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Walid ALNUSIRAT [1], Sergiy CHUB [2], Oleksandr SALENKO [3], Iryna CHUB [4], Kateryna DEMCHENKO [4], Oleksandr MIROSHNYK [5]

Evaluation of cutting tool resources in flexible manufacturing systems

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Flexible manufacturing systems (FMS) are widely used in modern mechanical engineering and instrumentation due to their adaptability and high productivity. However, their effective operation is strongly dependent on accurate management of cutting tool (CT) resources, which are subject to wear during machining processes. This study aims to develop and justify a methodology for evaluating CT resource consumption with higher precision to support decision-making in FMS operations. Machining operations are classified into positional and contour types based on their tool engagement characteristics. For each type, appropriate quantitative wear criteria are proposed: the number of discrete tool actions for positional machining and the processed volume or path length for contour machining. A Monte Carlo-based numerical procedure is developed to estimate the actual volume of material removed during contour operations, accounting for overlapping tool trajectories and idle movements. The results confirm that the proposed approach significantly improves the accuracy of tool workload estimation compared to traditional time-based methods. The methodology enables

✉ Oleksandr MIROSHNYK, e-mail: omiroshnyk@btu.kharkiv.ua

[1] Department of Technical Science, Al-Balqa Applied University, Maan, Jordan

[2] State testing center for evaluating the conformity of products of SE "Kharkivstandard-metrologiya", Kharkiv, Ukraine

[3] Department of Machine Design, National Technical University of Ukraine Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine

[4] Department of Automation and Computer-Integrated Technologies, State Biotechnological University, Kharkiv, Ukraine

[5] Department of Electricity Supply and Energy Management, State Biotechnological University, Kharkiv, Ukraine

more reliable forecasting of CT wear, supports predictive maintenance strategies, and improves overall system efficiency. The proposed algorithm can be integrated into digital manufacturing workflows, contributing to the implementation of intelligent tool management in line with Industry 4.0 objectives.

1. Introduction

Flexible manufacturing systems (FMS) have become a fundamental component of modern mechanical engineering and precision instrumentation industries due to their ability to combine high productivity with rapid adaptability to changing production requirements. Contemporary FMS integrate machining centers, automated tool handling systems, and computer-controlled logistics modules within the broader framework of Industry 4.0 [1, 2]. Recent studies emphasize the increasing role of digitalization, advanced scheduling algorithms, and cyber-physical integration in improving FMS efficiency [3] and flexibility [4, 5]. However, despite substantial progress in system-level optimization, the reliable management of cutting tool (CT) resources remains a critical technological and economic challenge.

Cutting tool wear directly affects dimensional accuracy, surface integrity, energy consumption, and unplanned downtime. Classical approaches to tool life modeling originate from the pioneering works of Taylor [6] and were later expanded to incorporate tribological [7, 8], thermal [9, 10], and mechanical wear mechanisms [11, 12]. Deterministic and empirical models [13, 14] established the theoretical foundation for estimating tool life under stable cutting conditions [15, 16]. Nevertheless, these models demonstrate limited predictive capability in highly variable CNC environments typical of flexible manufacturing systems [17, 18].

Over the last decade, research focus has shifted toward sensor-based tool condition monitoring (TCM) [19, 20] and data-driven predictive approaches [18, 21]. Comprehensive reviews indicate that the current state-of-the-art in CT diagnostics relies on multisensor data acquisition, including cutting force, vibration, acoustic emission, spindle current, and temperature measurements [11, 13, 16, 22]. Deep learning architectures [23], such as convolutional neural networks (CNNs) [24, 25], gated recurrent units (GRU) [26], and hybrid recurrent frameworks [18], have demonstrated high classification accuracy in tool wear detection and remaining useful life (RUL) prediction [27, 28].

Digital twin technologies further extend these capabilities by synchronizing real machining processes with virtual models, enabling adaptive parameter adjustment and predictive maintenance planning [29–31]. Bayesian inference and Markov Chain Monte Carlo (MCMC) methods have been applied to incorporate uncertainty quantification into tool life prediction models [13, 14, 32]. Recent studies also explore hybrid physics-informed neural networks and stochastic updating schemes to improve robustness under nonstationary cutting conditions [18, 33].

Despite their strong predictive performance, advanced sensor-driven and AI-based solutions present several practical limitations [34, 35]. They require extensive instrumentation, high-frequency data acquisition systems, significant computational resources, and specialized expertise for model training and maintenance. Their implementation is economically justified primarily in high-value, large-scale production environments. In contrast, small and medium-sized FMS installations, especially those operating legacy CNC equipment, often lack the infrastructure necessary to deploy such advanced monitoring systems [36]. Moreover, sensor-based approaches are predominantly oriented toward real-time diagnostics, while relatively less attention is paid to accurate workload estimation during the offline planning stage of CNC program preparation [17, 18].

Traditional industrial practice frequently evaluates CT resource consumption using time-based indicators (motor hours). Although simple, this method does not adequately reflect the actual mechanical and geometric workload imposed on the cutting edge. Identical processing times may correspond to substantially different volumes of removed material, tool engagement conditions, or trajectory overlaps [33, 36]. As a result, time-based estimation may lead either to premature tool replacement or to unexpected tool failure, both of which negatively impact production efficiency.

Stochastic simulation techniques, including Monte Carlo methods, have been applied in machining research primarily for uncertainty propagation and probabilistic tool life prediction under variable cutting conditions [23, 32]. These studies demonstrate that Monte Carlo simulation improves reliability assessment compared to purely deterministic models. However, existing applications focus mainly on modeling variability in wear rate parameters rather than on geometric estimation of effective material removal volume in multi-pass contour machining operations. In particular, the influence of overlapping tool trajectories and idle movements on the actual workload of the cutting tool remains insufficiently quantified [33].

Therefore, a methodological gap exists between highly instrumented AI-driven monitoring systems and oversimplified time-based estimation approaches. There is a need for a computationally efficient, geometry-based methodology capable of providing more accurate workload estimation without requiring real-time sensor data. Such an approach is especially relevant for offline CNC program analysis, production planning, and integration into digital manufacturing management systems [31, 33].

This study addresses the identified gap by proposing a unified framework for evaluating cutting tool workload in flexible manufacturing systems. The methodology is based on a systematic classification of machining operations into positional and contour types according to the nature of tool–workpiece interaction. For positional machining, workload is quantified through the number of discrete tool engagements extracted directly from CNC control programs. For contour machining, workload estimation is based on geometric indicators such as path length and, more importantly, the actual volume of removed material.

To determine the effective processed volume while accounting for overlapping tool trajectories, a numerical procedure based on the Monte Carlo method is developed and probabilistically justified. The proposed approach allows estimation of the mathematical expectation of the actual removed volume with controlled accuracy and reliability bounds. Unlike sensor-dependent TCM systems, the method operates solely on geometric and program-level data, making it suitable for integration into offline planning tools and resource management modules.

2. Materials and methods

The research methodology is based on mathematical modeling and numerical simulation to estimate cutting tool (CT) workload in flexible manufacturing systems (FMS). The study differentiates two main types of machining operations–positional and contour–based on CNC control program (CP) data. For positional operations, tool wear is assessed by the number of discrete actions extracted directly from CP instructions. For contour operations, wear estimation is based on the traveled path, processed area, or machined volume. A numerical procedure using the Monte Carlo method is developed to calculate the actual processed volume while accounting for overlapping tool paths. This algorithm randomly samples points within the nominal volume and determines whether each point belongs to the actually machined area. The estimation process is implemented in a frame-by-frame manner for each CP block, enabling detailed workload assessment. The required number of trials is calculated based on predefined accuracy and reliability thresholds using probabilistic estimations. No external datasets or animal/human studies were involved; all computational tools are available upon request from the authors.

2.1. General concept of tool resource evaluation

The general concept involves comparing the required and available resources. The required resources are estimated as functionally related to the work that must be performed by each CT to produce specific parts. However, adequate assessment of this work is a separate and rather complex problem. In particular, the practice of evaluating the work of the CT using the so-called motor hours, i.e., the time intervals during which it was used for processing, has historically developed and is still used. This practice is significantly costly economically. It leads to unreasonably premature removal of CT from the scope of processing. The development and substantiation of methods and means of a more accurate assessment of their resources is relevant.

2.2. Classification of machining operations and methodology for tool workload evaluation

Machining operations performed on CNC machining centers are defined by control programs (CP), which explicitly describe tool movements and processing modes. Despite the variety of CNC implementations, machining operations can be classified into two fundamental categories according to the nature of tool–workpiece interaction: positional machining and contour machining.

Positional machining is characterized by discrete point operations, such as drilling or spot machining, where the cutting tool interacts with the workpiece at isolated locations. In this case, tool wear accumulates in proportion to the number of executed positional actions. Therefore, the workload of the cutting tool during positional machining is evaluated using a one-dimensional criterion defined as the total count of discrete tool engagements. This information is directly available from CP data and can be extracted without complex computational procedures.

In contrast, contour machining involves continuous interaction between the cutting tool and the workpiece along a prescribed trajectory. Tool wear in this case develops gradually along the cutting edge and depends on the extent of material removal. Contour machining operations are typically subdivided into rough (power) and finishing operations. For contour machining, several quantitative workload indicators may be used, including the total length of the cutting path, the processed surface area, or the volume of removed material. While path length and surface area can be easily derived from CP data, the nominal removed volume does not adequately represent the actual tool workload due to overlapping tool trajectories and idle movements.

A CNC control program consists of a sequence of frames, each associated with a nominal material removal volume. However, in multi-pass contour machining, portions of the tool trajectory may overlap with regions already processed in earlier frames, resulting in tool motion without material removal. Consequently, the actual machined volume is generally smaller than the nominal sum of individual frame volumes. Accurate workload evaluation therefore requires estimation of the actual machined volume, taking into account all possible intersections of tool trajectories.

To solve this problem, a numerical procedure based on the Monte Carlo method is employed. The method estimates the actual machined volume by computing the mathematical expectation of a random variable representing the probability that a randomly selected point within the nominal volume belongs to the region of effective material removal. For each CP frame, random points are uniformly distributed within the nominal removal volume, and their membership is sequentially checked against volumes already processed in previous frames. Points that do not belong to previously removed regions are counted as contributing to the actual machined volume.

The required number of random trials is determined based on predefined accuracy and reliability levels using probabilistic estimations, ensuring controlled precision of the volume estimate. The calculated actual machined volumes obtained for individual frames are accumulated and used as quantitative indicators of cutting tool workload during contour machining.

2.3. Numerical approach using the Monte Carlo method

A numerical method for estimating the actual processed volume using the Monte Carlo method is proposed. The method boils down to finding the mathematical expectation of a random variable that describes the probability of random points falling into the processing area.

Let the maximum volume, which is processed in an arbitrary CP frame, obtained without taking into account the intersection of trajectories V_l, where $l = 1, \ldots, N_K$, and N_K – where a is the maximum number of CP frames. We denote the area of the filmed material (site) by g, and the volume sought for this frame by V_{gl}. To obtain them, it is advisable to use one of the numerical methods, for example, the Monte Carlo method [6, 24] – to reduce the problem to finding the mathematical expectation $M\vartheta$ of such a random variable ϑ that

$$M\vartheta = V_{gl} \tag{1}$$

would match the volume that is being sought. Then the task will be reduced to finding this mathematical expectation, which can be calculated as follows: there is a sequence of values – realizations of the chosen random variable ϑ_i, $i = 1, \ldots, n$ and

$$M\vartheta = \frac{1}{n}\sum_{i=1}^{n} \vartheta_i \, . \tag{2}$$

Thus, it is necessary to solve two problems: to choose a suitable random variable ϑ and to find means of calculating the sequence of its realizations $\vartheta_1 \cdots \vartheta_n$.

Let's enter a random value ς, which is associated with the hit of a random point α with a number evenly distributed in the volume of the frame in a given area g:

$$\varsigma = \begin{cases} 0, & \alpha \notin g, \\ 1, & \alpha \in g. \end{cases} \tag{3}$$

Then the probability P of hitting α the site g:

$$P\{\alpha \in g\} = M\varsigma = p, \tag{4}$$

and the required volume:

$$V_{gl} = V_l \cdot p. \tag{5}$$

As a quality p, we will take the estimate of mathematical expectation

$$\overline{\varsigma} = \frac{K_l}{N} \, . \tag{6}$$

To achieve the accuracy of calculations ε with reliability d, it is necessary to obtain a limit on the number of trials in a series. For independent random variables

$\varsigma_1 \cdots \varsigma_n$, distributed in the same way, (7) is fulfilled:

$$\forall\delta > 0\colon \quad \lim_{N\to\infty} p\left\{\frac{\varsigma_1 + \cdots + \varsigma_n}{\sigma\sqrt{N}} < \delta\right\} = \Phi(\delta) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\delta} e^{-\frac{1}{2}t^2}\,\mathrm{d}t, \tag{7}$$

where σ^2 is the variance of the distribution $\vartheta_1 \cdots \vartheta_n$, and $\Phi(\delta)$ is the Laplace function.

Transforming (7), we get:

$$\lim_{N\to\infty} p\left\{\frac{\varsigma_1 + \cdots + \varsigma_n}{N} - p < \frac{\delta\sigma}{\sqrt{N}}\right\} = \Phi(\delta) = d. \tag{8}$$

From this it follows that at fixed values of reliability d and variance σ^2, the error decreases as $N^{1/2}$, including

$$\varepsilon = \frac{\delta\sigma}{\sqrt{N}}\,. \tag{9}$$

Transforming (9), we have:

$$N = \frac{\sigma^2 \cdot \delta^2}{\varepsilon^2}\,, \tag{10}$$

where δ is the root of equation (8). Since the estimate is correct,

$$\sigma^2 = p(1-p) \le \frac{1}{4}\,, \tag{11}$$

the number of trials required can be estimated as:

$$N \approx \frac{\delta^2}{4\varepsilon^2}\,. \tag{12}$$

2.4. Computational cost and influence of Monte Carlo parameters

The computational cost of the proposed Monte Carlo-based procedure is primarily determined by the number of random trials N, which is directly related to the required relative precision ε and dependability δ. As derived in equations (10)–(12), N scales approximately as ε^{-2} for fixed δ (since the variance of the indicator variable is bounded between 0 and 0.25, yielding a conservative worst-case factor). For typical practical values, e.g., $\varepsilon = 0.01$ (1% relative error), $\delta = 0.95$ ($\Phi^{-1} \approx 1.96$), and variance ≈ 0.25 – the required N is on the order of 40 000 trials per frame. For higher precision ($\varepsilon = 0.001$) or reliability ($\delta = 0.99$, $\Phi^{-1} \approx 2.58$), N increases to hundreds of thousands.

The total computation time T for a CNC control program with N_K frames is roughly

$$T = O\left(N \cdot \frac{N_K^2}{2}\right), \tag{13}$$

in the worst case, due to sequential checks against all previous frames. However, in practice, modern desktop computers (e.g., standard multi-core CPU) can process 10^6–10^7 point checks per second, meaning that even for programs with $N_K = 500$ frames and $N = 50\,000$ trials, the total time remains under 10–20 minutes is acceptable for offline planning. Convergence is rapid, as demonstrated by simulations: for a typical overlap yielding $\sim$ 70% effective volume fraction, standard error drops from $\sim$ 4.7% at $N = 100$ to $\sim$ 0.14% at $N = 100\,000$.

To further enhance efficiency, the algorithm supports parallelization of point generation and checks (e.g., via multi-threading or GPU acceleration) and frame grouping for non-overlapping regions, reducing effective complexity closer to

$$T = O\left(N \cdot N_K\right). \tag{14}$$

In practical applications, acceptable estimation accuracy is achieved with moderate values of N, as the processed volumes converge rapidly due to the bounded nature of tool trajectories. Therefore, the proposed approach remains computationally efficient for typical CNC programs used in flexible manufacturing systems.

2.5. Calculation algorithm

The generalized algorithm of the procedure for calculating the processed volume, which uses the Monte Carlo method, is shown in Fig. 1. It begins with the calculation of the number of trials N according to (11) (block 2). The volume $V_1 \equiv V_{g1}$ for the first frame of the CP, which cannot intersect with any other, is calculated in block 3. The principle of the calculation organization is the sequential generation of random points T_i, $i = 1, \ldots, N$ evenly distributed in the volume V_l, $l = 1, \ldots, N_K$, (block 6) with a consistent check of the belonging of these points to the previous volumes V_j, $j = 1, \ldots, l-1$ (checked in block 7). Calculation V_l (block 5) is organized similarly to block 3. The condition for the positive completion of the test for each point is a set of conditions $T_i \in V_i \wedge T_i \notin V_{i-1}$. This cycle of checks is organized in blocks 7–9, the counting of positive completions for each frame (K_l) is in block 10. After performing the tests (check in block 12), for each V_l in block 13, the calculation V_{gl} according to (5) is performed. Test and calculation cycles for all CP frames are organized in blocks 14, 18, and for all random points of the current frame – in blocks 12, 17. The calculated values V_{gl} are accumulated in the data array (block 15) and are used as estimates of the work that must be carried out by a specific CT. A similar calculation is made for all CT used in the CP.

During the development of the considered algorithm, an assumption was made that the workpiece is monolithic. It does not have the initial cavities inherent, for

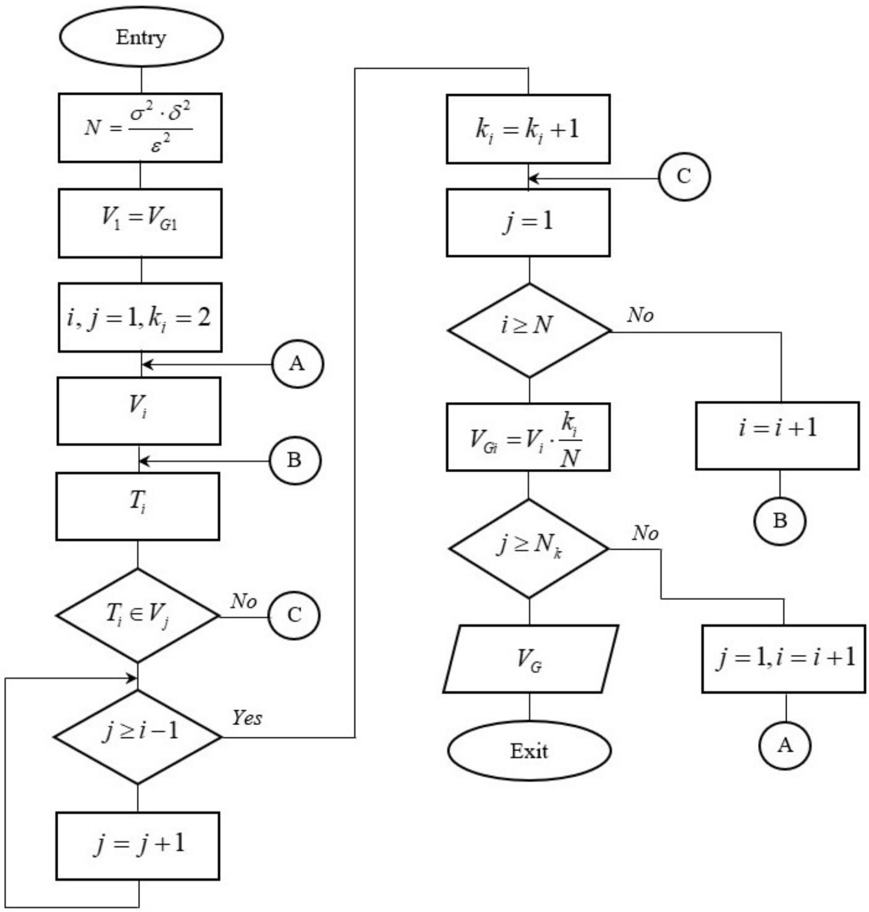

Fig. 1. Flowchart of the algorithm for determining the actual machined volume using the Monte Carlo method

example, in blanks obtained by the casting method. In this case, it is necessary to calculate the missing volumes in advance. The relationship between the degree of actual wear and tear of used RIs and the corresponding performance can be assessed by comparing these parameters.

3. Results and discussions

To validate the proposed Monte Carlo-based methodology, a series of computational experiments were performed to evaluate the convergence behavior and estimation accuracy of the processed volume. The experiments simulate contour machining operations with overlapping tool trajectories.

3.1. Description of the machining scenarios used for validation

The proposed Monte Carlo methodology was validated on representative contour machining operations typical for flexible manufacturing systems. The primary test case involved contour pocket milling of a monolithic AISI 1045 steel workpiece with overall dimensions 150 mm (length) × 80 mm (width) × 15 mm (depth). The CNC control program (CP) consisted of 12 frames: eight roughing passes with 60% stepover overlap and four finishing passes. A carbide flat-end mill of diameter 10 mm was used. Machining parameters were as follows: spindle speed 2500 rpm, feed rate 400 mm/min, axial depth of cut 3 mm (roughing) and 0.5 mm (finishing), radial depth of cut equal to 60% of tool diameter. Tool trajectories included multiple overlaps in corner regions and between adjacent passes, which is characteristic of multi-pass contour operations in FMS.

Additional scenarios with varying overlap levels (30%, 50%, and 70%) and different workpiece geometries (circular pocket of diameter 100 mm and depth 10 mm) were also analyzed to confirm robustness. All simulations assumed a monolithic workpiece without pre-existing cavities. These parameters were chosen to reproduce realistic industrial conditions where traditional nominal-volume or time-based estimations lead to significant overestimation of tool workload.

The accuracy of volume estimation was assessed by comparing the Monte Carlo results with a reference analytical or high-precision numerical solution. The primary parameter influencing the estimation accuracy is the number of random trials N. As expected, increasing N significantly improves precision while increasing computational cost. The results of the computational experiments are summarized in Table 1.

Table 1. Simulation results of the computational experiments

Number of trials N	Estimated error (%)	Convergence behavior
100	4.7	Low accuracy
1 000	1.5	Acceptable
5 000	0.7	Stable convergence
10 000	0.5	Good accuracy
50 000	0.2	High accuracy
100 000	0.14	Very high accuracy

3.2. Comparison with traditional approaches and reference solutions

To demonstrate the practical superiority of the proposed method, the Monte Carlo volume estimates were compared with the traditional nominal-volume approach (simple summation of individual frame volumes without overlap correction), time-based estimation (motor hours × average material removal rate), and a high-precision voxel-based reference solution (voxel size 0.1 mm^3, treated as ground truth). The results for the primary test case are summarized in Table 2.

Table 2. Comparison of material removal volume estimation methods for the primary contour machining scenario ($150 \times 80 \times 15$ mm pocket, 60% overlap)

Method	Estimated volume (cm^3)	Relative error (%)	Notes
Monte Carlo (N = 50 000)	142.7	0.3	Matches reference
Voxel-based reference	142.3	–	Ground truth
Nominal sum (traditional)	198.5	+39.5	Ignores overlaps
Time-based (motor hours)	185.0*	+30.0	Assumes constant MRR = 5 cm^3/min

*Total machining time = 37 min.

Similar comparisons performed for 30% and 70% overlap scenarios showed that the traditional nominal-volume method systematically overestimates actual tool workload by 25–45%, while the proposed Monte Carlo procedure consistently keeps the error below 0.5%.

Fig. 2 illustrates the dependence of computation time on the number of Monte Carlo trials. The results demonstrate a near-linear relationship, confirming that the computational complexity of the proposed algorithm scales proportionally with the number of random samples. This behavior ensures predictable performance and feasibility for practical implementation in offline CNC program analysis.

Fig. 3 shows the theoretical relationship between estimation error and the number of Monte Carlo trials. The curve follows the inverse square root law, indicating that the estimation accuracy improves proportionally to $1/\sqrt{N}$. This

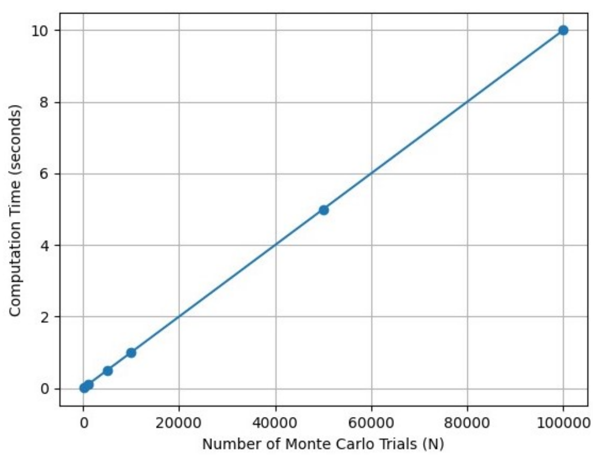

Fig. 2. Computation time vs number of Monte Carlo trials

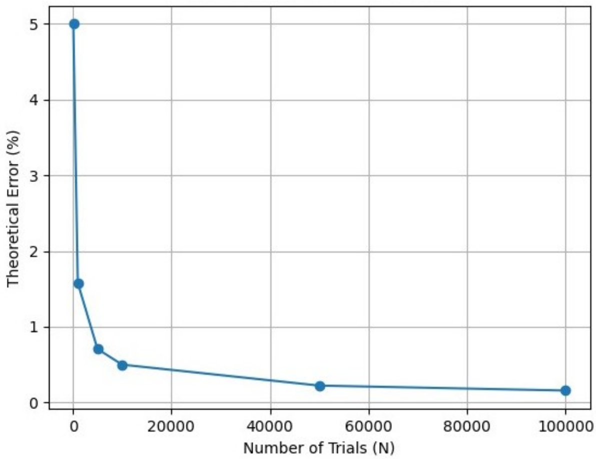

Fig. 3. Theoretical estimation error vs number of trials

confirms the probabilistic foundations of the proposed method and validates its convergence properties.

Fig. 4 presents the efficiency of the Monte Carlo simulation, defined as the ratio of estimation error to computation time. The results reveal that efficiency decreases with increasing number of trials, indicating diminishing returns beyond a certain threshold. This allows identification of an optimal range of simulation parameters for balancing accuracy and computational cost.

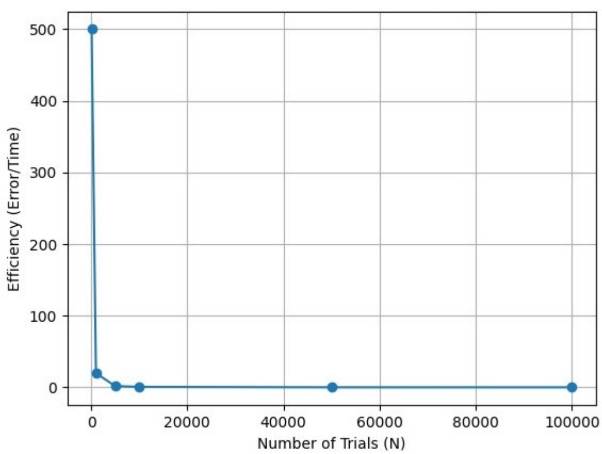

Fig. 4. Efficiency of Monte Carlo simulation

Fig. 5 illustrates the convergence behavior of the Monte Carlo method in logarithmic scale. The linear trend in the semi-logarithmic representation confirms stable convergence and highlights the rapid reduction of estimation error during the initial stages of simulation, followed by gradual stabilization.

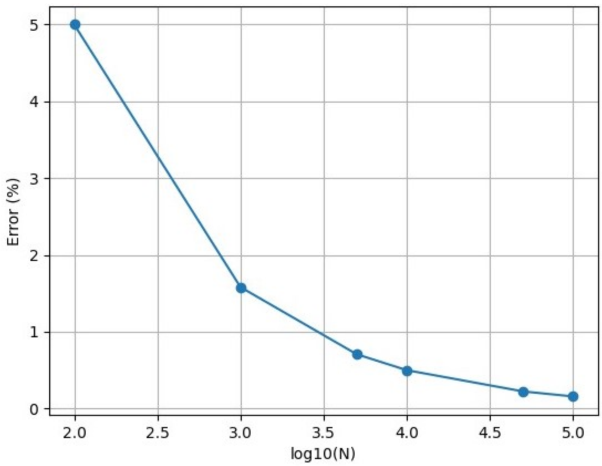

Fig. 5. Log-scale convergence behavior

The figures illustrating the convergence of the Monte Carlo estimation demonstrate the relationship between the number of trials and the relative error of the calculated volume. As the number of random samples increases, the estimation error decreases significantly, confirming the theoretical inverse square root dependency. The most substantial improvement in accuracy occurs at lower values of N, while further increases result in progressively smaller gains, indicating diminishing returns in computational efficiency.

The computation time versus number of trials graph reflects a nearly linear relationship between simulation cost and the number of generated random points. This behavior is expected because each additional trial requires an independent geometric verification. The results confirm that the proposed method remains computationally feasible even for relatively large values of N, making it suitable for offline analysis of CNC programs.

The error versus the square root of the number of trials graph highlights the theoretical foundation of the Monte Carlo method. The observed linear trend in this representation confirms that the estimation error is proportional to $1/\sqrt{N}$. This validates both the correctness of the implemented algorithm and the probabilistic assumptions used in the methodology.

The accuracy versus computation time graph provides a practical interpretation of the trade-off between precision and computational cost. As computation time increases, accuracy improves rapidly at first and then stabilizes. This behavior allows identification of an optimal operating region where sufficient accuracy is achieved without excessive computational expense.

The logarithmic convergence graph presents the same error behavior on a logarithmic scale of the number of trials, making it easier to analyze convergence over several orders of magnitude. The smooth decreasing trend confirms stable and predictable convergence of the Monte Carlo estimator, which is essential for reliable application in engineering calculations.

The figures illustrating the convergence of the Monte Carlo estimation (Table 1, Figs. 2–5) demonstrate the relationship between the number of trials and the relative error of the calculated volume. As the number of random samples increases, the estimation error decreases significantly, confirming the theoretical inverse square root dependency.

The comparison with traditional methods and the reference voxel-based solution (Table 2, Fig. 6, Fig. 7) proves that the proposed geometry-based Monte Carlo approach substantially outperforms existing industrial practices. While the nominal-volume summation and time-based (motor-hour) methods overestimate the actual material removal by 25–45% due to unaccounted trajectory overlaps and idle movements, the Monte Carlo estimator achieves sub-0.5% accuracy with a controllable computational cost.

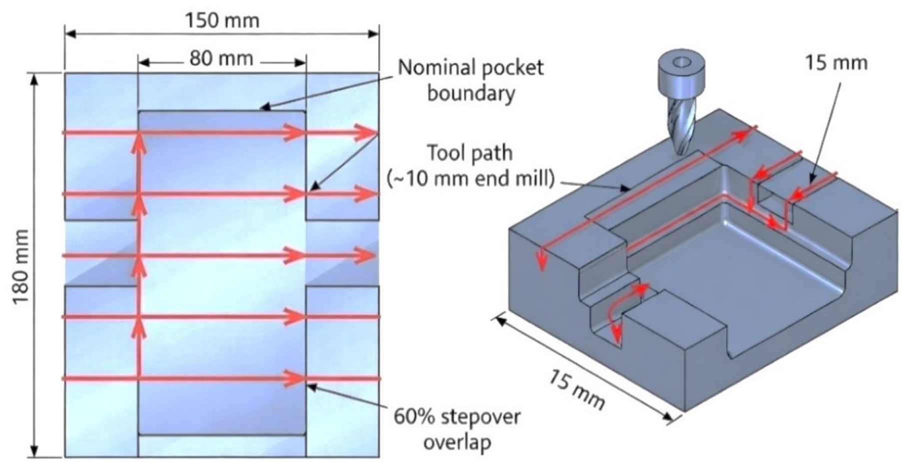

Fig. 6. Tool-path visualisation and actual machined volume (shaded region) for the primary test case with 60% overlap

From a practical standpoint, this overestimation in traditional approaches leads to premature tool replacement, increased tooling costs, and reduced overall equipment effectiveness in FMS. The proposed methodology eliminates this systematic error by providing a reliable quantitative indicator of actual tool workload (effective machined volume) directly from CNC program data.

The results also confirm the method's robustness across different overlap levels and workpiece geometries. The linear scaling of computation time with the number of trials and frames (Fig. 2) ensures that even large industrial CNC programs ($N_K > 500$) can be analyzed offline within minutes on a standard desktop computer.

Thus, the developed Monte Carlo procedure not only validates its internal convergence properties but also demonstrates clear superiority over conventional

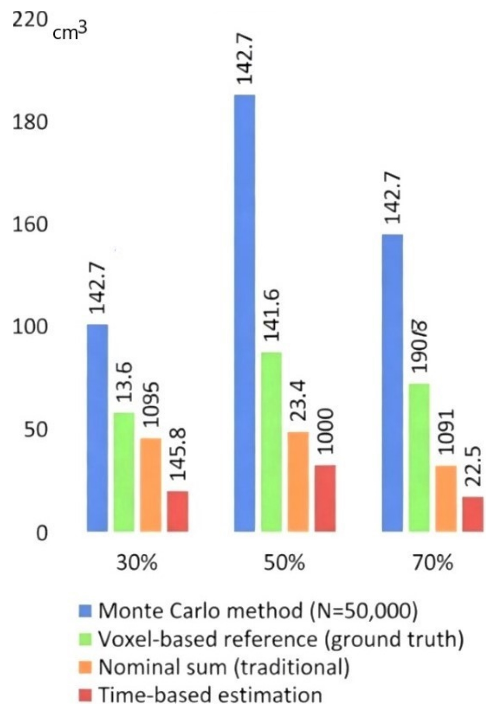

Fig. 7. Bar chart comparing estimated volumes across the three methods for different overlap levels (30%, 50%, 70%)

time-based and nominal-volume approaches, making it a practical tool for accurate cutting tool resource evaluation in flexible manufacturing systems.

From a computational standpoint, the proposed methodology demonstrates favorable scalability characteristics. The algorithm processes CNC programs sequentially, frame by frame, which allows it to handle large control programs without excessive memory requirements. The linear dependence of computational complexity on both the number of Monte Carlo trials and the number of CP frames enables straightforward adaptation to programs of varying size and complexity.

For large-scale CNC programs, the calculation can be further optimized by parallel generation of random points or by grouping frames with non-overlapping machining regions. These measures make the approach suitable for integration into digital manufacturing environments and offline process planning systems.

Thus, from the perspective of workload evaluation, the main types of machining operations on “Machining Center” type machines are classified into positional and contour machining.

For each type of machining, a specific evaluation criterion is established. In particular, for positional machining, the criterion is the number of active point operations. For contour machining, depending on its subtype, the criteria include: the path length traveled by CT, the machined volume, or the machined surface area.

To obtain estimates of the processed volume, a numerical procedure based on the Monte Carlo method has been proposed and substantiated.

Provided that functional relationships exist between the workload estimates and the level of CT resource consumption, the obtained data can be used to develop appropriate assessment and forecasting procedures for determining the necessary and sufficient resources.

If functional relationships between CT workload estimates and resource consumption levels are confirmed, the proposed methodology can be extended to develop predictive algorithms for resource estimation and optimization.

The conducted research is aimed at developing an effective methodology for assessing the resource consumption of CT within the framework of the FMS. These systems are characterized by the ability to perform a variety of machining operations under numerical control (NC), which provides rapid reconfiguration and adaptability in modern mechanical.

One of the contributions of this study is the division of machining operations within FMS into two main varieties based on their technological characteristics and impact on tool wear: positional machining and contour machining. Positional machining refers to discrete point operations, such as drilling, where the tool processes the workpiece at specific points. In contrast, contouring involves continuous movement of the tool along a predetermined path, such as milling, which results in gradual and distributed wear of the active tool edge.

A specific quantitative criterion for CT wear is proposed for each category of machining operations. In the case of positional machining, wear is estimated through the number of certain point machining operations. In contouring, the wear of the CT is estimated by the length of the cutting path traveled, and by the volume or surface area of the material removed, depending on the characteristics of the workpiece.

The proposed methodology is based on several assumptions that define its current scope of applicability. In particular, it is assumed that the workpiece is monolithic and does not contain initial cavities, as is typical for cast or additively manufactured blanks. In such cases, the volumes corresponding to pre-existing cavities must be identified and excluded from the nominal processing volume prior to Monte Carlo evaluation.

In addition, the current implementation focuses on geometric estimation of material removal and does not explicitly account for cutting force variations, tool material properties, or thermomechanical effects. These factors may influence the actual wear rate and can be incorporated in future extensions of the model by coupling the workload estimates with experimentally identified wear functions.

The extended validation, including detailed machining scenarios and direct comparison with traditional and high-precision reference methods, confirms the reliability and industrial applicability of the proposed approach.

4. Conclusions

For a more accurate assessment of the actual load and consumption of CT resources during contour processing, a numerical procedure based on the Monte Carlo method has been proposed and substantiated. This method allows for estimating the actual processing volume by randomly selecting points within the nominal processing area, and estimating their inclusion in the areas of actual material removal, taking into account potential overlaps of trajectories. The applied stochastic modeling approach significantly increases the accuracy of workload estimation compared to traditional approaches based on time or nominal trajectory length.

While recent Industry 4.0 advancements emphasize real-time, sensor-driven monitoring using AI and digital twins, the proposed sensor-independent geometric method provides a practical, low-cost tool for offline workload forecasting and integration into planning systems. The proposed methodology provides a framework for forecasting CT consumption in CNC-based FMS environments. By correlating estimated load indicators with previous data on tool degradation or failure, it becomes possible to develop predictive maintenance schedules and optimize tool replacement strategies. This helps increase operational efficiency, reduce unplanned downtime, and make more efficient use of CT resources. In addition, the proposed generalized algorithm can be integrated into digital manufacturing workflows. The research results can be applied not only at the modeling and planning stages, but also for real-time monitoring and adaptive control in intelligent manufacturing systems when combined with emerging technologies.

Although the Monte Carlo procedure introduces stochastic computation, the required accuracy can be achieved with a controllable number of trials, ensuring predictable computational cost and scalability for industrial-scale CNC programs.

Thus, the study substantiates a new and sufficiently reliable methodology for assessing the resource of CT in the FMS. The proposed approach can be seen as a step towards implementing intelligent tool management systems, which will contribute to the achievement of the broader goals of Industry 4.0 and predictive manufacturing. The comprehensive validation performed on realistic multi-pass contour scenarios, together with direct quantitative comparison against traditional time-based and nominal-volume methods, demonstrates that the Monte Carlo-based workload estimation significantly improves accuracy and can be readily integrated into existing FMS planning software.

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