the range $[4, 5.5]$; the (4) displays chaotic behavior where $l_1 > 0$, $l_2 = 0$ and $l_3 < 0$.

Dynamic behaviors and Lyapunov exponents of the new 3-D system(4) for different values of the parameter a are summarized in Table 2.

Table 2: Summary Table for the dynamic behavior of the new system (4) for various values of the parameter b , where $(a, c, p, q) = (4.1, 1.3, 0.9, 0.3)$

Dynamics	Parameter b	LEs		
		l_1	l_2	l_3
Chaotic	4	0.02472	0	-0.06143
	4.2	0.02357	0	-0.06248
	4.6	0.03358	0	-0.09416
	4.9	0.05577	0	-0.1645
	5	0.05234	0	-0.1727
	5.2	0.05617	0	-0.2212
	5.5	0.04097	0	-0.2930

4.3. Varying the parameter c

Figure 12 shows the Lyapunov exponents (LE) spectrum and the bifurcation diagram of the new 3-D chaotic system (4) with respect to parameter c . We fix the values the parameters: a, b, p, q as: $(a, b, p, q) = (4.1, 5, 0.9, 0.3)$. We can identify the dynamic behavior of the system (4) when the parameter c varies in the range $[1, 2.20]$ as follows: we define the two intervals A and B as $A = [1, 1.10]$ and $B = (1.10, 2.20]$.

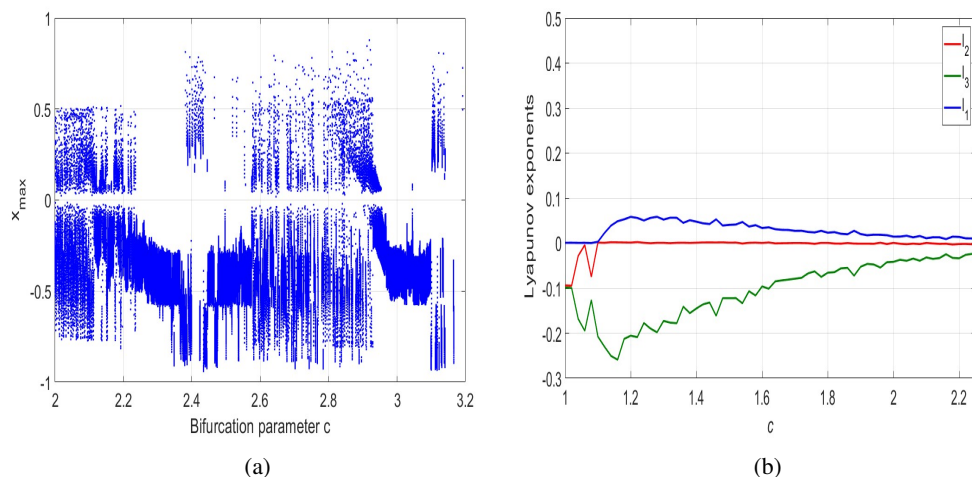


Figure 12: (a) bifurcation diagram and (b) LE spectrum of (4) for $c \in [1, 2.20]$

Table 3: Summary Table for the dynamic behavior of the new system (4) for various values of the parameter c , where $(a, b, p, q) = (4.1, 5, 0.9, 0.3)$

Dynamics	Parameter c	LEs		
		l_1	l_2	l_3
Periodic	1	0	-0.09393	-0.09855
	1.02	0	-0.09452	-0.09962
	1.04	0	-0.02825	-0.1682
	1.08	0	-0.07473	-0.01263
	1.10	0	0	-0.2075
Chaotic	1.20	0.05840	0	-0.2055
	1.34	0.05641	0	-0.1410
	1.46	0.05293	0	-0.1614
	1.60	0.03364	0	-0.09601
	1.80	0.02320	0	-0.06558
	2	0.01450	0	-0.0415

When $c \in A$, we can see from Figure 12 that the system (4) exhibits periodic behavior where the maximal LE is zero, i.e. $l_1 = 0$, $l_2 < 0$ and $l_3 < 0$.

When $c \in B$, we can see from Figure 12 that the system (4) exhibits chaotic behavior where the maximal LE is positive, i.e. $l_1 > 0$, $l_2 = 0$ and $l_3 < 0$.

Dynamic behaviors and Lyapunov exponents of the new 3-D system(4) for different values of the parameter c are summarized in Table 3.

4.4. Varying the parameter p

Figure 13 shows the Lyapunov exponents (LE) spectrum and the bifurcation diagram of the new 3-D chaotic system (4) with respect to parameter p . We fix the values the parameters: a, b, c, q as: $(a, b, c, q) = (4.1, 5, 1.3, 0.3)$. We can identify the dynamic behavior of the system (4) when the parameter p varies in the range $[0, 4]$ as follows: we define the two intervals A and B as $A = [1.18, 1.32) \cup (1.32, 4]$ and $B = [0, 1.18)$.

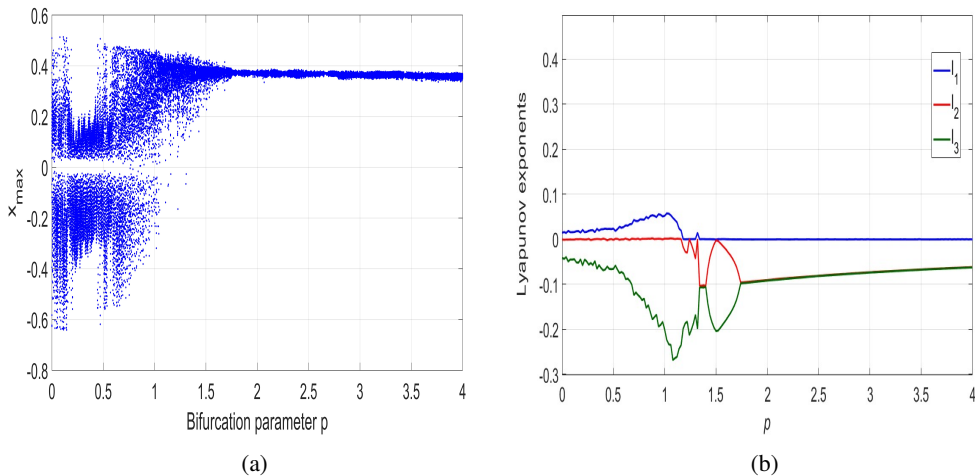


Figure 13: (a) bifurcation diagram and (b) LE spectrum of (4) for $p \in [0, 4]$

When $p \in A$, we can see from Figure 13 that the system (4) exhibits periodic behavior where the maximal LE is zero, i.e. $l_1 = 0$, $l_2 < 0$ and $l_3 < 0$.

When $p \in B$, we can see from Figure 13 that the system (4) exhibits chaotic behavior where the maximal LE is positive, i.e. $l_1 > 0$, $l_2 = 0$ and $l_3 < 0$.

Dynamic behaviors and Lyapunov exponents of the new 3-D system(4) for different values of the parameter p are summarized in Table 4.

Table 4: Summary Table for the dynamic behavior of the new system (4) for various values of the parameter p , where $(a, b, c, q) = (4.1, 5, 1.3, 0.3)$

Dynamics	Parameter p	LEs		
		l_1	l_2	l_3
Periodic	1.50	0	-0.002421	-0.2035
	2	0	-0.09029	-0.09136
	2	0	-0.0806	-0.08141
	3	0	-0.0730	-0.07367
	3.50	0	-0.06674	-0.06737
	4	0	-0.06133	-0.06228
Chaotic	0	0.01494	0	-0.0412
	0.20	0.02108	0	-0.05784
	0.50	0.02391	0	-0.06545
	0.70	0.04319	0	-0.1236
	0.90	0.05234	0	-0.1727
	1.04	0.05716	0	-0.2330
	1.16	0.01959	0	-0.2350
	1.32	0.01497	0	-0.1988

4.5. Varying the parameter q

Figure 14 shows the Lyapunov exponents (LE) spectrum and the bifurcation diagram of the new 3-D chaotic system (4) with respect to parameter q . We fix the values the parameters: a, b, c, p as: $(a, b, c, p) = (4.1, 5, 1.3, 0.9)$. We can

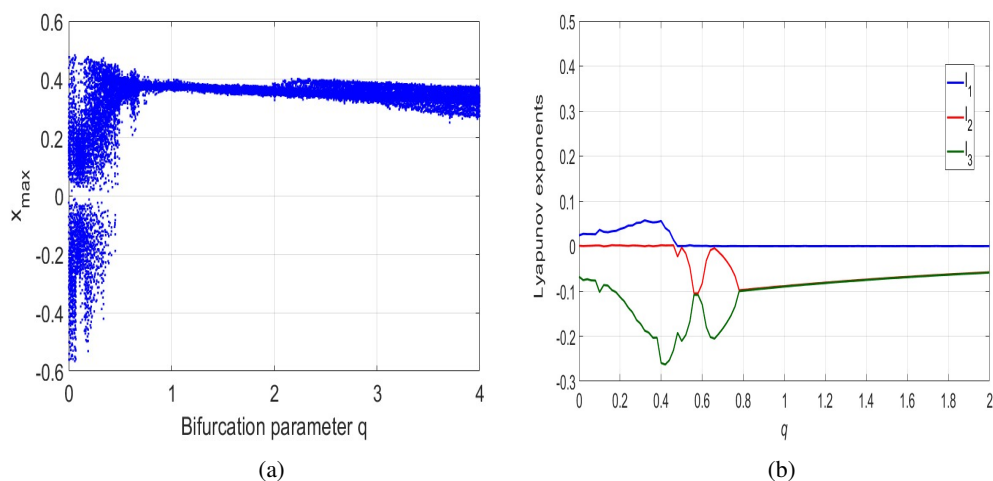


Figure 14: (a) bifurcation diagram and (b) LE spectrum of (4) for $q \in [0, 2]$

identify the dynamic behavior of the system (4) when the parameter q varies in the range $[0, 2]$ as follows: we define the two intervals A and B as $A = [0, 0.48)$ and $B = [0.48, 2]$.

Table 5: Summary table for the dynamic behavior of the new system (4) for various values of the parameter q , where $(a, b, c, p) = (4.1, 5, 1.3, 0.9)$

Dynamics	Parameter q	LEs		
		l_1	l_2	l_3
Chaotic	0	0.02391	0	-0.06776
	0.10	0.03626	0	-0.1022
	0.24	0.0456 2	0	-0.1366
	0.34	0.05465	0	-0.1921
	0.40	0.05596	0	-0.2595
	0.44	0.03325	0	-0.2534
Periodic	0.60	0	-0.08294	-0.1294
	1	0	-0.08884	-0.0897
	1.50	0	-0.07097	-0.07174
	2	0	-0.05785	-0.0587

When $q \in A$, we can see from Figure 14 that the system (4) exhibits chaotic behavior where the maximal LE is positive, i.e. $l_1 > 0$, $l_2 = 0$ and $l_3 < 0$.

When $q \in B$, we can see from Figure 14 that the system (4) exhibits periodic behavior where the maximal LE is zero, i.e. $l_1 = 0$, $l_2 < 0$ and $l_3 < 0$.

Dynamic behaviors and Lyapunov exponents of the new 3-D system(4) for different values of the parameter p are summarized in Table 5.

5. Multistability of the new 3-D chaotic system with a curve equilibrium

A notable feature of chaotic dynamical systems is their ability to exhibit multistability, wherein multiple attractors coexist under the same set of system parameters. This phenomenon holds significant relevance for both theoretical investigations and practical applications of chaotic dynamical systems. The co-existence of several attractors enhances our understanding of the complex and diverse dynamical behaviors inherent to such systems.

Let X_1 and X_2 be two different initial conditions for the new 3-D chaotic system (4). Figure 15 demonstrates the coexistence of two chaotic attractors with different initial values where $a = 4.1$, $b = 5$, $c = 1.3$, $p = 0.9$, $q = 0.3$ and initial

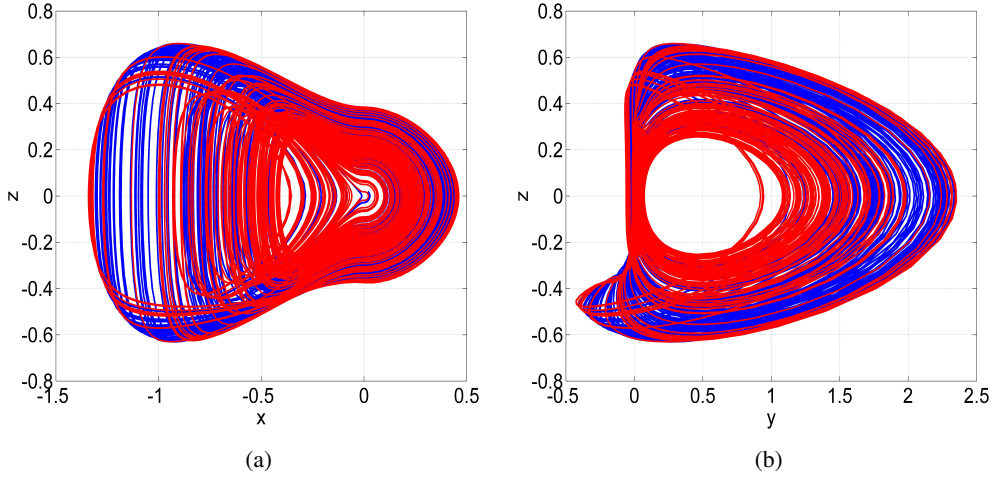


Figure 15: Coexistence of two chaotic attractors with different initial values where $a = 4.1$, $b = 5$, $c = 1.3$, $p = 0.9$ and $q = 0.3$: The initial states are chosen as $X_1 = (0.02, 0.01, 0.02)$ (blue orbit) and $X_2 = (-0.02, 0.02, -0.01)$ (red orbit)

states $X_1 = (0.02, 0.01, 0.02)$ (blue orbit) and $X_2 = (-0.02, 0.02, -0.01)$ (red orbit).

6. Circuit simulation of the new 3-D chaotic system

In this section, the new 3D chaotic system (4) is realized by the NI Multisim 14.2 platform. The electronic circuit design of the 3D chaotic system (4) is shown in Figure 16 in which TLO84ACD is selected as OPAMP and the multipliers are of type AD633.

Using Kirchhoff's electrical circuit laws, we derive the circuit model for the rescaled 3D chaotic (4) as follows:

$$\begin{cases} C_1 \dot{x} = z, \\ C_2 \dot{y} = \left(\frac{1}{R_2}x - \frac{1}{R_3}y - \frac{1}{R_{74}}x^2 - \frac{1}{R_5}y^2 - \frac{1}{R_6}xz \right) z, \\ C_3 \dot{z} = \frac{1}{R_7}x^2 + \frac{1}{R_8}y^2 - \frac{1}{R_9}|x|. \end{cases} \quad (14)$$

Here, x , y , z are the voltages across the capacitors, C_1 , C_2 , C_3 respectively. The values of components in the circuit are selected as:

$$C_1 = C_2 = C_3 = 1 \text{ nF}, \quad (15)$$

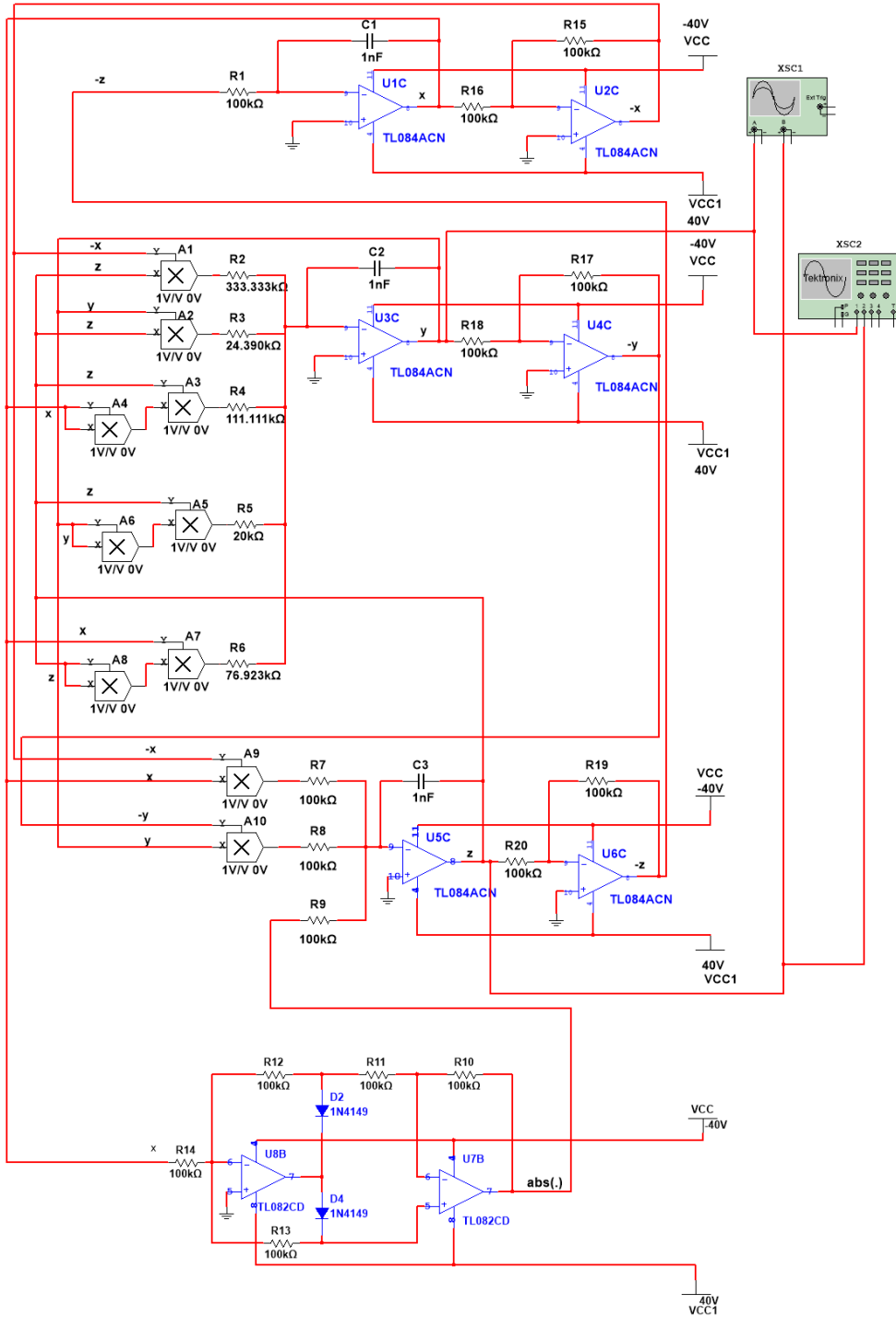


Figure 16: Circuit design of the new 3-D chaotic system (14)

$$R_1 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = 10 \text{ k}\Omega, \quad (16)$$

$$R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = 10 \text{ k}\Omega, \quad (17)$$

$$R_2 = 333.33 \text{ k}\Omega, \quad R_3 = 24.390 \text{ k}\Omega, \quad R_4 = 111.111 \text{ k}\Omega, \quad (18)$$

$$R_5 = 20 \text{ k}\Omega, \quad R_6 = 76.923 \text{ k}\Omega. \quad (19)$$

Multisim outputs of the circuit via XSC1 oscilloscope are presented in Figures 17–19.

Multisim outputs of the circuit via Tektronix oscilloscope are presented in Figures 20–22.

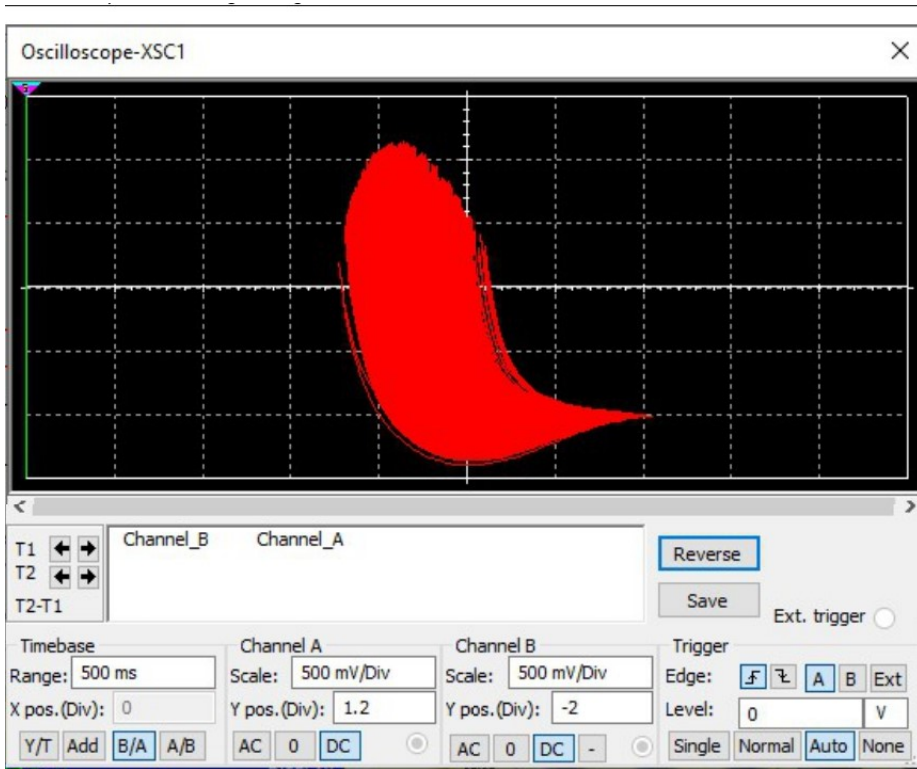


Figure 17: Chaotic attractor in (x, y) plane of the new 3-D chaotic system (14) via XSC1 oscilloscope

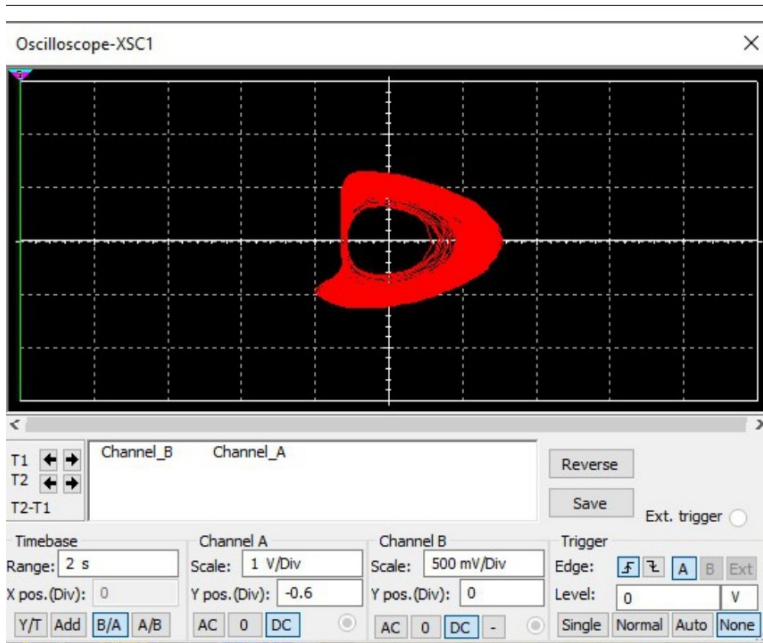


Figure 18: Chaotic attractor in (y, z) plane of the new 3-D chaotic system (14) via XSC1 oscilloscope

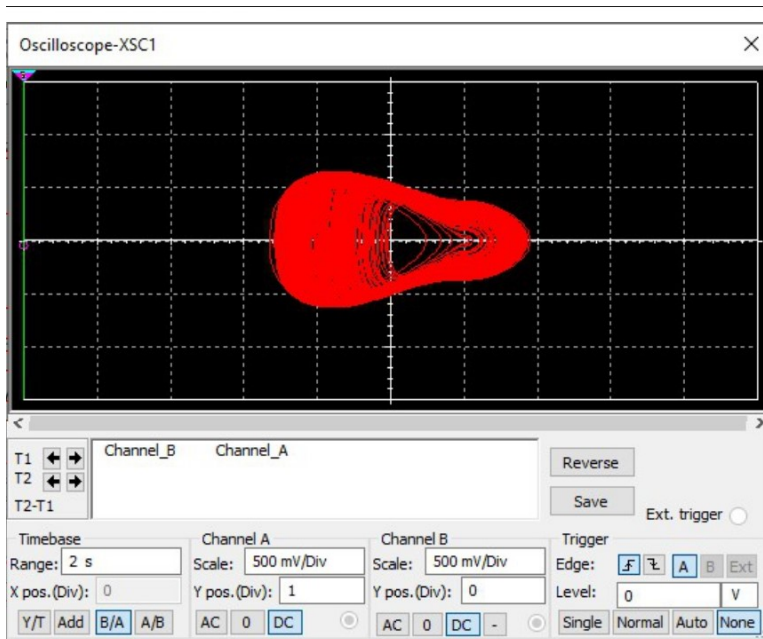


Figure 19: Chaotic attractor in (x, z) plane of the new 3-D chaotic system (14) via XSC1 oscilloscope

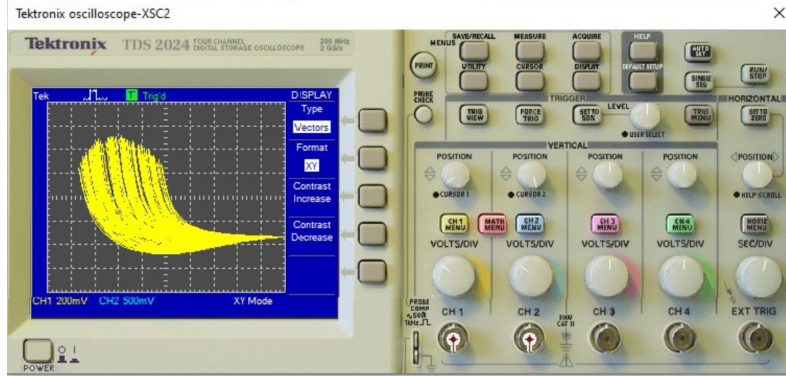


Figure 20: Chaotic attractor in (x, y) plane of the new 5-D chaotic system (14) via Tektronix oscilloscope

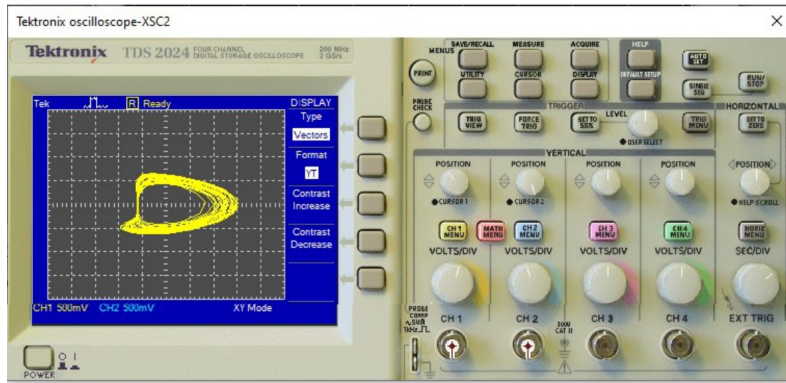


Figure 21: Chaotic attractor in (y, z) plane of the new 5-D chaotic system (14) via Tektronix oscilloscope

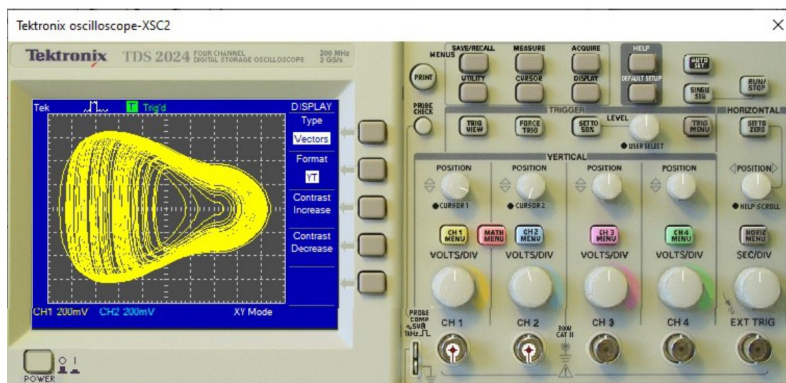


Figure 22: Chaotic attractor in (x, z) plane of the new 5-D chaotic system (14) via Tektronix oscilloscope

7. Complete synchronization of the new 3-D chaotic systems with curve equilibrium points

As a control application, we we deploy the integral sliding mode control (ISMC) for the complete synchronization of a pair of new 3-D chaotic systems with curve equilibrium points. ISMC is a well-known control technique which has many important applications in science and engineering [30].

As the transmitter system, we take the new 3-D chaotic dynamics with a curve equilibrium given as follows:

$$\begin{cases} \dot{x}_1 = z_1, \\ \dot{y}_1 = (qx_1 - ay_1 - px_1^2 - by_1^2 - cx_1z_1)z_1, \\ \dot{z}_1 = x_1^2 + y_1^2 - |x_1|. \end{cases} \quad (20)$$

We denote the state of the transmitter system (20) as $X_1 = (x_1, y_1, z_1)$.

As the receiver system, we take the new 3-D chaotic dynamics equipped with integral sliding mode controls (ISMC), which is given as follows:

$$\begin{cases} \dot{x}_2 = z_2 + u_x, \\ \dot{y}_2 = (qx_2 - ay_2 - px_2^2 - by_2^2 - cx_2z_2)z_2 + u_y, \\ \dot{z}_2 = x_2^2 + y_2^2 - |x_2| + u_z. \end{cases} \quad (21)$$

We denote the state of the receiver system (21) as $X_2 = (x_2, y_2, z_2)$. Moreover, we denote $u = (u_x, u_y, u_z)$. Thus, u is the integral sliding mode control (ISMC) which is to be designed so as to completely synchronize the respective states of the transmitter system (20) and the receiver system (21).

We define the synchronization error as $e = X_2 - X_1$. In other words, we define

$$\begin{cases} e_x = x_2 - x_1, \\ e_y = y_2 - y_1, \\ e_z = z_2 - z_1. \end{cases} \quad (22)$$

We derive the error dynamics as given below:

$$\begin{cases} \dot{e}_x = e_z + u_x, \\ \dot{e}_y = q(x_2z_2 - x_1z_1) - a(y_2z_2 - y_1z_1) - p(x_2^2z_2 - x_1^2z_1) \\ \quad - b(y_2^2z_2 - y_1^2z_1) - c(x_2z_2^2 - x_1z_1^2) + u_y, \\ \dot{e}_z = x_2^2 - x_1^2 + y_2^2 - y_1^2 - |x_2| + |x_1| + u_z. \end{cases} \quad (23)$$

In the IMSC design, an integral sliding surface is assigned with every synchronization error variable as follows:

$$\begin{cases} S_x = e_x + m_x \int_0^t e_x(\xi) d\xi, \\ S_y = e_y + m_y \int_0^t e_y(\xi) d\xi, \\ S_z = e_z + m_z \int_0^t e_z(\xi) d\xi. \end{cases} \quad (24)$$

A simple calculation results in the following dynamics:

$$\begin{cases} \dot{S}_x = \dot{e}_x + m_x e_x, \\ \dot{S}_y = \dot{e}_y + m_y e_y, \\ \dot{S}_z = \dot{e}_z + m_z e_z. \end{cases} \quad (25)$$

We consider the integral sliding mode controls defined as follows:

$$\begin{cases} u_x = -e_z - m_x e_x - \mu_x \operatorname{sgn}(S_x) - K_x S_x, \\ u_y = -q(x_2 z_2 - x_1 z_1) + a(y_2 z_2 - y_1 z_1) + p(x_2^2 z_2 - x_1^2 z_1) \\ \quad + b(y_2^2 z_2 - y_1^2 z_1) + c(x_2 z_2^2 - x_1 z_1^2), \\ \quad - m_y e_y - \mu_y \operatorname{sgn}(S_y) - K_y S_y, \\ u_z = -x_2^2 + x_1^2 - y_2^2 + y_1^2 + |x_2| - |x_1| \\ \quad - m_z e_z - \mu_z \operatorname{sgn}(S_z) - K_z S_z. \end{cases} \quad (26)$$

In Eq. (26), $m_x, m_y, m_z, \mu_x, \mu_y, \mu_z, K_x, K_y$ and K_z are positive constants.

From (23) and (26), we simplify the closed-loop error dynamics as follows:

$$\begin{cases} \dot{e}_x = -m_x e_x - \mu_x \operatorname{sgn}(S_x) - K_x S_x, \\ \dot{e}_y = -m_y e_y - \mu_y \operatorname{sgn}(S_y) - K_y S_y, \\ \dot{e}_z = -m_z e_z - \mu_z \operatorname{sgn}(S_z) - K_z S_z. \end{cases} \quad (27)$$

Next, we state and prove the main ISMC result for the complete synchronization of the transmitter system (20) and the receiver system (21).

Theorem 1. *The integral sliding mode control (ISMC) law defined by Eq. (26) achieves global chaos synchronization between the transmitter system (20) and the receiver system (21) for all values of the initial states $X_1(0), X_2(0) \in \mathbb{R}^3$, where $m_x, m_y, m_z, \mu_x, \mu_y, \mu_z, K_x, K_y$ and K_z are selected as positive constants.*

Proof. We consider the candidate Lyapunov function defined by

$$V(S_x, S_y, S_z) = \frac{1}{2} (S_x^2 + S_y^2 + S_z^2). \quad (28)$$

It is very easy to check that V is a quadratic and strictly positive definite function on $\mathbb{R}^3$.

A direct calculation shows that

$$\begin{aligned} \dot{V} = & S_x [-m_x \operatorname{sgn}(S_x) - K_x S_x] + S_y [-m_y \operatorname{sgn}(S_y) - K_y S_y] \\ & + S_z [-m_z \operatorname{sgn}(S_z) - K_z S_z]. \end{aligned} \quad (29)$$

Simplifying (29), we get

$$\dot{V} = -m_x |S_x| - K_x S_x^2 - m_y |S_y| - K_y S_y^2 - m_z |S_z| - K_z S_z^2. \quad (30)$$

Thus, $\dot{V}$ is a strictly negative definite function defined on $\mathbb{R}^3$.

Using Lyapunov stability theory, we deduce that $S_x(t) \rightarrow 0, S_y(t) \rightarrow 0$ and $S_z(t) \rightarrow 0$ asymptotically as $t \rightarrow \infty$.

Hence, it follows that $e_x(t) \rightarrow 0, e_y(t) \rightarrow 0$ and $e_z(t) \rightarrow 0$ asymptotically as $t \rightarrow \infty$.

This completes the proof. $\square$

For MATLAB simulations, we take the parameter values as in the chaotic case, viz.

$$a = 4.1, \quad b = 5, \quad c = 1.3, \quad p = 0.9, \quad q = 0.3. \quad (31)$$

We take the values of ISMC parameters as follows:

$$m_x = m_y = m_z = 8, \quad \mu_x = \mu_y = \mu_z = 0.4, \quad K_x = K_y = K_z = 12. \quad (32)$$

For the transmitter system (20), we take

$$x_1(0) = 0.4, \quad y_1(0) = 1.1, \quad z_1(0) = 0.5. \quad (33)$$

For the receiver system (21), we take:

$$x_2(0) = 0.7, \quad y_2(0) = 0.3, \quad z_2(0) = 1.2. \quad (34)$$

Figure 23 shows the time-plot of the synchronization error between the transmitter system (20) and the receiver system (21).

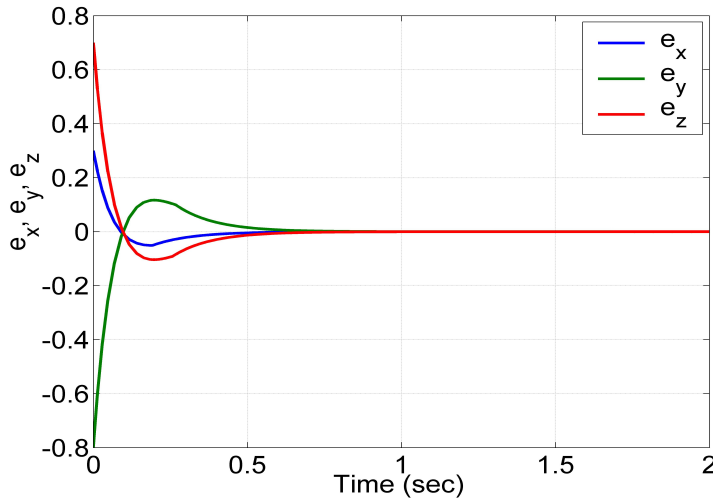


Figure 23: Time-plot of the synchronization error between the transmitter system (20) and the receiver system (21)

8. Conclusions

In this study, a new three-dimensional chaotic system was derived by adding two nonlinearities to Sambas chaotic system (2019) with a curve equilibrium. We established that the new chaotic system also features the same curve equilibrium as the Sambas chaotic system (2019), namely a pair of circles in the (x, y) -plane with centres in the x -axis and intersecting at the origin. It is established that the new chaotic system with a curve equilibrium has more complexity than the Sambas chaotic system (2019) with a curve equilibrium. Since the new chaotic system has a curve equilibrium, it exhibits hidden attractors. A detailed bifurcation analysis was prescribed for the new chaotic system with a curve equilibrium. Multistability within this chaotic system with curve equilibrium was evidenced by the coexistence of two distinct chaotic attractors, observed under identical parameters from the chaotic regime but with differing initial conditions. An analog electronic circuit for the proposed chaotic system with a curve equilibrium was designed and validated in Multisim, offering a practical pathway for hardware-based implementations of its chaotic behavior. Using integral sliding mode control (ISMC), complete synchronization of a pair of new chaotic systems taken as the master and slave systems was achieved. Simulations using MATLAB R2024b were shown to illustrate the main results of this paper. The proposed chaotic system with a curve equilibrium will be useful for applications in science and engineering.

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