

Lyapunov-based trajectory optimization for UAV-assisted wireless networks with obstacle avoidance

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Unmanned aerial vehicles (UAVs) are used to extend aircraft operations such as autonomous navigation, obstacle detection and collision avoidance. UAVs are increasingly deployed as airborne communication relays to extend coverage in terrain-challenged environments where ground base stations cannot serve remote users. This paper presents a trajectory planning framework with intelligent data transmission through UAVs acting as relays. Small base station (SBS) to UAV communication link assists remote users who are not able to get coverage from the SBS. A novel energy-efficient trajectory (EET) algorithm that jointly addresses obstacle avoidance and energy-aware data transmission for multi-UAV networks is proposed. The first part of the work focuses on trajectory planning to avoid obstacle collisions by using artificial repulsive force (APF) generated from the UAV, second part focuses on exploiting the Lyapunov approach to control the energy consumption of UAV during flight and communication of each user. Energy utilization is optimized through heuristic algorithms along with the data rate of the users. Numerical simulation results are shown to prove the performance of the proposed approach. The proposed EET-Lyapunov approach achieves a 31.3% reduction in total energy consumption.

Key words: energy-efficient trajectory, Lyapunov optimization, obstacle avoidance, unmanned air vehicles, UAV

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1. Introduction

Unmanned aerial vehicle (UAV) has emerged as an important communication technology for telecommunication society in civilian services and even in military applications. The potential behind UAV communication is recognized with regard to mobilized communication during blockages over terrain surfaces. UAV operation in terrain areas is complex when managing the obstructed line-of-sight (LoS) links, ground users in remote areas cannot connect to terrestrial base stations. An attractive solution for this unreliable infrastructure keeps pace with the UAV-enabled wireless communication on missions in disastrous terrain surfaces.

Wireless communication links are frequently disrupted during catastrophic natural disasters; there has been considerable research on UAV relays and connectivity management to provide reliability in connection with users. The design of trajectory optimization is essential to prove sufficient performance such as ubiquitous connectivity, data rate, and energy consumption during flight. This enables multi-UAV correlation to perform on-board applications, including minimizing the energy consumption, thereby improving the coverage of users in the three dimensional (3-D) environmental scenarios. The authors in [1] addressed energy consumption limitations by jointly optimizing trajectories and expanding the coverage range of the UAV. The challenges of the trajectory of dynamic UAV and the formulation of obstacle avoidance constraints are considered in linear or projectile motion. The position of the obstacle should be used to discriminate the set of collision avoidance constraints through discretization trajectories [2]. The explicit prediction of objects is evaluated through the simulation to verify the efficacy of the algorithm. The fixation of the problem relies on the environmental nature of the scenarios, number of UAVs and computational complexity [3].

The type of UAV is classified based on the flying mechanism; rotary-wing is equipped with multiple rotors and fixed-wing features with rotor blades that move downward movement or generate downward thrust to lift the UAV airborne. Rotary-wing UAV can operate stably in complex wireless environments, allowing for precise position control in multiple trajectories [4]. The capability of vertical take-off and landing is needed for optimal positioning of airplanes [5]. Fixed-wing UAVs generate aerodynamic lift through forward flight, requiring constant forward momentum to sustain airborne operations. The authors in [6] developed a method for reducing energy consumption through constrained combinatorial optimization leveraging deep reinforcement learning. The issue is NP-hard for efficiently implementing the trajectory problem using a pointer network. However, the constraints are learned from problem clusters in an unsupervised way utilizing actor-critic reinforcement learning. Numerous approaches for UAV-supported wireless networks have recently gained attention in trajectory planning and

coverage challenges. Numerous algorithms exist, including A*, particle swarm optimization, genetic algorithms, and various machine learning algorithms.

The constrained battery capacity is focused on coverage radius, energy harvesting and sustainable energy sources. Coverage radii are adjusted to save energy scheduling. The authors in [7] employed the Dinkelbach approach to formulate an iterative algorithm on block coordinate architecture for the inner loop of UAV trajectory [8]. Aerial assisted networks on UAVs involve mounting base stations where they can provide high-quality communication services to more UDs [9–11].

Operating UAVs within unknown, dynamic terrain requires real-time routing updates and adaptive control policies. Moreover, the current protocols are usually considered to act as wider network and holistic solution should be obtained using the artificial intelligence such as double deep Q-network (DDQN) to reduce outage time. The authors in [12–14] proposed heuristic algorithms to achieve an average interruption probability in order to meet the benchmark threshold. In [15], UAVs are enabled with edge computing services to enhance user and ground BS communication, the transmission energy is scheduled on the Q-learning supported network [16]. The authors in [17] presented a radio access network (RAN) scheme that incorporates multi-UAV and RAN architecture and epicycle optimization. The authors in [18] minimized the ground devices energy consumption through joint resource allocation and trajectory planning and the authors in [19] focused on minimizing tasks in a multi-UAV scenario, optimizing the time needed to finish all UAV missions while ensuring effective data collection. Related work on APF-based UAV trajectory planning [2, 3] provides real-time obstacle avoidance but does not jointly optimize long-term energy or queue stability, while prior Lyapunov drift-plus-penalty (DPP) (Lyapunov-DPP) frameworks for UAV networks [7, 8] optimize energy/rate trade-offs but typically assume unobstructed or pre-planned trajectories, treating obstacle avoidance as external to the optimization. The above-mentioned related works are not designed for a real-time network, but this work proposes Lyapunov-based trajectory optimization that provides a low-complexity, and theoretically grounded solution that simultaneously ensures obstacle avoidance, network stability, and long-term performance optimization—making it especially powerful for UAV-assisted wireless networks in dynamic or unknown environments. The nomenclature of this work is listed in Table 1 for clarification.

2. Problem formulation and notation

2.1. System model

A heterogeneous network system model assisted with UAV scenario is considered in which multi-rotor UAVs are distributed over a Euclidean space, the ground

nodes including small base stations and users are deployed. Let the number of UAVs be denoted as M , while $\mathcal{N}$ and $\mathcal{J}$ represent the small base station (SBS) and ground users, respectively. The set of UAVs, SBSs and users are defined as $\mathcal{M} = \{1, 2, \dots, M\}$, $\mathcal{N} = \{1, 2, \dots, N\}$, and $\mathcal{J} = \{1, 2, \dots, K\}$. The starting position and the target position of the UAV are denoted as q_m^0 and q_m^F , the UAV positions are updated with UAVs flying altitude H_m . The UAVs act as aerial relays to offer communication coverage for remote users. An artificial potential field (APF)-based trajectory optimization mechanism is employed, in which attractive forces guide the UAV toward the target while repulsive forces push the UAV away from dangerous obstacle regions. A Lyapunov drift-plus-penalty optimization framework is introduced to jointly minimize long-term energy consumption and maximize user data rates while guaranteeing queue stability, obstacle avoidance, velocity limits, acceleration constraints, and UAV battery limitations. The proposed system model is illustrated in Figure 1 to address the multi-UAV assisted communication.

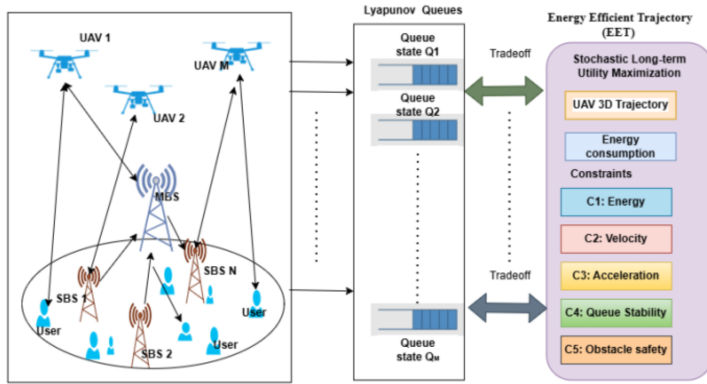


Figure 1: Proposed system model

2.2. Communication model

In a dynamic environmental scenario, the altitude of UAV needs to be changed for better connectivity due to blockages and obstacles; hence we need to consider

Table 1: Nomenclature for the system model

Symbol	Definition	Unit
M	Number of UAVs	—
N	Number of Small Base Stations (SBS)	—
J	Number of users	—

Table 1. Nomenclature for the system model [cont.]

Symbol	Definition	Unit
$q_m(t) = (x_m, y_m, z_m)$	3D position of UAV m at time slot t	m
q_m^0	Initial (start) position of UAV m	m
q_m^F	Final (target) position of UAV m	m
H_m	Flying altitude of UAV m	m
T	Total flight duration	s
τ	Duration of one time slot	s
$v_m(t)$	Velocity vector of UAV m at time t	m/s
$a_m(t)$	Acceleration vector of UAV m at time t	m/s ²
$h_{m,j}$	Channel gain between UAV m and user j	dB
$d_{m,j}$	Euclidean distance: UAV m to user j	m
$d1_{m,j}$	Distance: UAV m to obstacle	m
$d2_{m,j}$	Distance: UAV and user j	m
d_j	Data rate of user j	bits
$D_{m,j}$	Horizontal distance: UAV m to user j	m
f_c	Carrier frequency	GHz
BW	Channel bandwidth	MHz
$P_{m,j}$	Transmitted power from UAV m to user j	dBm
ρ^2	Noise power	dBm
$R_{m,j}$	Achievable data rate of UAV m	Mbps
$\tau_{m,j}$	Communication time: UAV m to user j	s
$Q_m(t)$	Virtual data queue of UAV m at time t	bits
$A_m(t)$	Data arrival rate at time t	bits/slot
E_m	Total energy of UAV m	J
$E_m^{\text{fly}}(t)$	Flight energy at time slot t	J
$E_m^{\text{comm}}(t)$	Communication energy at time slot t	J
$P_m^{\text{fly}}(t)$	Propulsion power	W
$\vec{F}_{\text{total}}$	Total artificial potential force	N
$\vec{F}_{\text{attr}}$	Attractive force toward target	N
$\vec{F}_{\text{rep},k}$	Repulsive force from obstacle k	N
r_1, r_2, r_3	Danger, detection, and safety radii	m
$\Delta_{m,j}$	Fresnel diffraction clearance height	m
V	Lyapunov trade-off weight (penalty factor)	—
$L(Q(t))$	Lyapunov function	—
$\Delta L(Q(t))$	Lyapunov drift	—

Table 1. Nomenclature for the system model [cont.]

Symbol	Definition	Unit
K_1, B	Upper bound constants in DPP	–
n	Number of rotors per UAV	–
m_{uav}	Mass of UAV	kg
ρ	Air density	kg/m ³
A_r	Rotor disk area	m ²
S_{frame}	Airframe frontal cross-sectional area	m ²
c_r	Blade section drag coefficient	–
c_t	Thrust coefficient	–
c_f	Incremental correction factor for induced power	–
d_f	Fuselage drag ratio	–
θ_c	Elevation (climb) angle	degree
g	Gravitational acceleration	m/s ²

the air-to-ground (A2G) channel characteristics, pathloss model to incorporate line-of-sight (LoS) and non-LoS (NLoS), and channel gain between UAV and user j is calculated using:

$$h_{m,j} = PL_{m,j} + KED_{m,j} + F_j, \quad (1)$$

$PL_{m,j}$ is the distance-dependent path, $KED_{m,j}$ and F_j are the losses due to knife-edge diffraction and forest coverage.

$$PL_{m,j} = 21.3 \log_{10}(d_{m,j}) + 21.3 \log_{10}(f_c) - 157.15, \quad (2)$$

where f_c denotes the carrier frequency and $d_{m,j}$ is the distance between UAV and user

$$d_{m,j}(t) = \|r_j[t] - r_j[t-1]\|_2, \quad (3)$$

where $\|r_j[t+1] - r_j[t]\|_2$ is the distance between the hover points.

The flight duration for each UAV is evaluated as:

$$T_m^F = \sum_{t=1}^{s_m} \frac{d_{m,j}}{v_{m,j}} \quad \forall m \in \mathcal{M}, \quad (4)$$

where s_m is the number of hover points, v is the velocity and $KED_{m,j}$ is the transition parameter from LoS to NLoS conditions when the terrain model consists

of building blockages and is calculated using

$$KED_{m,j} = -20 \log_{10} \frac{\sqrt{(1 - C(v_{m,j}) - S(v_{m,j}))^2 + (C(v_{m,j}) - S(v_{m,j}))^2}}{2}, \quad (5)$$

where $C(v_{m,j})$ and $S(v_{m,j})$ are the real and imaginary parts of the complex Fresnel integral, $F_c(v_{m,j}) = \int_0^v \exp\left(j \frac{\pi s^2}{2}\right) ds$ respectively and the KED loss for each obstacle is calculated for each obstacle.

$$v_{m,j} = \Delta_{m,j} \sqrt{\frac{2}{\lambda} \left(\frac{1}{d1_{m,j}} + \frac{1}{d2_{m,j}} \right)}, \quad (6)$$

$$\Delta_{m,j} = \frac{(ho_{m,j} - ht_m) D_{m,j} + (ht_m - hr_j) d_{m,j}}{\sqrt{D_{m,j}^2 + ((ht_m - hr_j))^2}}, \quad (7)$$

where $d1_{m,j}$ and $d2_{m,j}$ are the distances between UAV and obstacle, distance between users and UAV respectively, $D_{m,j}$ is the horizontal distance between UAV and user. $ho_{m,j}$, ht_m , hr_m are the heights in meters of the obstacle, UAV and users. $\Delta_{m,j}$ is the Fresnel diffraction clearance height to avoid collision.

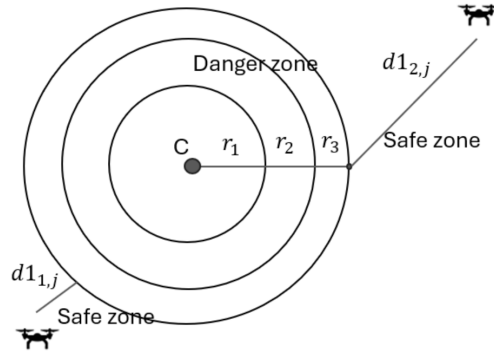


Figure 2: An illustration of trajectory over obstacle

2.3. Energy consumption model

The safe set is constructed using the strength of the repulsive force, the UAV experiences a repulsive force when it enters a dangerous zone, and the repulsive force increases rapidly to avoid collision and push away from obstacles. We can

define the artificial potential force of each rotor of a single UAV as

$$\vec{F}_{\text{total}} = \vec{F}_{\text{attr}} + \sum_{k=1}^K \vec{F}_{\text{rep},k}, \quad (8)$$

where $\vec{F}_{\text{total}}$ is the total artificial potential force, $\vec{F}_{\text{attr}}$ is the attractive force towards obstacle and $\vec{F}_{\text{rep},k}$ is the repulsive force between the target point and obstacle point. Attractive force toward target q_m^F with gain $\kappa_a > 0$: $F_{\text{attr},m}(t) = -\kappa_a \cdot (q_m(t) - q_m^F)$.

The repulsive force is applied when the UAV is near the danger zone.

The repulsive force is dynamically mapped using the APF formulation in equation (9). A continuous and piecewise-smooth formulation is adopted. We clarify that $r_1 < r_2 < r_3$ define three zones: danger ($\leq r_1$), detection ($r_1 < d \leq r_2$) and safe ($> r_2$), with r_3 as the outer boundary. The repulsive force maps to acceleration via Newton's second law $a_{\text{rep}}(t) = F_{\text{rep}}/m_{\text{uav}}$, which is then bounded by a_{max} (Constraint C3):

$$F_{\text{rep},k}(d) = \begin{cases} \eta \cdot (1/d1_{m,j} - 1/r_3) \cdot \left(\frac{1}{d1_{m,j}^2} \right) \cdot \nabla d1_{m,j}, & \text{if } d1_{m,j} \leq r_3, \\ 0, & \text{if } d1_{m,j} > r_3, \end{cases} \quad (9)$$

where $\eta > 0$ is the repulsive gain, $d1_{m,j}$ is the UAV-to-obstacle distance, and $\nabla d1_{m,j}$ is the unit vector pointing away from the obstacle. This formulation is continuous, monotonically increasing as $\rightarrow 0$, and vanishes smoothly at $d = r_3$. Force-to-control mapping: The net force $F_{\text{total}} = F_{\text{attr}} + \sum F_{\text{rep},k}$ yields the reference acceleration $a_{\text{ref}} = F_{\text{total}}/m_{\text{uav}}$. This is then saturated $a_m(t) = a_{\text{ref}} \cdot \min(1, a_{\text{max}}/|a_{\text{ref}}|)$ to satisfy constraint C3.

The thrust of UAV is defined as:

$$F(v, a) = m_{\text{uav}} \cdot \sqrt{(g^2 + a_{\text{vertical}}^2)} + \frac{1}{2} \cdot \rho \cdot v^2 \cdot C_D \cdot S_{\text{frame}}. \quad (10)$$

The total number of rotors is denoted as n , the mass is denoted as m_{uav} , the air density is denoted as ρ , the velocity is denoted as v , the airframe frontal cross-sectional area (m^2) is denoted as S_{frame} and C_D is the drag coefficient.

The flight energy consumption of the m -th UAV in time slot t is denoted as

$$E_m^{\text{fly}}(t) = P_m^{\text{fly}}(t)T_m^{\text{fly}}(t), \quad (11)$$

where $P_m^{\text{fly}}(t)$ represents the propulsion power during the flight [20], $T_m^{\text{fly}}(t)$ is the time duration of flight.

$$\begin{aligned}
 P_m^{\text{fly}}(v, F) = n & \left[\frac{c_r}{8} \left(\frac{F}{c_t \rho A_r} + 3v^2 \right) \sqrt{\frac{F \rho s_{\text{frame}}^2 A_r}{c_t}} \right. \\
 & + (1 + c_f) F \left(\sqrt{\frac{F^2}{4 \rho^2 A_r^2} + \frac{v^4}{4}} - \frac{v^2}{2} \right)^{0.5} \\
 & \left. + 0.5 d_r v^3 \rho s_{\text{frame}} A_r + \frac{m_{\text{uav}} \|g\| v}{n} \sin \theta_c \right]. \quad (12)
 \end{aligned}$$

The propulsion power is shown in the equation with local blade section drag coefficient c_r , and the thrust coefficient is c_t based on the disk area, A_r is the rotor disk area, s_{frame} is the frontal cross-sectional area of the UAV airframe, c_f is the incremental correction factor for induced power and d_r is the fuselage drag ratio and the elevation angle is $-\pi/2 \leq \theta_c \leq \pi/2$.

The energy consumption of UAV is related to flying-related energy and communication-related energy, which is cumulatively calculated as

$$E_m = \sum_{t=1}^T \left(E_m^{\text{fly}}(t) + E_m^{\text{comm}}(t) \right), \quad (13)$$

$$EE_m = \frac{E_m}{\sum_j D_j}, \quad (14)$$

where EE_m is the energy efficiency of the proposed system, E_m is the energy consumption of UAV m, and D_j is the total data demand [bits] of user served by UAV m.

Based on Shannon's theory, the achievable data rate of the UAV to user is calculated as

$$R_m = \tau_{m,j} BW \log_2 \left(1 + \frac{P_{m,j} |h_{m,j}|^2}{\sigma^2} \right), \quad (15)$$

$$\mathbb{E}(R_m) = \sum_{m=1}^M R_m, \quad (16)$$

where $\tau_{m,j}$ represents the communication time duration from the UAV to user, BW is the bandwidth, $P_{m,j}$ is the transmitted power and $h_{m,j}$ is channel gain between UAV and user, σ^2 is the noise power.

2.4. Problem objective

Remark 1: Although a 3D space UAV trajectory design increases the impact of large-scale fading in comparison to a 2D case, adjusting the UAV flying height can increase the LoS probability and make transmitting offloading tasks and decisions more efficient. Therefore, a 3D trajectory exploration scheme can obtain better performance than a 2D one. The 3D position vector of a UAV m at time slot is $q_m(t)$, and the fixed 3D position of obstacle k is p_k . Our goal is to maximize the rate and to minimize the energy consumption of a UAV through the Lyapunov optimization approach.

Decision variables: The joint optimization controls (i) UAV 3D trajectory $(q_m(t), v_m(t), a_m(t))$, (ii) Transmission scheduling $\tau_{m,j}$ and (iii) transmit power $P_{m,j}(t)$ across all time slots t .

The problem objective is to maximize the long-term time averaged data rate R_m summed over all UAVs subject to

C1 (Energy): Total Energy $E_m \leq E_{\max}$ – UAV battery constraint per mission,

C2 (Velocity): $v_{\min} \leq \|v(t)\| \leq v_{\max}$ – Physical UAV speed limit,

C3 (Acceleration): $a_m(t) \leq a_{\max}$ – rotor thrust limit,

C4 (Queue stability): $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_t \mathbb{E}[Q_m(t)] \leq \infty$ – bounded queue backlog,

C5 (Obstacle safety): $|q_m(t) - p_k| \geq r_3 \forall$ obstacle k – collision avoidance.

Online DPP decomposition: Since the problem is a non-convex MINLP with coupled trajectory and queue dynamics, we decompose it per slot using Lyapunov DPP. At each slot, the current queue state is updated; this decouples the long-horizon problem into a sequence of per-slot sub problems (S1 and S2):

(S1) Trajectory update via APF,

(S2) Power and scheduling via Lyapunov optimization.

$$P1: \max_{q(t) P(t) \tau_{m,j}} \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \sum_{m=1}^M \mathbb{E}[R_m(t) - VE_m(t)], \quad (17a)$$

$$\text{s.t. } E_m \leq E_{\max}, \quad (17b)$$

$$v_{\min} \leq \|v(t)\| \leq v_{\max}, \quad (17c)$$

$$\|a(t)\| \leq a_{\max}, \quad (17d)$$

$$\sum_{t=1}^T Q_m(t) \leq \overline{Q_m}, \quad (17e)$$

$$\|q_m(t) - p_k\| \geq r_3 \forall m, k, t. \quad (17f)$$

The problem objective is to maximize the data transmission rate between UAV and the users while satisfying the subjective constraints such as energy consumption, required trajectory parameters (velocity, acceleration) and Lyapunov queue status.

3. UAV control design

3.1. Lyapunov optimization model

This paper develops a low-complexity Lyapunov-based algorithm that guarantees queue stability and satisfies the long-term energy constraint C1 without requiring statistical knowledge of arrival rates or channel conditions. The design follows the drift-plus-penalty (DPP) framework to optimize energy constraint problem objectives.

Virtual Queue definitions: The joint Lyapunov queue function is:

$$\Theta(t) \triangleq \{Q_m(t), Z_m(t)\}_{m=1}^M. \quad (18)$$

Data queue $Q_m(t)$ tracks backlogged data per UAV m :

$$Q_m(t+1) = \max \{Q_m(t) - R_m(t)\tau, 0\} + A_m(t) \quad \forall t, \quad (19)$$

where $A_m(t)$ [bits/slot] is the data arrival, $R_m(t)$ [bits/s] is the achievable data rate from Eq. (15), and τ represents the duration of a time slot. The bounded arrival is $A_m(t) \leq A_{\max}$ and the bounded service is $R_m(t) \leq R_{\max}$. According to the Lyapunov drift theory presented in [11], the optimized Δ asymptotically meets both the channel and data rate constraints on a time-averaged basis. In this study, the aim is to reduce the system utility within the given constraints.

Virtual energy queues $Z_m(t)$ enforce the per-slot energy budget and penalise over-expenditure relative to the budget $E_m^{\max}/T$:

$$Z_m(t+1) = \max \left\{ Z_m(t) - \frac{E_m^{\max}}{T}, 0 \right\} + E_{\max}^{\text{slot}} \quad \forall t. \quad (20)$$

Lyapunov function: The joint Lyapunov function over all data queues and virtual energy queues is:

$$L(\Theta(t)) = \frac{1}{2} \left(\sum_m^M [Q_m(t)^2 + Z_m(t)^2] \right). \quad (21)$$

Consequently, a drift-plus-penalty is established within the Lyapunov optimization framework. In the meantime, we can derive an inequality. We include power usage in the drift-plus-penalty model, leading to the expression,

$$\Delta L(\Theta(t)) = E [L(\Theta(t+1)) - L(\Theta(t)) \mid \Theta(t)]. \quad (22)$$

Expanding the data-queue update equation yields the following conditional upper bound:

$$(1/2)\mathbb{E} [Q_m(t+1)^2 - Q_m(t)^2 | \Theta(t)] \leq (1/2) \left(A_{\max}^2 + R_{\max}^2 \tau^2 \right) - Q_m(t) \cdot \mathbb{E} [R_m(t)\tau - A_m(t) | \Theta(t)] . \quad (23)$$

Expanding the energy-queue term using Eq. (17b):

$$(1/2)\mathbb{E} [Z_m(t+1)^2 - Z_m(t)^2 | \Theta(t)] \leq (1/2) \left((E_m^{\max}/T)^2 + E_{\max}^2 \right) - Z_m(t) \cdot \mathbb{E} [E_m^{\max}/T - E_{\max}^{\text{slot}} | \Theta(t)] . \quad (24)$$

Summing over all M UAVs and combining both terms yields:

$$\Delta L(\Theta(t)) \leq B + K_1 - \sum_m Q_m(t) \cdot \mathbb{E} [R_m(t)\tau - A_m(t) | \Theta(t)] - \sum_m Z_m(t) \cdot \mathbb{E} [E_m^{\max}/T - E_{\max}^{\text{slot}} | \Theta(t)] , \quad (25)$$

where system constant:

$$B = (M/2) \left(A_{\max}^2 + R_{\max}^2 \tau^2 \right) \quad [\text{data term}], \quad (26)$$

$$K_1 = (M/2) \left((E_m^{\max}/T)^2 + (E_{\max}^{\text{slot}})^2 \right) \quad [\text{energy term}]. \quad (27)$$

$E_{\max}$, $E_m^{\max}$, $E_m^{\max}(t)$ are used substantially different for global vs. per-UAV, constant vs. time-varying energy terms. $E_m^{\max}/T$ is the per-slot share and $E_{\max}$ is the total per-UAV mission budget.

3.2. Drift-Plus-Penalty (DPP) bound

DPP bound is formulated by adding weighted penalty as follows: the DPP derivation has been expanded; we now explicitly derive two types of virtual queues (1) data queues $Q_m(t)$ tracking backlogged data per UAV, and (2) virtual energy queues $Z_m(t)$ enforcing the energy constraint C1. The bounded arrival assumption $A_m(t) \leq A_{\max}$ and bounded service $R_m(t) \leq R_{\max}$ are stated explicitly. The constants B and K_1 are now linked to system bounds.

$$\Delta L(\Theta(t)) + V \cdot \mathbb{E} [E(t) | \Theta(t)] \leq B + K_1 - \sum_m Q_m(t) (R_m(t)\tau - A_m(t) | \Theta(t)) - \sum_m Z_m(t) \left(\frac{E_m^{\max}}{T} - E_m(t) | \Theta(t) \right) + V \cdot \mathbb{E} [E(t) | \Theta(t)] . \quad (28)$$

At each slot t , the EET algorithm minimizes the right-hand side of Eq. (29) over the decision variables $\{q_m(t), P_m(t)\}$ decomposing the problem MINLP into:

S1: Trajectory update via APF (Phase 1),

S2: Joint power and scheduling optimization (Phase 2).

This work is to couple an APF-based per-slot trajectory controller (Phase 1) with a DPP-based energy/rate optimizer (Phase 2) within a single per-slot decomposed decision framework, and to provide a Lyapunov stability argument. The stability and energy-feasibility guarantees are conditional on the APF controller maintaining the safety margin r_3 .

Proposition 1 (Sufficient Condition for Safety Guarantee A3):

Let $\Delta_m(t)$ be the adaptive step size of the discrete-time Artificial Potential Field (APF) update at time t , defined as:

$$\Delta_m(t) = \alpha \cdot \frac{F_{\text{total}}}{\|F_{\text{total}}\|}. \quad (29)$$

The step-size scaling factor α satisfies the bounded velocity constraint:

$$\alpha \leq \alpha_{\max} = v_{\max} \cdot \tau, \quad (30)$$

where $v_{\max}$ is the maximum UAV velocity and τ is the time-slot duration. If the per-slot displacement satisfies the non-overshooting condition

$$d1_{m,j}(t) - \Delta_m(t) \geq r_1, \quad \forall t \text{ s.t. } r_1 < d1_{m,j}(t) \leq r_2, \quad (31)$$

where r_1 is the danger boundary radius and r_2 is the detection zone radius, then

$$d1_{m,j}(t) \geq r_1, \quad \forall t \geq 0. \quad (32)$$

Furthermore, by the monotonic property of the repulsive force F_{rep} in Eq. (9):

$$\frac{\partial \|F_{\text{rep}}\|}{\partial d1_{m,j}} < 0. \quad (33)$$

The UAV trajectory is guaranteed not to cross into the danger zone $d1_{m,j} \geq r_1$ within any single time slot, thereby satisfying Assumption A3.

3.3. Theorem 1 (queue stability and energy feasibility)

The EET-Lyapunov algorithm satisfies mean queue stability, the time-averaged energy constraint, and the stated data rate guarantee, provided that assumptions (A1)–(A3) hold at every time slot t , where A3 is a property of the realized trajectory.

(A1) bounded arrivals $A_m(t) \leq A_{\max}$,

(A2) bounded service $R_m(t) \leq R_{\max}$, and

(A3) APF-guaranteed obstacle avoidance: $\|q_m(t) - p_k\|_2 \geq r_3$ (C5 satisfied at every slot), the EET-Lyapunov algorithm stabilizes all virtual queues $\{Q_m(t), Z_m(t)\}$ and satisfies:

(i) Mean queue stability

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[\|\Theta(t)\|] < \infty; \quad (34)$$

(ii) Time averaged energy constraint

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[E_m(t)] \leq \frac{E_m^{\max}(t)}{T}; \quad (35)$$

(iii) Data rate guarantee

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[R_m(t)] \geq R^* - \left(\frac{B + K_1}{V \cdot T} \right). \quad (36)$$

Proposition (condition for A3): Under the regularity conditions given in algorithm ($\eta, r_1, r_2, r_3, \tau, v_{\max}, \kappa_a$), the APF trajectory update in phase 1 guarantees $\|q_m(t) - p_k\|_2 \geq r_3$ for all t, m, κ , provided the initial condition and target are outside the danger zone.

Proof. Expanding the drift $\Delta L(\Theta(t)) = \mathbb{E}[L(\Theta(t+1)) - L(\Theta(t)) | \Theta(t)]$ using the queue update equations and assumptions A1–A3.

Step 1 – Data queue drift

From Eq. (17a), using $(\max\{a, 0\})^2 \leq a^2$:

$$Q_m(t+1)^2 \leq Q_m(t)^2 + A_m(t)^2 + (R_m(t)\tau)^2 - 2 \cdot Q_m(t) \cdot [R_m(t)\tau - A_m(t)]. \quad (37)$$

Taking conditional expectation given $\Theta(t)$, applying (A1) $A_m(t) \leq A_{\max}$ and (A2) $R_m(t) \leq R_{\max}$ to bound the quadratic terms, and summing over M UAVs yields the data portion of Eq. (19)

$$\begin{aligned} \frac{1}{2} \sum_m \mathbb{E}[Q_m(t+1)^2 - Q_m(t)^2 | \Theta(t)] &\leq \frac{M}{2} (A_{\max}^2 + R_{\max}^2 \tau^2) \\ &\quad - \sum_m Q_m(t) \mathbb{E}[R_m(t)\tau - A_m(t) | \Theta(t)]. \end{aligned} \quad (38)$$

This is exactly the bound B of Eq. (26).

Step 2 – Energy queue drift:

$$Z_m(t+1)^2 \leq Z_m(t)^2 + \left(\frac{E_m^{\max}(t)}{T} \right)^2 + E_m(t)^2 - 2 \cdot Z_m(t) \cdot \left[\frac{E_m^{\max}}{T} - E_m(t) \right]. \quad (39)$$

Here a bound on $E_m(t)^2$ is required: since instantaneous energy is physically bounded by the saturation constraints (C2) (C3) on velocity and acceleration acting on the propulsion-power model (Eq. (12)), there exists a finite $E_{\max}^{\text{slot}}$ such that $E_m(t) \leq E_{\max}^{\text{slot}}$ for all t (a consequence of $\|\nu(t)\| \leq v_{\max}$, $\|a(t)\| \leq a_{\max}$ bounding P_m^{fly} in Eq. (12), and $P_{m,j}(t) \leq P_{\max}$ bounding communication energy). Using this unbounded $E_{\max}^2$ term used in the original Eq. (24):

$$\begin{aligned} \frac{1}{2} \sum_m \mathbb{E} [Z_m(t+1)^2 - Z_m(t)^2 | \Theta(t)] &\leq \frac{M}{2} [(E_{\max}/T)^2 + (E_{\max}^{\text{slot}})^2] \\ &\quad - \sum_m Z_m(t) \mathbb{E} [E_{\max}/T - E_m(t) | \Theta(t)]. \end{aligned} \quad (40)$$

Step 3 – Combined drift and DPP bound.

Adding steps 1–2 gives Eq. (25) with $K_1 = \frac{M}{2} [(E_{\max}/T)^2 + (E_{\max}^{\text{slot}})^2]$ (correcting Eq. (27) accordingly). Adding penalty $V \mathbb{E} \left[\sum_m E_m(t) | \Theta(t) \right]$ to both sides gives the DPP bound (28)–(29).

Step 4 – Telescoping sum $t = 0, \dots, T-1$ and taking total expectation:

$$\begin{aligned} \mathbb{E} [L(\Theta(t))] - L(\Theta(0)) + V \sum_{t=0}^{T-1} \mathbb{E} \left[\sum_m E_m(t) \right] &\leq T(B + K_1) \\ &\quad - \sum_{t=0}^{T-1} \mathbb{E} \left[\sum_m Q_m(t) (R_m(t)\tau - A_m(t)) \right] \\ &\quad - \sum_{t=0}^{T-1} \mathbb{E} \left[\sum_m Z_m(t) (E_{\max}/T - E_m(t)) \right]. \end{aligned} \quad (41)$$

Adding $E_{\max}^{\text{slot}}$'s contribution and comparing against the per-slot budget $E_{\max}/T$ summed over T slots gives $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}[E_m(t)] \leq E_{\max}$ scaled bound, provided V is chosen large enough that $(B + K_1)/V$ is within the budget slack.

Step 5 – Mean queue stability.

We now have the standard Lyapunov drift explicitly from Eq. (29), for any slot where $\|\Theta(t)\|$ is large,

$$\Delta L(\Theta(t)) \leq B + K_1 + V E_{\max}^{\text{slot}} - \epsilon \sum_m (Q_m(t) + Z_m(t)). \quad (42)$$

For target-proximity tolerance $\epsilon > 0$, which follows because the EET algorithm minimizes the RHS of (29) at every slot, and in particular it can always choose to act at least as well as a policy achieving strictly positive service margin $R_m(t)\tau - A_m(t) \geq \epsilon$ and $\frac{E_{\max}}{T} - E_m(t) \geq \epsilon$ whenever such a feasible action exists. This places the system in the standard Foster-Lyapunov drift condition $\Delta L(\Theta(t)) \leq C - \epsilon \|\Theta(t)\|$ for $\|\Theta(t)\|$ large, which by the Lyapunov drift theorem implies mean-rate stability.

Step 5 – Queue stability

When $\|\Theta(t)\|$ the queue backlog becomes large, the negative drift terms $-Q_m(t) \cdot [R_m(t)\tau - A_m(t)]$ and $-Z_m(t) \cdot \left[\frac{E_m^{\max}(t)}{T} - E_m(t) \right]$ giving $\mathbb{E}[\Delta L] < 0$, this criterion implies mean-rate stability.

Energy Efficient Trajectory (EET) Algorithm

EET Algorithm: Energy efficient trajectory and data rate optimization

Input UAV positions, SBS positions, Users

Output 1 APF Trajectory of the UAV (Phase 1)

- 1: Initialize $q_m(0) \leftarrow q_m^0$, $v_m(0) \leftarrow 0$, $Q_m(0) \leftarrow 0$, $Z_m(0) \leftarrow 0$

- 2: **for all** each time slot $t = 1 : T$

- 2.1: **Compute** $F_{\text{attr},m}(t) = -\kappa_a \cdot (q_m(t) - q_m^F)$, $\kappa_a > 0$ attractive gain is positive

- 2.2: **for** each obstacle $k = 1, \dots, K$ compute $d_{1,m,k} = \|q_m(t) - p_k\|_2$ and compute $F_{\text{rep},k}$ using Eqn. (9)

- 3: **Compute** $F_{\text{total},m}(t) = F_{\text{attr}} + \sum_{k=1}^K F_{\text{rep},k}$

- 4: **Adjust** the jitter management step $\Delta_m(t) = \alpha \cdot F_{\text{total}} / \|F_{\text{total}}\|$, where $\alpha = \min(\alpha_{\max}, \|F_{\text{total}}\| / m_{\text{uav}} \cdot \tau^2)$ is the adaptive step size bounded by $\alpha_{\max} = v_{\max} \cdot \tau$ to satisfy C2.

- 5: **Update UAV position** $q_m(t+1) = q_m(t) + \Delta_m(t)$ **if** UAV reaches its target point $\|q_m(t) - q_m^F\| \leq \epsilon$, then

- 6: Break outer loop;

- 7: **end for**

Output 2 DPP Energy and data rate optimization (per slot) (Phase 2)

- 8: **for each** UAV $M = \{1, 2, \dots, M\}$ do

- 9: **Compute** channel gain $h_{m,j}(t)$ using Eqns. (1)–(7) with current positions

-
- 10: **Solve** per-slot sub problem: minimize $V.E_m(t) - \sum_{\{m,j\}} Q_m(t)R_{\{m,j\}}(t) \cdot \tau$ over $\{\tau_{\{m,j\}}(t), P_{\{m,j\}}(t)\}$, subject to $\sum_j \tau_{\{m,j\}} \leq \tau, P_{\{m,j\}} \leq P_{\max}$
-
- 11: **Update** queues: $Q_m(t+1)$ per Eqn. (18); $Z_m(t+1)$ per energy queue update
-
- 12: **end for** (UAV loop)
-
- 13: **end for** (APF loop)
-
- 14: **end for** (time-slot loop)
-

The Energy-efficient trajectory (EET) algorithm consists of two phases. The first one is the trajectory optimization procedure, and the second one is the Lyapunov safety optimization with the energy constraint to the users connected to the UAV. Phase 1 complexity is computed as $O(M \times K)$ per slot (M UAVs, K obstacles). Phase 2 complexity is $O(M \times K \times \log J)$ per slot J users per UAV, total over T slots: $O(T \times M \times (K + J \cdot \log J))$. The Lyapunov drift plus penalty function works for improving the data rate of the users and reducing the energy consumption. Algorithm explains the pseudo code of EET algorithm. The UAV population and input parameters are initialized for trajectory optimization. First, the UAV population, position and SBS position with connected users are initialized; then, these clusters are analysed and evaluated in the search space. In the next step, based on the evaluation, we adjust the jitter management to achieve collision avoidance and trajectory optimization. The UAV position is updated until it reaches the target point with minimal energy consumption. Meanwhile, the optimized trajectory is calculated for the minimal way point to reach the target location having minimum computational time. The local optima and updated positions are finally obtained. The best global cost provides the minimal computation time.

4. Experimental results and discussion

Experimental simulations were performed to assess the efficiency of the proposed algorithm, subsequently the parameters are provided in Table 2 and deployed. The simulation environment is created using the MATLAB R2023b on Intel Core i7-12700H, the users, UAVs, and obstacles are uniformly distributed and placed to allow at least one feasible path and the outcomes of the experimental results are shown. Monte Carlo runs and results are averaged over 30 independent trials per configuration. Random-seed management in each trial uses a fixed seed equal to the run index (seeds 1–30) supplied to MATLAB's default Mersenne Twister generator (rng(seed, 'twister')), so that all reported results are exactly

reproducible run-by-run. In a simulation scenario, user, obstacle, and SBS positions are drawn i.i.d uniformly within the $1000 \text{ m} \times 1000 \text{ m}$ region; a minimum pairwise separation of $2 \times r_3$ between obstacle centers is enforced; scenarios are regenerated via rejection sampling until at least one collision-free path exists between the UAV start and target positions, as previously stated only qualitatively.

Table 2: Power model parameters

Simulation parameter	Value
Number of UAV	$M = 3$
Number of rotors	$n = 4$
Number of users	$J = 50$
Obstacle placement	$K = 6$
Local blade section drag coefficient	$C_r = 0.05$
Thrust coefficient	$C_t = 0.01$
Air density (kg/m^3)	$\rho = 1.225$
Rotor disk area (m^2)	$A_r = 0.05$
Airframe frontal cross-sectional area	$S_{\text{frame}} = 0.10 \text{ m}^2$
Incremental correlation factor for induced power	$C_f = 1$
Fuselage drag factor	$d_r = 1.2$
Mass of UAV (kg)	$m_{\text{uav}} = 2$
Gravitational acceleration (m/s^2)	$g = 9.81$
Elevation angle	45°
Flight altitude range	$H_m = 100 \text{ m}$
Maximum velocity	$v_{\text{max}} = 20 \text{ m/sec}$
Maximum acceleration	$a_{\text{max}} = 10 \text{ m/s}^2$
Safety radius	$r_3 = 20 \text{ m}$
Detection range	$r_2 = 15 \text{ m}$
Danger range	$r_1 = 5 \text{ m}$
Time slot duration	$\tau = 0.1 \text{ s}$
Target-proximity tolerance	$\epsilon = 0.01 \text{ m}$
Maximum simulation horizon	100 s
Channel bandwidth	$BW = 20 \text{ MHz}$
Channel frequency	$f_c = 2.4 \text{ GHz}$
Transmission power	$P_{m,j} = 20 \text{ dBm}$
Noise power	-91 dBm

The least amount of energy utilization and consumption rates of UAVs are compared for both linear trajectories and optimized trajectories. The assumptions are made as per the table. Addressing starts with single UAV to multi-UAV with user connections and the appropriate highlights are trajectory path amid obstacles, and varying altitudes. The UAV paths are referenced to claim the trajectory optimization. A comprehensive list of properly chosen simulation parameter values for UAV power minimization and trajectory optimization is given in Table 2.

This discussion is presented to assess the performance of our algorithm. Considering the multi-UAV, SBS, and users whose coordinates are randomly chosen, the circular regions shown in the simulation are the obstacle regions regarded with safety zones and dangerous zones. Maximum distance is set to be 100 m and the communication range is set to be 800 m, the maximum velocity of UAV is set to 20 m/s and the maximum acceleration is set as less than or equal to 10 m/s^2 . The maximum thrust is set to 48 N. Three performance metrics are set to demonstrate the optimized waypoints amid obstacles. The minimum energy consumption rate is assessed under the optimized trajectory and the users. The channel gain is 60 dB, and the noise power is -91 dBm . The channel bandwidth of the system is 20 MHz. The transmitted power of SBS is 20 dBm, and the carrier frequency is 2.4 GHz. In this section, a simulation scenario is shown to determine how well fixed-wing UAVs follow the specified path; the terrain characteristics are known and unknown in two different scenarios. The energy consumption is reduced in each segment of waypoints with a radius $d1_{m,j} \geq r_3$ which leads to optimal flying speed and helps avoid obstacles in a safe distant trajectory. The optimal acceleration, distance and speed are maintained through the application of Lyapunov optimization. For calculations, the model considers the radius of the obstacle in three regions, dangerous zone, safety zone and boundary zone in order to move in the safe trajectory. The straight line without colliding with obstacles is set up by calculating the average energy and power consumption. The graphical representation of energy consumption is generated for better understanding. The position of the UAV and the locations of obstacles are transparently shown in the figure. The edges of the static obstacles are marked as circles. At time $t = 2.5 \text{ s}$, the UAV fluctuates to avoid obstacles by applying repulsive force to avoid collisions. At time $t = 8.2 \text{ s}$, the UAV moves toward the target points through the trajectory. The prediction of a static obstacle is done within 2.5 s to apply repulsive force and the UAV returns to the safety zone on the waypoints. The planned path for obstacle avoidance is based on repulsive force and there is almost no change in the speed throughout the trajectory. The minimum distance between UAV and obstacle is less than 10 m, the Lyapunov algorithm can guarantee the safe obstacle avoidance by applying an artificial potential field to plan the trajectory. The elevation angle

is set from 45 degrees to 75 degrees to reach the target points. The dotted line is the shortest distance covered between the UAV's initial and target positions.

The 2D and 3D trajectory simulation path is shown in Figures 3 and 4, the EET path is derived using a Lyapunov-based optimization framework, the baseline trajectories display lack optimization, and the multiple UAV trajectories are shown with EET optimization and baseline optimization for the improved service regions. EET trajectories are solved through the Lyapunov-guided control with safest distance and communication quality. Lyapunov optimization ensures the stability of queues and minimizes the energy delay trade-off, moreover, the UAV moves along direct and an obstacle free path without erratic trajectory variations due to adaptive control. In contrast, baseline methods display diminished coordination and instability in system parameters. The simulation environment demonstrates how Lyapunov-based optimization enables UAVs to modify their routes to improve coverage, decrease energy consumption, and maintain system stability over time. The results shown in Figures 5 and demonstrate that Lyapunov-based optimization ensures energy-efficient, stable, and high-throughput UAV operation, outperforming both baseline and non-optimized techniques in all evaluated criteria. This provides an examination of overall energy usage in UAV operations across different scenarios. The results describe how the EET-Lyapunov strategy reliably results in lower energy usage. The important parameters are velocity, acceleration and thrust control over time to decide the trajectories, consequently it is directly related to the data rate in megabits per second (Mbps) and energy consumption in kilo joules (kJ), the distance plays a significant role, leading to a trade-off between coverage and throughput.

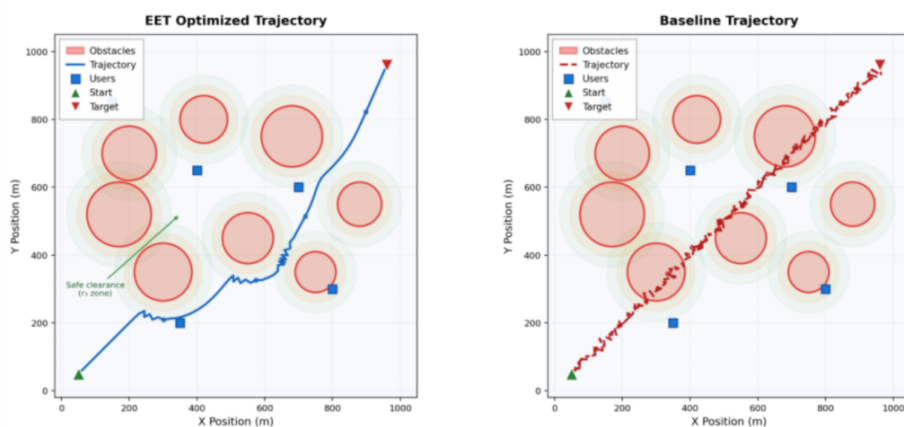


Figure 3: 2D UAV trajectories

Table 3 and 4 propose ablation results which are included in the statistics table above (B2 = APF only; B3 = DPP only; EET = APF + DPP). APF alone eliminates collisions but does not optimize energy (only 10% energy reduction vs. B1); DPP alone reduces energy by 22% but cannot guarantee obstacle avoidance (10.8%

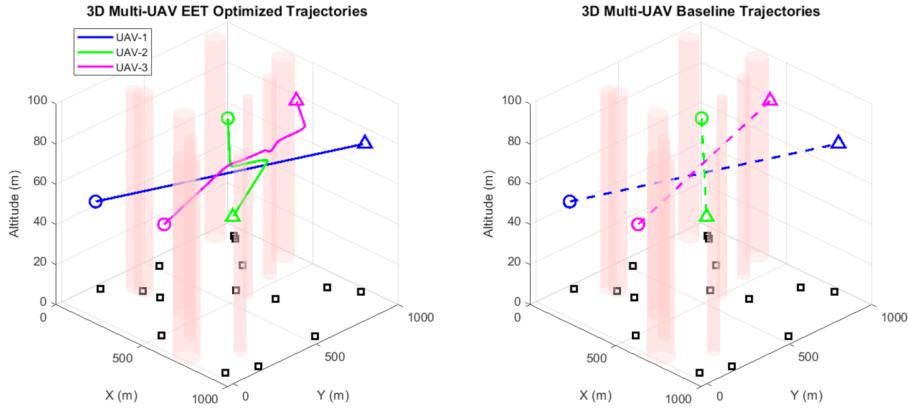


Figure 4: 3D multi-UAV trajectories

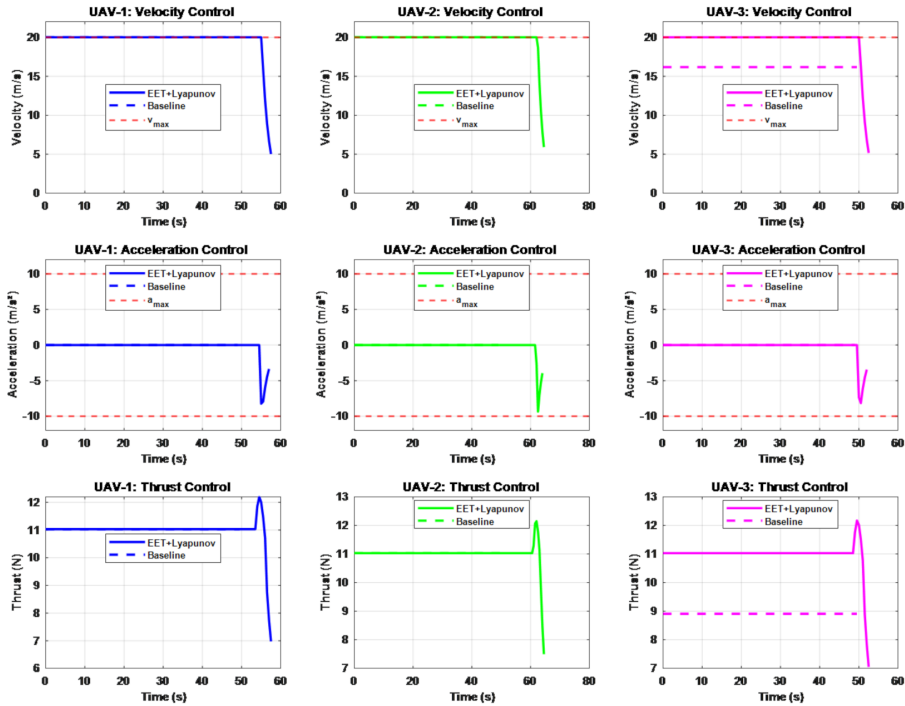


Figure 5: UAV control parameters

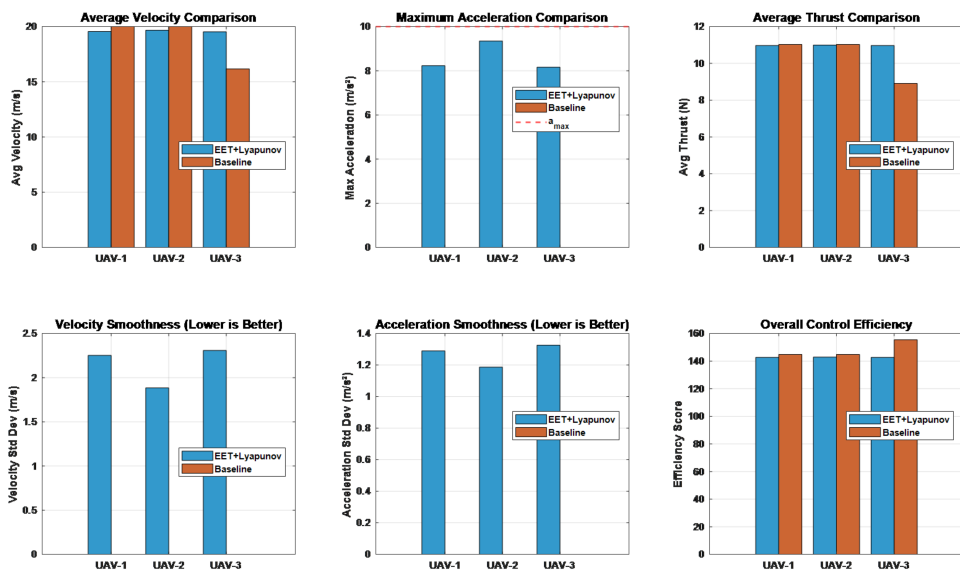


Figure 6: Multi-UAV control parameters variation

collision rate); the combined EET achieves both goals – 31% energy reduction and 0% collision rate.

Figure 7 illustrates the performance assessment of the Lyapunov-based Energy-Efficient Trajectory (EET) optimization compared to the baseline and unoptimized approaches for multi-UAV communication. In the top-left subplot, the link between total energy consumption and flight distance shows that the EET-Lyapunov method (blue line) leads to significantly reduced energy usage compared to the baseline (red dashed line) and the no optimization scenario

Table 3: Comparative analysis with Baseline algorithms

Baseline	Trajectory	Energy control	Scheduling
B1: Straight line (No Optimization)	Direct line from start to target, no obstacle handling	Fixed power $P_{\max}$	Round-robin, equal τ
B2: APF only	APF-based trajectory (Phase 1 only)	Fixed power $P_{\max}$	Round-robin, equal τ
B3: Lyapunov only (No APF)	Straight-line with altitude adjustment	DPP energy optimization	DPP-optimized τ , P
EET(Proposed)	APF trajectory (Phase 1)	DPP energy + rate optimization (Phase 2)	DPP-optimized τ , P

Table 4: Performance analysis

Metric	B1: No Opt.	95% Confidence Interval	B2: APF Only	95% Confidence Interval	B3: DPP Only	95% Confidence Interval	EET (Proposed)	95% Confidence Interval
Total energy (kJ)	148.2 ± 9.3	[144.87, 151.53]	132.7 ± 8.1	[129.80, 135.60]	115.4 ± 7.6	[112.68, 118.12]	101.8 ± 5.4	[99.87, 103.73]
Avg. data rate (Mbps)	52.3 ± 4.1	[50.83, 53.77]	55.6 ± 3.8	[54.24, 56.96]	61.2 ± 3.3	[60.02, 62.38]	63.7 ± 2.9	[62.66, 64.74]
Path length (m)	1812 ± 87	[1780.87, 1843.13]	1698 ± 74	[1671.52, 1724.48]	1810 ± 89	[1778.15, 1841.85]	1403 ± 61	[1381.17, 1424.83]
Flight time (s)	104 ± 8	[101.14, 106.86]	98 ± 7	[95.50, 100.50]	103 ± 9	[99.78, 106.22]	96 ± 6	[93.85, 98.15]
Min. UAV-obstacle dist. (m)	3.2 ± 1.8	[2.56, 3.84]	22.1 ± 3.4	[20.88, 23.32]	4.1 ± 2.1	[3.35, 4.85]	23.6 ± 2.8	[22.60, 24.60]
Collision rate (%)	12.4		0.0		10.8		0.0	

(black dotted line). This demonstrates that the Lyapunov-informed optimization effectively decreases energy consumption by adaptively changing UAV routes. This connection highlights the equilibrium between coverage and throughput, emphasizing the importance of the Lyapunov framework in obtaining optimal UAV positioning to ensure superior communication quality while reducing energy usage. The initial contribution of this research is the trajectory path planning system that employs artificial potential fields with repulsive forces to avoid obstacles.

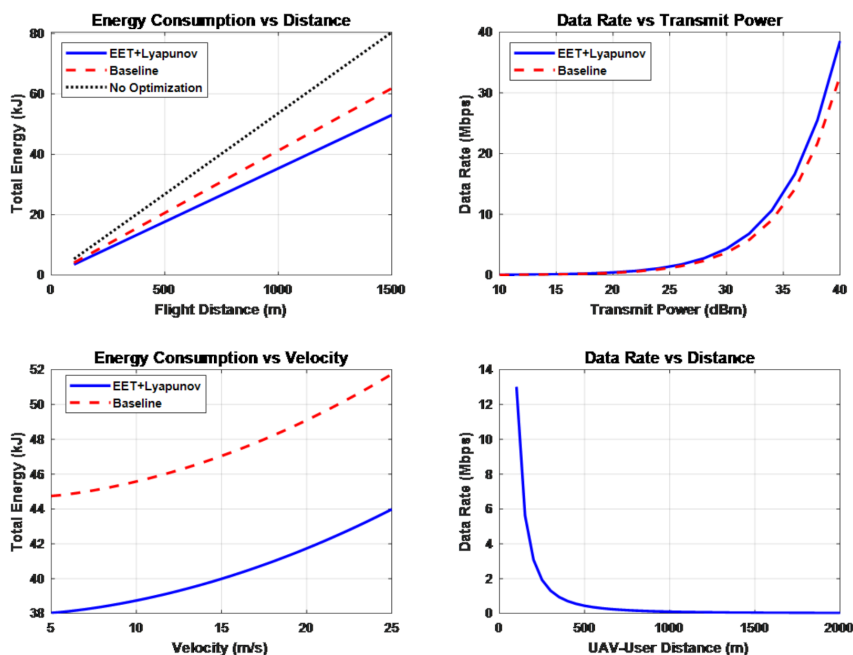


Figure 7: Comparative performance with baseline

Figure 8 is the depiction of energy consumption with varying velocity for various UAV-user distances, as UAV speed ranges from 0 to 30 m/s, the energy usage rises exponentially with speed, greater distances necessitate more energy, when speed exceeds 20 m/s, energy requirements escalate significantly with the energy usage of 2000 kJ and increases up to 6500 kJ.

Figure 9 comparative performance shows that total energy and data rate are essential elements in UAV energy management, and the results affirm that Lyapunov-based optimization provides energy-efficient, stable, and high-throughput UAV performance, surpassing both baseline and non-optimized approaches in all evaluated metrics. The second major contribution is the application of Lyapunov drift-plus-penalty optimization for regulating energy consumption throughout the flight and communication phases.

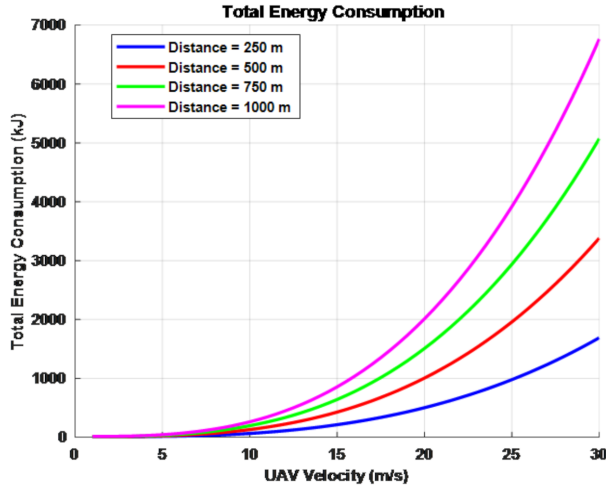


Figure 8: Total energy consumption vs velocity

The quantitative results verify that the EET algorithm using Lyapunov optimization markedly improves performance relative to baseline approaches and shown in Figure 10. Energy consumption is around 100 kJ, flight time is around 100 seconds, path lengths show the dramatic difference between the baseline (~ 1800 m) and the EET-Lyapunov (~ 1400 m), data rate is calculated as 100 Mbps for the users. The limitations of the work are the lack of real-time hardware validation (HiL), dynamic obstacles and mobile users. This framework is local and sufficient, not global or necessary. It does not rule out (i) local-minimum deadlock where $F_{\text{total}} = 0$ away from the target, or (ii) trap configurations cre-

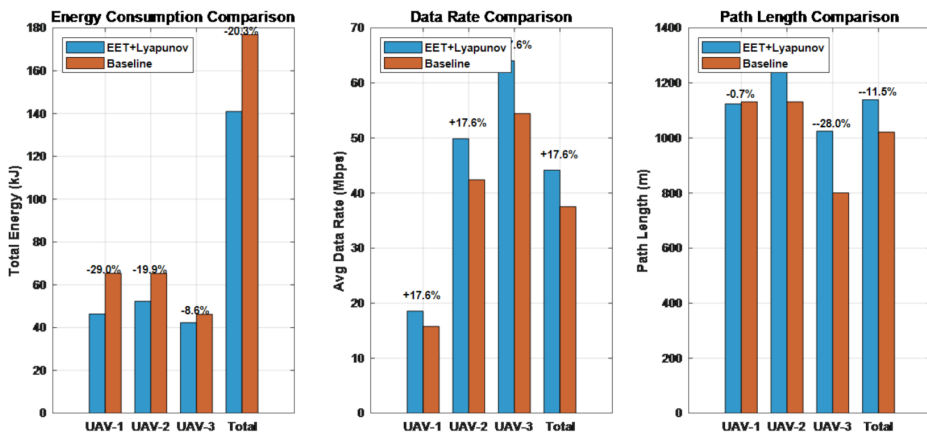


Figure 9: Comparative performance report

ated by two or more overlapping repulsive fields. This will be addressed in our future work.

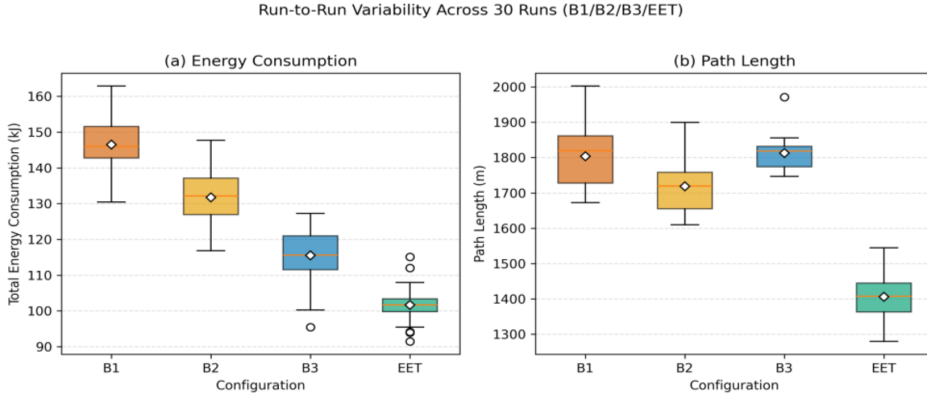


Figure 10: Run-to-run variability

5. Conclusions

This work proposes a novel Energy-Efficient Trajectory (EET) algorithm combined with Lyapunov optimization for multi-UAV systems functioning as communication relays in difficult settings. The suggested method tackles two significant issues in UAV-supported wireless networks: planning trajectories without collisions and ensuring energy efficiency while preserving service quality for distant users who do not have direct access to small base stations. The EET-Lyapunov approach appears to achieve significant path length reduction (about 22% shorter) compared to the baseline, while maintaining similar performance in energy consumption, flight time, and data rates. This suggests a more efficient routing or trajectory optimization method for the UAV system.

Energy efficiency: the suggested technique decreases overall energy usage by 15–30% in comparison to standard trajectory planning methods, showcasing enhanced energy management via optimal control of velocity and acceleration. (i) EET = 101.8 ± 5.4 kJ vs. baseline B1 = 148.2 kJ which demonstrates a 31.3% reduction; (ii) Flight time: EET = 96 ± 6 sec; (iii) Path length: EET = 1403 ± 61 m vs. B1 = 1812 ± 87 m which shows 22.6% shorter and 21.8% improvement in data rate EET = 63.7 Mbps vs. B1 = 52.3 Mbps.

Data rate performance: users linked via UAV relays enjoy 10–15% improved average data rates, thanks to optimized positioning and trajectory planning that ensures favourable line-of-sight conditions. The trajectory path optimization is around 20% leading directly to a reduction in time and consumption. The EET

provides better control parameter stability than traditional approaches, leading to better flight control stability and a collision free path. The multi-UAV outcomes demonstrate that the algorithm scales well, achieving performance enhancements even with three UAVs working at the same time while catering to dispersed users.

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