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What was wrong with Fryze's approach to power theory development?

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Abstract: Prof. Stanisław Fryze, by developing his power theory (PT), made a fundamental contribution to studies on energy transfer in electrical circuits. After that, these studies were continued by scientists and engineers for several decades, and several new power theories were developed. It means that despite his major contribution, his PT still has failed to provide answers to several important questions asked about physical phenomena that associate the electric energy transfer and their mathematical description, as well as the methods of increasing its effectiveness by compensation. This article discusses the causes of the failure of Fryze's approach to PT development. One of them was the rejection of the concept of harmonics. The extension of his PT to three-phase circuits has failed because he did not try to reveal the power properties of such circuits as a single unit, but expressed them in terms of the power properties of separate supply phases of such circuits.

Key words: frequency domain, physical phenomena, power definitions, power factor, time domain

1. Introduction

The development of power theory (PT) of electrical circuits with distorted currents was initiated in 1892 by Ch. Steinmetz, who, in an experiment with an arc bulb supplied from a source of sinusoidal voltage, observed [1] that apparent power *S* of such a load is higher than active power *P*. It was known at that time that a phase shift between the supply voltage and load current increases apparent power *S*, but oscilloscopes were not yet invented. Therefore, that experiment justified Steinmetz's question about the phase shift between the voltage and the current and the relation between active *P* and apparent *S* powers. Identification of physical phenomena responsible for apparent power *S* increase above active power *P* is the main cognitive reason for PT development.

Several scientists responded to Steinmetz's question. First, there were studies on power factor (PF) [2], which later shifted to studies on powers [3, 5–7], in particular, on powers at a distorted supply voltage [8–11]. Paper [15], published in 1932, compiles 75 papers with results of studies on distorted quantities and their measurements. These studies were dominated by German scientists [6, 7, 10, 11], who created a sort of a background for Fryze's research on PT and affected the terminology he adopted.

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This paper is aimed at a mathematically-oriented validation of Fryze's power theory. When a circuit is described in terms of Kirchoff's Laws, its sources and RLC elements are regarded as idealized devices with a strict voltage-current relationship, the power theory of such a circuit can be regarded as a mathematical theory, associated with physical phenomena, specified by

strict mathematical terms. The result of the validation of such a mathematical theory can be only "***true***" or "***false***". There are no other options. The history of theory development and related opinions are irrelevant for a mathematical theory validation. Because not all Readers of this paper could be aware of that, the justification for the above conclusion can be found in the Popper book [37]. The validation of a theory requires that the criterion of its correctness be specified. In the case of the power theory, the first scientists who initiated its development, Steinmetz, Budeanu, and Fryze, asked: (i) ***What physical phenomena are responsible for the increase of apparent power S above active power P value?*** Later, a second question occurred: (ii) ***How can the difference between S and P be eliminated?*** The conclusion of whether a power theory answers these two questions correctly or not provides the criterion for its validation. Apart from this validation, the paper has an essay-like format that enables the author to present elements of the history of the PT development and express various opinions on that.

Reading this article, the reader can observe that the term "***power theory***" is used in two meanings. First, it is used to associate a particular explanation and description of energy transfer with a particular scientist who developed it and suggested its use in electrical engineering, as, for example, "***Budeanu's PT***", "***Fryze's PT***" or "***Tenti's PT***". In the second meaning, the term "***power theory***" is used as the state of the present knowledge on energy transfer-related properties of electrical circuits, a sort of database [32], composed of statements, not challenged as erroneous, on these properties, expressed verbally or by mathematical descriptions. The author hopes that, having the context of its use, the reader will recognize its meaning easily.

2. New concepts introduced to power theory by Fryze

In his studies on energy transfer in electrical circuits, its explanation and description in mathematical terms, Fryze introduced [12, 14] several new concepts to PT. They are compiled below.

1. ***Load current decomposition***. Before Fryze, apparent power S was the major subject of possible decompositions into various power components, starting from active power P. Fryze argued that, at a fixed supply source, the load, through its current, $i(t)$, affects the load voltage. The load voltage changes with the change in the load properties. This is a response to the current changes. Therefore, according to Fryze, the description of the load power properties should originate from its current, $i(t)$, decomposition. The apparent power decomposition should be regarded as secondary to the current decomposition.
2. ***Time domain approach.*** Fryze argued that harmonics are not physical, but only mathematical entities, so for the description of such an important physical phenomenon in electrical circuits, as energy transfer, tools that are not physically founded should not be used. Instead

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of harmonics, all quantities in PT should be defined, according to Fryze's opinion, in terms of time-varying voltages and currents, $u(t)$ and $i(t)$, or their functionals, such as integrals.

3. ***Orthogonality.*** This concept, used in various branches of mathematics, was applied by Fryze to PT development. According to him, the components obtained in the load current decomposition should be mutually orthogonal. When this condition is satisfied, the rms value of the load current will depend on the rms value of these components individually, independently of each other. This orthogonality requires that the mutual scalar products of these components are zero.
4. ***Importance of physical interpretation.*** Fryze emphasized the importance of the physical interpretation of all entities used in PT for describing energy transfer in electrical circuits. This physical perspective was likely a result of his status as an Honorary Member of the Polish Physics Society. The importance of such interpretation was very strongly articulated in Fryze's debate with Staniewicz [13].
5. ***Warning that some entities in PT could be mathematical but not physical entities.*** Fryze used to warn scientists involved in PT development that harmonics are not physical but only mathematical entities. The same can apply to power quantities.

Some of these concepts are clearly visible in Fryze's articles, even by his plain declarations, some of them as, for example, orthogonality or scalar product, are not visible without a detailed study of Fryze's concept, but they do exist in his development of the power theory. This theory is drafted below.

3. Fryze's power theory

Mathematical symbols used by Fryze a century ago might be a bit confusing for present-day readers. Their change to symbols used presently can make mathematics in [12, 14] and [28] easier to follow. The changes (>>) of main symbols are compiled below:

$$U_t >> u(t), \quad U >> \|u\|, \quad J_t >> i(t), \quad J >> \|i\|, \quad P_w >> P, \quad P_s >> S,$$

$$P_b >> Q_F, \quad J_{wt} >> i_a(t), \quad J_w >> \|i_a\|, \quad J_{bt} >> i_{rF}(t), \quad J_b >> \|i_{rF}\|.$$

Below is the summary of Fryze's approach to describing the power properties of electrical circuits at distorted supply voltage, as presented in [12, 14].

Let a single-phase load be supplied with a distorted but periodic voltage $u(t)$. Fryze was not specific in [12] as to the load properties, however. We can only guess that it was composed of RLC elements and, as emphasized by the author, rotating switches. We do not know whether it was a linear load or not. Fortunately, it did not affect the presented reasoning.

Fryze's reasoning in [12] was oriented toward obtaining two decompositions. Decomposition of the load voltage $u(t)$, and another one, decomposition of the load current $i(t)$. These two goals interlace mutually, causing a loss of reasoning clarity. There is no major difference between

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these two reasonings; therefore, in this article, their draft is confined to only one of them, namely, the load current decomposition. This draft runs as follows.

A fictitious element can be extracted from the load under consideration, the element that would be responsible for the energy consumption of the whole load, with a unity power factor $\lambda = P/S$. Because of that, it must be a purely resistive element, with constant conductance, G. The conductance of that fictitious element was named an "***equivalent conductance***", G_e, of the load. Such a fictitious resistive element draws a current, named by Fryze an "***active current***", $i_a(t)$, from the supply source

$$i_a\,(t) = G_e\, u\,(t). \tag{1}$$

Such a resistive element consumes the whole energy delivered to the load at the supply voltage $u(t)$ when its conductance satisfies the relationship

$$G_e\,||u||^2 = P, \tag{2}$$

hence the equivalent conductance must be equal to

$$G_e = \frac{P}{||u||^2} \tag{3}$$

Since this fictitious resistive element consumes the whole energy delivered to the load, its current $i_a(t)$ is the only useful part of the load current. The residuum of the load current

$$i\,(t) - i_a\,(t) \stackrel{\text{df}}{=} i_{rF}\,(t) \tag{4}$$

is useless for energy transfer from the supply source to the load. It was named by Fryze a "***reactive current***". To differentiate this definition from the reactive current in other PTs, the index "*F*" emphasizes in this article that this is Fryze's definition. In such a way, the load current was decomposed into two components

$$i\,(t) = i_a\,(t) + i_{rF}\,(t). \tag{5}$$

Although the term "***scalar product***" of periodic quantities $x(t)$ and $y(t)$ with period T, and its symbol (x, y)

$$(x,y) \stackrel{\text{df}}{=} \frac{1}{T}\int_0^T x(\,t\,)\; y(\,t\,)\mathrm{d}t, \tag{6}$$

as well as the term "***orthogonal***" were not explicitly used in [12], Fryze proved that the scalar product of the active and reactive currents $(i_a,\, i_{rF})$ is equal to zero, so that these two current' components are mutually orthogonal. Consequently, their rms value satisfies the relationship

$$||\,i\,||^2 = ||\,i_a\,||^2 + ||\,i_{rF}\,||^2. \tag{7}$$

When this equation is multiplied by the square of the supply voltage rms value $||u||$, Fryze's power equation

$$S^2 = P^2 + Q_F^2 \tag{8}$$

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is obtained, with

$$S \overset{\text{df}}{=} ||\, u \,||||\, i \,||, \qquad P \overset{\text{df}}{=} ||\, u \,||||\, i_a \,||, \qquad Q_F \overset{\text{df}}{=} ||\, u \,||||\, i_{rF} \,||, \tag{9}$$

where Q_F is the reactive power according to Fryze's definition.

4. Impact of Fryze's approach upon power theory development

The simplicity of Fryze's approach and the concepts he introduced to PT have appealed to scientists involved in studies on power properties of electrical circuits with distorted voltages and currents. Fryze's PT, published originally in Polish, first affected studies on PT, continued by Rosenzweig [20] and Nowomiejski [21], his students and followers in Poland. Its publication in German [14] made this PT known internationally [16-19, 36, 40], and it affected the Standard DIN [22]. The active current has become a permanent component of power theories as developed by Rosenzwaig, Nowomiejski, Kusters & Moore [24], as well as by the Conservative PT (CPT) developed by Tenti [34]. It was also adopted in the Currents' Physical Components (CPC) - founded PT, developed by Czarnecki [29, 41], although a different current component was recognized as the active current of electrical loads. Moreover, it has become a component of the FBD (Fryze-Buchholz-Depenbrock) Method, developed by Depenbrock [31]. Moreover, Fryze's opinion that the time domain approach is superior to the frequency domain approach, permeated studies on PT development for decades. All of these justified the author's opinion, presented in [33], that "***nobody affected PT development to the degree comparable with Fryze***". Despite that, studies on power properties of electrical circuits did not end with the development of Fryze's PT. Studies on PT development were continued, and several new Schools of Power Theory were developed. This was because researchers in this area have concluded that Fryze's approach fell short in answering major questions and eventually failed. It has major deficiencies, explained below, that have degraded its value as the PT of electrical circuits. The concept of Fryze's reactive current $i_{rF}(t)$ was not as popular [25] as active current $i_a(t)$ and did not occur in PTs developed by Shepherd, Kusters, Tenti, or Czarnecki.

5. Cognitive deficiencies of Fryze's power theory

Power theory of electrical circuits is the subject of interest and has been developing for decades, for two main reasons. The first of them is purely cognitive, the second is very practical. Let's start with the first reason. We, as electrical engineers or scientists, would like to identify and understand physical phenomena or physical properties of electrical circuits that degrade the effectiveness of energy transfer from its sources to customer loads. Next, these physical phenomena or properties must be described in sufficiently strict mathematical terms. Fryze strongly emphasized the importance of this understanding in his debate with Staniewicz [13]. Unfortunately, this is only partially visible in his PT. The active current $i_a(t)$ is indeed associated

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with the physical phenomenon of permanent energy transfer from the supply source to the load. However, the only property of the reactive current $i_{rF}(t)$, used by Fryze for its separation from the load current, is its uselessness for this transfer. This is an important characteristic of reactive current $i_{rF}(t)$ but tells us nothing about physical phenomena in the load or about its properties for which this component occurs in the load's current. Fryze's PT does not explain what phenomena cause this current to be useless. The same conclusion applies to reactive power Q_F. Fryze's PT does not explain the causes for which it occurs in the power equation (8), other than the observation that apparent power S is higher than active power, P. We expect from power theories that they will explain to us the difference in values between active and apparent powers, P and S. Moreover, all PTs developed, as strongly suggested by Fryze, in the time domain, have failed to reveal physical phenomena responsible for power factor degradation. As to Kusters and Moore's PT [24], this was demonstrated in [27] and [29]. The same, for Akagi's Instantaneous Reactive Power *p-q* Theory [26], was done in [35]. For Tenti's Conservative PT (CPT) [34], it was demonstrated in [39].

The "***scattered***" and "***reactive***" components of the load current, associated with physical phenomena in the load, were identified [41] in the framework of the CPC-based PT, founded on the frequency domain approach. We know that scattered current, one of the useless components of the load current, occurs because of a change of the load conductance with harmonic order, n. However, the terms "***harmonic***" and "***harmonic order***" do not even belong to the vocabulary of the time domain. How to explain the physical meaning of that current without these terms? The same is true for reactive current, another useless component of the current. It is composed of current's harmonic components that are shifted with respect to voltage harmonics by $\pi/2$. How to explain its meaning without using "***harmonics***" and their phase-shift? Moreover, in circuits with nonlinear loads, or loads with time-varying parameters, the load current can contain [41] a component, named a "***load-generated current***". It is composed of current harmonics that transfer energy, permanently, from the load back to the supply source, where this energy is dissipated on the supply source's internal resistance. This is a physical phenomenon that can be identified and described in the frequency domain.

Functions specified in the time domain and the frequency domain are related by Fourier Transforms, namely

$$F(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,\mathrm{d}t, \tag{10}$$

$$f(t) = \int_{-\infty}^{\infty} F(j\omega)\,e^{j\omega t}\,d\omega, \tag{11}$$

thus, these domains are mutually equivalent, on the condition that these functions are continuous. Voltages and currents observed at real physical elements cannot have discontinuities; however, so there are no exceptions for the equivalency of description of real voltages and currents in these two domains. So, one could ask a question: does it mean that PT formulated in the frequency domain is equivalent to that in the time domain? Investigations and comparison of various power theories, as presented in [41], show that power properties of electrical circuits as described by power theories formulated in the frequency domain differ from those described in the time

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domain, however. For example, the reactive current, which in the CPC-based PT [40], is defined as a function of time in the frequency domain, meaning in terms of frequency properties of the load (B_n) and the supply voltage, ($\boldsymbol{U}_n$), namely

$$i_{rC}(t) \stackrel{\text{df}}{=} \sqrt{2}\text{Re}\sum_{n\in N} jB_n\,\boldsymbol{U}_n\,e^{j\,n\omega_1 t}. \tag{12}$$

This current can, using the Fourier Transform (10), be expressed as a function of frequency. It is not known, however, how it could be defined in the time domain, meaning, in terms of time-related properties of the load. The same applies to the scattered current, defined in the frequency domain as

$$i_{sC}(t) \stackrel{\text{df}}{=} (G_0 - G_e)U_0 + \sqrt{2}\text{Re}\sum_{n\in N}(G_n - G_e)\,\boldsymbol{U}_n e^{j\,n\omega_1 t}. \tag{13}$$

The load conductance for harmonic frequency, G_n, has no meaning in the time domain.

Although due to Fourier Transforms, the time domain and the frequency domain are mutually equivalent, we can conclude, observing the long history of PT development, and its formulations in both domains, that the time domain approach does not have the cognitive capacity of the frequency domain. Even Fryze's research suffered from this lower cognitive capacity of the time domain approach. Being aware that Budeanu's PT is erroneous, he was not able to prove that in the time domain.

6. Fryze's versus Budeanu's approach to PT

Prof. Fryze developed his PT as an opposing approach to the PT developed four years earlier, in 1927, by Prof. Budeanu [9]. For almost four decades, these two approaches have dominated studies on energy transfer in electrical circuits.

While Fryze's PT was formulated in the time domain, Budeanu developed it in the frequency domain, by defining the reactive power in terms of the load voltage and current harmonics, namely

$$Q_B \stackrel{\text{df}}{=} \sum_{n=1}^{\infty} U_n\,I_n \sin\varphi_n, \tag{14}$$

where U_n and I_n are the rms values of the n^{th} order voltage and current harmonics, while φ_n is the phase shift between them. Since

$$\sqrt{P^2 + Q_B^2} \;\le\; S. \tag{15}$$

Budeanu introduced a new power quantity to PT

$$D \;\stackrel{\text{df}}{=}\; \sqrt{S^2 - P^2 - Q_B^2}, \tag{16}$$

which he called a "***distortion power***". Thus, Budeanu's power equation has the form

$$S^2 = P^2 + Q_B^2 + D^2. \tag{17}$$

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Budeanu interpreted reactive power Q_B as a measure of the effect of the energy oscillations between the supply source and the load, on apparent power S increase, while distorted power D is interpreted as a measure of the similar effect of the supply voltage $u(t)$ and the load current $i(t)$ mutual distortion.

These interpretations have turned out to be erroneous, however. In paper [30], it was proven that energy oscillations can exist even at zero reactive power Q_B, thus there is no relationship between this power and energy oscillations. Also, it was proven in the same paper that the voltage and current waveforms can be mutually distorted even at zero distortion power D, or it can have a non-zero value in the lack of any mutual distortion between the load voltage and current waveforms. Thus, there is no relationship between the voltage and current mutual waveform distortion and distortion power D. Moreover, any other physical phenomena associated with Q_B and D powers were not revealed to this moment. Thus, there are no known physical phenomena in electrical circuits associated with Budeanu's reactive power Q_B and distortion power D. Physical interpretations of these two powers do not exist.

Fryze emphasized [12, 13] the importance of physical interpretations of energy transfer-related phenomena, however, he did not challenge Budeanu's interpretations of reactive and distortion powers Q_B and D. He was only concerned with the use of harmonics for their definition. Fryze concluded that harmonics do not exist physically, so they should not be used for describing such an important physical phenomenon as electric energy transfer. Even more, in paper [28], Fryze demonstrated that Budeanu's PT can be developed, as he suggested, starting at the level of the load current, not the apparent power decomposition, into current components associated with Budeanu's reactive and distortion powers, namely

$$i(t) = i_a(t) + i_{rB}(t) + i_D(t), \tag{18}$$

and that this can be done in the time domain, meaning without the use of harmonics. Unfortunately, this contributed to a validation of the entirely erroneous Budeanu's PT. There are no physical phenomena in electrical circuits associated with currents $i_{rB}(t)$ and $i_D(t)$, however. The impossibility of observing this major deficiency of Budeanu's PT was the price of rejection by Fryze of the frequency domain tools for energy transfer analysis and description. The falsification of Budeanu's PT, done by the author of this paper in [30], was obtained in the frequency domain. As to the author's of this paper best knowledge, such a falsification in the time domain is not known yet.

To show that Budeanu's reactive power is not associated with the phenomenon of energy oscillations, let us observe that the definition of this power (14) can be presented in the form

$$Q_B = \sum_{n=1}^{\infty} Q_n, \tag{19}$$

where Q_n is the reactive power of the n^{th} order harmonic

$$Q_n = U_n I_n \sin\varphi_n. \tag{20}$$

It is the general (positive or negative) amplitude of energy oscillations of the n^{th} order harmonic, i.e., with frequency $2nf_1$, where f_1 is the fundamental frequency. Maxima of these

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oscillations, $|Q_n|$, for different harmonics occur at different instances of time. Hence, the sum of these amplitudes (19), meaning reactive power Q_B, does not represent any physical phenomenon.

7. Fryze's PT and compensation

Originally, the research on methods of compensation at distorted supply voltage was confined to attempts to extend the method of compensation of reactive power Q, as used in systems with sinusoidal voltage and current, meaning to a capacitive compensation. The first results as to calculating the compensating capacitance, obtained in the frequency domain, were those of Shepherd and Zakikhani in 1972 [23], while those obtained in the time domain, by Kusters and Moore, in 1980 [24]. It occurred [27], however, that at a distorted supply voltage, a capacitor alone is not capable of compensating the whole reactive power. More complex reactance compensators are needed for that. The problem of the synthesis of such reactance compensators for circuits supplied with distorted voltage was eventually solved in [29].

In his approach to PT development, Fryze did not mention compensation issues, however. Unfortunately, deficiencies of Fryze's decomposition of the load current, discussed in the previous section, made the development of methods of reactance compensation impossible. It was because physical phenomena that were responsible for the presence of the reactive component $i_{rF}(t)$ in the load current were not revealed. Its definition (4), formulated in the time domain, does not provide any starting point for a reactance compensator synthesis. Such compensators can be synthesized, as shown in [29, 41], in the frequency domain. Their synthesis in the time domain, to the best of the author's knowledge, is not yet developed.

The author's opinion that Fryze's PT does not provide fundamentals for reactance compensator synthesis was supported in the paper [29], with a numerical illustration. Because it was published in Polish, that illustration is repeated below.

Let two loads, shown in Fig. 1 are supplied with the same distorted voltage

$$u = 1\,00\sqrt{2}(\sin\omega_1 t + \sin 3\omega_1 t)\ \text{V}, \quad \omega_1 = 1\,\frac{\text{rad}}{s}.$$

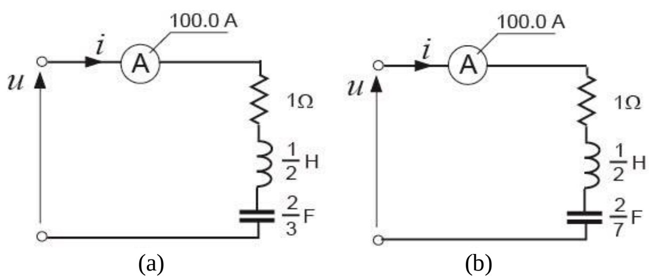

Fig. 1. Two single-phase loads, identical in terms of Fryze's PT

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These two loads are identical in terms of Fryze's powers, since these powers have the same value, namely

$$P = 10.0\ \text{kW}, \qquad Q_F = 10.0\ \text{kvar}, \qquad S = 4.0\ \text{kVA},$$

so that both loads have the same power factor

$$\lambda = \frac{P}{S} = 0.71.$$

The reactive power Q_F of the load shown in Fig. 1(a), can be compensated entirely by a reactance compensator, shown in Fig. 2(a), so that the PF can be improved to $\lambda = 1$, while this power of the second load cannot be compensated by any reactance compensator below value $Q_F = 8.0$ kvar. This compensator is shown in Fig. 2(b). The PF of this load cannot be improved above $\lambda = 0.79$. Unfortunately, this difference cannot be explained in the framework of Fryze's PT. Moreover, this theory does not provide the fundamentals for a reactance compensator synthesis and design. Thus, Fryze's PT has failed as a tool for reactance compensators synthesis.

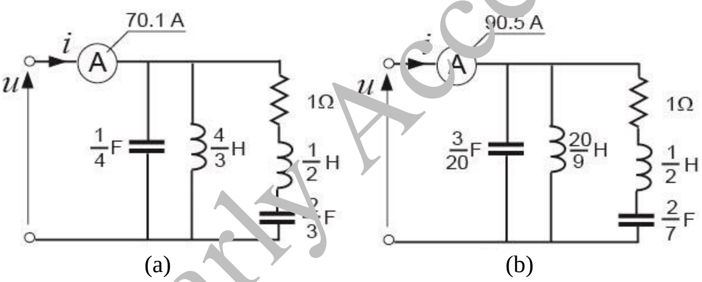

Fig. 2. Two single-phase loads with reactance compensators

The PF in electrical circuits can be improved, however, not only by a reactance compensator, but also by a controlled shunt current source that injects a compensating current $j(t)$ into the circuit. The basic structure of a single-phase circuit with such a compensator is shown in Fig. 3.

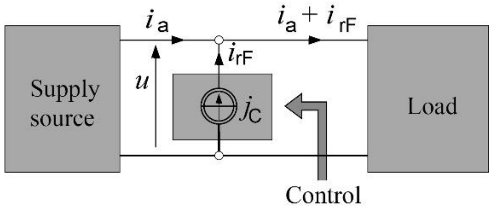

Fig. 3. A load with a controlled current source $j(t)$

In literature, for example [38], such compensators are referred to as "***active power filters***" and are used mainly in three-phase applications. These are not active, but passive devices,

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however, because the compensating current $j(t)$ is orthogonal to the load voltage $u(t)$, since their scalar product

$$(u, j) = (u,\, i_{rF}) = 0. \tag{21}$$

Moreover, they do not operate as filters, but as compensators. The compensating current $j(t)$ of such a compensator is shaped by transistors switched ON-OFF, with a frequency of several kHz. Therefore, in [41], such devices were referred to as "***switching compensators***".

According to the opinion in [38], the current decomposition, as suggested by Fryze, is a straightforward recipe for such a compensator control. Just separate, according to (4), the reactive current $i_{rF}(t)$ from the load current, and use it for the control of the current source $j(t)$, so after that, only the active current $i_a(t)$ will remain in the supply current $i'(t)$, thus, the compensated load will operate at unity PF. This conclusion was challenged in paper [29] as incorrect, however, and this opinion was supported in that paper by the following numerical illustration. Observe what will happen in the circuit shown in Fig. 4. Let us assume that a harmonics-generating load is supplied with a sinusoidal voltage

$$e_1 = 100\sqrt{2}\sin\omega_1 t \quad \text{V},$$

while the load, at such a supply, generates the 3rd order current harmonic

$$j_3 = 50\sqrt{2}\sin 3\omega_1 t \quad \text{A}.$$

The active power in the cross-section *x*-*x* is $P = 0$, so that there is no active current in this circuit, meaning

$$i_a(t) \equiv 0.$$

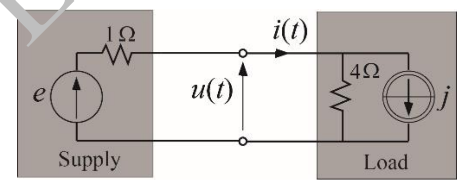

Fig. 4. Example of a circuit with a harmonics-generating load

Therefore, there is only Fryze's reactive current in that cross-section

$$i_{rF} = i = 20\sqrt{2}\sin\omega_1 t + 40\sqrt{2}\sin 3\omega_1 t \quad \text{A}.$$

When this current is used for controlling the current source shown in Fig. 3, then the active current would remain zero, so that after compensation, there would be no flow of energy from the supply source to the load. This is not expected from compensation, however.

In his paper [12], Fryze assumed that the load can contain periodic switches, meaning, harmonics-generating elements. His theory does not reveal, however, that in the presence of

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harmonic-generating loads, there is a phenomenon of two permanent flows of energy (i) one from the supply source to the load and the second, (ii) from the load to the source. Both these flows contribute to the active current, defined by Fryze, but only the first one (i) is useful. The remaining one (ii) only increases the supply current rms value, so it should be compensated. Therefore, in the frame of CPC-based PT, the active current is not defined according to Fryze's power theory.

To keep energy flow, the compensator should eliminate the current harmonic generated in the load $j_3(t)$ instead. To do this, the compensator should detect the current $j_3(t)$, but not Fryze's reactive current $i_{rF}(t)$, and use it for the compensator control. Only when the supply source is ideal, meaning with zero internal impedance, so in the circuit under consideration

$$i_{rF} = j_3 = 50\sqrt{2}\sin 3\omega_1 t \quad \text{A}.$$

For a real source, meaning with non-zero internal impedance, this formula would be satisfied only approximately. Thus, Fryze's PT provides indeed a tool for only approximate compensation with switching compensators. Nonetheless, such an approximation could be satisfactory for practical applications. We do not usually expect total compensation, but only a sufficient reduction of unwanted currents.

8. Fryze's PT and harmonics

Most of the deficiencies of Fryze's PT resulted from his attitude to the concept of harmonics. He was aware that a current in a conductor with a periodic ON-OFF switch is distorted and can be described in terms of harmonics. He rightly concluded that a current harmonic, as a continuous quantity, cannot flow in an open conductor, meaning at the switch in the OFF state, however. This means that a harmonic cannot exist as a physical quantity. It is only a mathematical entity. Fryze was right at this point, but science cannot exist without such mathematical entities. Complex numbers are among the most visible mathematical entities that do not exist physically, and despite that, Fryze used them extensively.

Instead of the frequency domain, founded in circuits with nonsinusoidal voltages and currents, on the concept of harmonics, Fryze strongly advocated the description of energy transfer phenomena in the time domain. There are physical phenomena in electrical circuits

that are observable in the frequency domain. These are, as revealed in terms of CPC [41], the phase shift between the voltage and current harmonics, change of the load conductance with harmonic order, generation of current harmonics in the load in the presence of non-linearity or periodically switched elements. Such load-generated harmonics can cause bi-directional flow of energy; not only from the supply source to the load, but also from the load back to the supply source. Rejecting the concept of harmonics, Fryze failed to reveal any of these three phenomena.

Reactance compensators in circuits with distorted voltages and currents are specified in terms of the load susceptance for harmonic frequencies. When a power theory does not use the concept

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of harmonics, these susceptances in the frame of such a theory are not specified, so reactance compensators cannot be synthesized using Fryze's PT. The same is true for all PTs formulated in the time domain.

9. The extension of Fryze's power theory to three-phase circuits

In his research on PT, as presented in [12, 14], Fryze was focused on single-phase circuits, but in the paper [28], published posthumously in 1985, he extended his PT to three-phase and even to multi-phase circuits.

It was known from Steinmetz's research, published [3] in 1917 and Lyon's research, published [4] in 1920, that load imbalance degrades the PF, λ. Steinmetz even invented a reactance balancing compensator, known as the "***Steinmetz circuit***," that reduces detrimental effects of load imbalance upon the supply source and can improve PF to unity.

In paper [28], Fryze extended his PT of single-phase circuits to three-phase circuits. He applied his concept of the load current decomposition to individual phases *R*, *S*, and *T* of three-phase circuits. Unfortunately, this phase-by-phase approach did not reveal the load imbalance as the load property that degrades the power factor of three-phase loads. At such an approach, as presented in paper [28], a purely resistive load has phase power factors, λ_R, λ_S, and λ_T equal to one, while the power factor λ of the whole load can be lower than one. In Fryze's PT, the true value of the power factor of three-phase loads remains unknown.

Three-phase circuits are sometimes regarded as aggregates of three separate single-phase circuits, so there is a tendency, as demonstrated in [28], for deriving power properties of the resultant three-phase circuit from the properties of single-phase circuits. The aggregated three-phase circuit loses, however, some power properties of single-phase circuits. This is because it is not possible to find in three-phase circuits any energetically confined space to which electric energy is delivered by an individual, single supply line, as is the case in single-phase circuits. The instantaneous power $p(t)$ of a single-phase circuit is defined as

$$p(t) \stackrel{\mathrm{df}}{=} \frac{\mathrm{d}\,W(t)}{\mathrm{d}t} = u(t)\,i(t), \tag{22}$$

where $W(t)$ is the electric energy delivered to the load. Because in three-phase circuits, such a space to which energy is delivered by a single line cannot be specified, the voltage-current product, say for *R* line

$$u_R\,(t)\,i_R\,(t) \neq p_R\,(t) \tag{23}$$

is not instantaneous power. Moreover, any single point in three-phase circuits can be grounded and serve as a reference for the voltage definition and measurement. Therefore, individual *R*, *S*, or *T* lines do not have $p(t)$, *P*, *S*, or Q_F powers. Their value can change with the change of the grounding point. Consequently, powers and the power factors of individual lines λ_R, λ_S, or λ_T do not have any meaning. These powers and the power factor can be attributed only to the entire

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three-phase loads. Consequently, Fryze's PT does not identify load imbalance as the cause of the power factor degradation, which is known from Lyon and Steinmetz papers [4] and [3], thus it describes power properties of three-phase circuits erroneously.

10. Conclusions

The power theory of circuits with distorted voltages and currents developed by Fryze and his approach to its development had major implications for studies on energy transfer in electrical circuits. Several concepts he introduced have become permanent components of the research towards PT development. Unfortunately, some of his opinions had detrimental effects on this research. The most important of them was his attitude towards harmonics. Fryze concluded that harmonics are not physical entities and should not be used for describing energy transfer, which is a physical phenomenon. According to Fryze, such a phenomenon should be described only in the time domain, meaning without any use of harmonics. The frequency domain, which is based on the concept of harmonics, is not acceptable for that. Unfortunately, none of the followers of that opinion has revealed physical phenomena or physical features of the circuit that are responsible for the increase of the apparent power S above the active power P value. Moreover, none of them has created fundamentals for the design of reactance compensators.

The history of the PT development shows that the time domain approach has proved to be less effective for studies on power properties of electrical circuits than the frequency domain approach. Just the passionate rejection by Fryze of the frequency domain caused his approach to power theory development eventually to fail. He rejected the use of harmonics because they do not exist as physical entities. Fryze was against using the concept of harmonics, at the same time, using complex numbers for circuits analysis and energy transfer description, although such numbers are, like harmonics, only mathematical but not physical entities. Thus, his opinion against using harmonics was not well justified.

All scientists who worked on the power theory development, including Fryze, attempted to respond to the cognitive question: "***What are the physical reasons beyond the inequality of the apparent and active powers?***" According to the Currents' Physical Components-based PT [41], there are six such reasons in three-phase loads, while three in single-phase ones. Unfortunately, Fryze's PT did not reveal even a single one of them. Thus, Fryze's PT has failed to identify the physical phenomena that degrade the effectiveness of the energy transfer in electrical circuits. Consequently, it failed to create fundamentals for reactance compensation. The load current decomposition into useful and useless components provides only a very shallow insight into the energy transfer properties of electrical circuits, not sufficient to create fundamentals for reactance compensator synthesis. It could be satisfactory if one needs to comprehend the power properties of such circuits only superficially. It could be, however, sufficient for compensating the useless current only by means of a switching, but not by a reactance compensator. In the case of three-phase circuits, his approach has failed because he attempted to describe the power properties of such circuits by the power properties of single-phase circuits.

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