

Viacheslav LOVEIKIN ¹, Yuriy ROMASEVYCH ¹, Andriy LOVEIKIN ², Anastasiia LIASHKO ¹

Optimization of the simultaneous start mode of the luffing and the hoisting mechanisms during steady slewing of the jib crane

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The study presents a solution to the optimization problem of the simultaneous start mode of the mechanism for load hoisting, luffing, and jib section extension during steady slewing of the crane. A dynamic model of the jib system with 6 DOF is used to optimize the mechanisms' movement modes. The main movement of the mechanism for load hoisting, luffing, jib section extension and slewing the crane, as well as the oscillations of the drive elements of the hoisting mechanism and the load on a flexible suspension, are taken into account in the work. The complex dimensionless integral criterion, which is the drive mechanism forces (torques) RMS value, is used as the optimization criterion. The nonlinear constrained optimization problem is reduced to an unconstrained one and solved by an approximate method. The metaheuristic particle swarm method VCP-PSO, modified to solve problems of this class, is used to solve the optimization problem numerically. It is revealed that during the simultaneous startup of the mechanisms both high-frequency and low-frequency oscillations of the links are damped, and dynamic loads are reduced.

1. Introduction

An essential problem in jib cranes operations is to increase their productivity. Combining the performance of individual operations is one way to solve this problem. However, the dynamic loads acting on the structural elements of the crane

✉ Anastasiia LIASHKO, e-mail: liashko@nubip.edu.ua

¹National University of Life and Environmental Sciences of Ukraine, Kyiv, Ukraine

²Taras Shevchenko National University of Kyiv, Kyiv, Ukraine



jib system increase. In particular, this occurs during starting and braking. Therefore, there appear the problem of reducing the dynamic loads that arise during the start-up process when several mechanisms operate simultaneously. Optimization of their movement modes is one of the ways to reduce dynamic loads on the drive elements and the structure of the jib system during simultaneous operation of the mechanisms. The optimization of the simultaneous start-up of the mechanisms for slewing the jib, extending its section, and hoisting the load during a steady slewing increases productivity and reduces the loads in the crane elements during the operation of jib cranes.

In a recent study [1], an optimization problem concerning the simultaneous motion of two mechanisms – load hoisting and jib slewing – during the start-up process of a jib crane was solved under constraints on the driving torques of the actuators. Both mechanisms were described by linear functions, which connect actuators impact to the movement of the load suspension point. The present study considers a more complex jib system configuration in which the jib consists of a base section and an extendable section. In this case, the simultaneous operation of four mechanisms must be considered: load hoisting, luffing, jib section extension in the plane of load outreach variation, and crane slewing. In the case of simultaneous operation of both mechanisms, the relationship between the position of drive mechanisms and that of the load suspension point can only be described by a nonlinear function. The proposed approach to solving the problem of eliminating vibrations in the crane structural elements and reducing dynamic loads during the simultaneous operation of the mechanisms, under constraints on the driving forces (torques) and drive power, is based on optimizing the simultaneous motion regimes of the mechanisms. In order to solve the optimization problem, one needs to develop a mathematical model describing the dynamics of the simultaneous motion of the mechanisms, to formulate an optimization criterion, and to develop an algorithm for minimization of vibrations under the specified constraints.

The optimization of the simultaneous start-up of the load- hoisting, jib- hoisting, and jib section extension mechanisms during steady crane slewing will improve the productivity and reduce dynamic loads of jib cranes.

According to some authors [2, 3], crane productivity increases with the simultaneous operation of crane mechanisms, but dynamic loads increase significantly. Mathematical modelling of a jib crane was carried out in [2]. The dynamics of the simultaneous movement of the mechanisms for load hoisting, changing its outreach, and the crane slewing were also investigated in this article. The dynamic motion of a double-jib crane with a nonlinear control system for drive mechanisms was researched in [3]. Crane overloads were identified in [3]. A control system that minimises these loads was also developed in [3]. The authors of [4] conducted a dynamic analysis of an overhead crane with several hoisting mechanisms and performed a control using a sliding mode approach. This made it possible to reduce the load on the crane mechanisms. Multibody modelling of tower crane systems was carried out in [5]. The dynamic analysis of the movement of their mechanisms,

which revealed significant loads in the crane elements, was also carried out in this article. The dynamic analysis of a tower crane with a rotating jib using the Kane method, which showed an overload of crane mechanisms, was carried out in [6]. The authors performed the dynamic analysis of a container overhead crane, taking into account the effect of spider adhesion to the load, in [7]. In particular, the analysis revealed the deviation from the vertical of the flexible load suspension, which significantly impacts the dynamic loads acting on the crane and its performance. The accuracy of determining dynamic loads depends largely on modelling methods. The analysis of different approaches to modelling the dynamics of movement of hoisting and trolley movement was carried out in [8].

The impact of dynamic loads on the elements of jib cranes can be significantly reduced by using optimal control of the mechanism's movement. The authors of [9] proposed a performance-maximum optimization method for a polar ship crane by optimizing the load trajectory. The nonlinear planning of the optimal time trajectory of a load for overhead cranes with different lengths of its flexible suspension was carried out in [10]. This optimization made it possible to increase the crane's productivity. The dynamic analysis of an overhead crane with distributed beam masses was carried out in [11]. In this article, the authors also determined an optimal trajectory control of the hoisting mechanism represented by a model of the double mathematical pendulum. The authors of [12] obtained an energy-optimal solution for controlling a crane when moving the load on a resilient suspension represented by a dynamic system in the form of a double mathematical pendulum. They also presented the methods of designing such systems and the results of research. The minimisation of load oscillations on a flexible suspension by manipulating its length during the crane jib slewing was researched in [13]. The authors of [14] proposed an optimal method for controlling the mechanism of slewing a jib crane by switching the driving force, which reduced the load's moving time, energy consumption, and residual oscillations. The authors of [15] proposed optimal control by switching the driving force based on nonlinear feedback logic to improve the performance of ship tower cranes. The optimal control strategy for a jib crane aimed at minimising load oscillations on a flexible suspension, which increases crane productivity and load positioning accuracy, was developed in [16]. The authors of [17] optimized the parameters of crane mechanisms to reduce the oscillation of crane structural elements and drives. Metaheuristic optimization methods have been widely used to solve these problems [18]. The global optimization of crane mechanisms was proposed in [19]. In this article, the optimization criteria in the form of quadratic functions were developed based on a mathematical model of the mechanisms. The optimal parameters of crane mechanisms that increase the efficiency of their joint operation were obtained as a result of minimizing the functionalities. The authors of [20] optimized a crane trolley's start-up mode during steady-state slewing of a tower crane. The integral functional, used as an optimization criterion for reducing the load in the crane elements, was in the form of the RMS value of the drive torque and its rate of change over time. The complex integral criterion, applying the RMS

value of the power of the drive mechanisms during start-up, was used in [21] to optimize the mode of simultaneous start-up of the mechanisms for changing the outreach and slewing of a tower crane.

In [23–25], an algorithm was developed for formulating the equations of motion for systems composed of elements of different physical nature, in particular using the Lagrange equations of the second kind. Asymptotic solutions of third-order and higher-order approximations were obtained using multi-scale methods. Numerical solutions of the resulting equations were also presented, which were in agreement with those obtained using asymptotic methods. Based on the derived equations, the authors performed a dynamic analysis of the systems under consideration.

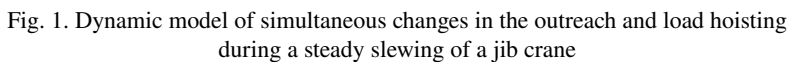
The presented methods for constructing and solving the differential equations of motion, as well as the performed dynamic analysis, can be applied to the problems of dynamic analysis and optimization of operating regimes of hoisting machine mechanisms.

The analysis of references has shown a considerable attention given by the researchers to the dynamics of crane movement, methods of control and optimization of parameters and modes of movement of the mechanisms. These problems are relevant and require further research in the optimization of jib systems, in particular, when the mechanism for load hoisting, luffing, and jib section extension move simultaneously during a steady crane slewing.

2. Optimization problem statement

In the process of optimizing the simultaneous start-up of the mechanism for load lifting, luffing, and jib section extension during a steady crane slewing, the jib system is represented by the dynamic model shown in Fig. 1. The jib consists of two sections. The first one is the main part, and the second one is the extendable part. The extension section is driven by a hydraulic cylinder located in the main jib section. The jib is rotated by a hydraulic cylinder 6, which is pivotally connected to the crane's slewing part and the main jib section 1. The load hoisting mechanism 4 includes a hydraulic motor, a gearbox, a drum 5 and a single chain hoist 3. The elastic rope of the hoist of the load hoisting mechanism is wound on the drum 5. In this research, we assume that the change in the load is a simultaneous operation of the mechanism for luffing and jib section extension. The dynamic model consists of absolutely rigid and elastic links. The rope of the load hoisting mechanism is assumed to have elastic properties in tension, and the jib root and extension sections, load, and drive elements are represented by absolutely rigid bodies. The hoist acts as a flexible load suspension and is represented by a mathematical pendulum that oscillates in the plane of the outreach change. The drive elements of the load hoisting mechanism are limited to the axis of rotation of the drum 5.

Based on the above, the dynamic model of the jib system with simultaneous jib slewing, extension of its section and load hoisting at a steady crane slewing is represented by a holonomic 6 DOF mechanical system (Fig. 1). The angular


$$\varphi = \varphi_0 + \omega t. \quad (1)$$

In this case, the jib system can be viewed as a mechanical system with 5 DOF.

We will create a mathematical model of the dynamics of the simultaneous movement of the mechanism for load hoisting, luffing, and jib section extension during a steady-state crane slewing using the Lagrange equations:

$$\begin{aligned} \frac{d}{dt} \frac{\partial T}{\partial \dot{\alpha}} - \frac{\partial T}{\partial \alpha} &= M_1 - \frac{\partial \Pi}{\partial \alpha}, & \frac{d}{dt} \frac{\partial T}{\partial \dot{\beta}} - \frac{\partial T}{\partial \beta} &= M_3 - \frac{\partial \Pi}{\partial \beta}, \\ \frac{d}{dt} \frac{\partial T}{\partial \dot{z}} - \frac{\partial T}{\partial z} &= F_2 - \frac{\partial \Pi}{\partial z}, & \frac{d}{dt} \frac{\partial T}{\partial \dot{u}} - \frac{\partial T}{\partial u} &= \frac{\partial \Pi}{\partial u}, \\ \frac{d}{dt} \frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x} &= -\frac{\partial \Pi}{\partial x}. \end{aligned} \quad (2)$$

The following notations are used here: T , Π are, respectively, the kinetic and potential energy of the jib system at the simultaneous change of the load hoisting, luffing, and jib section extension with a steady crane slewing; M_1 , M_3 are driving torques of the drives of the mechanisms for the jib and load hoisting, reduced to the axes of rotation of the jib and drum, respectively; F_2 is the driving force of the hydraulic cylinder of the jib section extension mechanism; t is the time coordinate; φ_0 is the initial crane rotation coordinate; $\omega = \text{const}$ is the angular velocity of the crane rotation.

The primary and extension sections of the jib perform a complex spatial motion. Taking this fact into account, we will determine their kinetic energy through the energy of elementary individual particles (Fig. 2). We will assume that the masses of the primary and extension jib sections are uniformly distributed along their lengths with linear masses ρ_1 and ρ_2 , respectively.

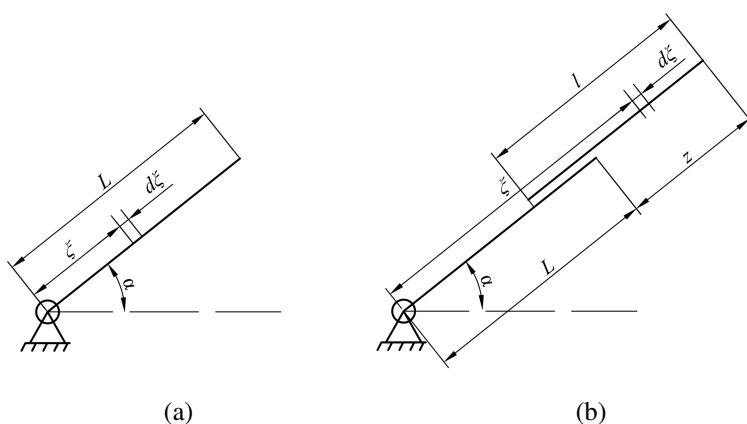


Fig. 2. Crane jib model: (a) the main jib section; (b) the extension section

The kinetic energy of an elementary particle of the main section of the jib (Fig. 2a) is determined with the following formula

$$dT_1 = \frac{1}{2} \rho_1 d\xi \left(\dot{\alpha}^2 \xi^2 + \omega^2 \xi^2 (\cos \alpha)^2 \right). \quad (3)$$

Integrating expression (3), along the entire length of the main section of the jib, we obtain the expression of its kinetic energy

$$T_1 = \frac{1}{2}\rho_1 \left(\dot{\alpha}^2 + (\omega \cos \alpha)^2 \right) \int_0^L \xi^2 d\xi = \frac{1}{2}J_1 \left(\dot{\alpha}^2 + (\omega \cos \alpha)^2 \right). \quad (4)$$

Similarly, the kinetic energy of an elementary particle of the extendable jib section is determined (Fig. 2b)

$$dT_2 = \frac{1}{2}\rho_2 d\xi \left(\dot{\alpha}^2 \xi^2 + \omega^2 \xi^2 (\cos \alpha)^2 + \dot{z}^2 \right). \quad (5)$$

Integrating expression (5), along the entire length of the extendable jib section, we obtain its kinetic energy

$$\begin{aligned} T_2 &= \frac{1}{2}\rho_2 \left(\dot{\alpha}^2 + (\omega \cos \alpha)^2 \right) \int_{L+z-l}^{L+z} \xi^2 d\xi + \frac{1}{2}\rho_2 \dot{z}^2 \int_{L+z-l}^{L+z} dz \\ &= \frac{1}{2}\rho_2 l \left(\dot{\alpha}^2 + (\omega \cos \alpha)^2 \right) \left[(L+z)^2 - (L+z)l + \frac{l^2}{3} \right] + \frac{1}{2}\rho_2 l \dot{z}^2 \\ &= \frac{1}{2} [J_2 + m_2(z+L)(z+L-l)] \left(\dot{\alpha}^2 + (\omega \cos \alpha)^2 \right) + \frac{1}{2}m_2 \dot{z}^2. \end{aligned} \quad (6)$$

The formula of the kinetic energy of the jib system at the simultaneous movement of the mechanisms for slewing the jib, extending its section and load hoisting at a steady-state crane slewing is obtained by adding the expressions (4) and (6), as well as the kinetic energy of the drive of the mechanism for load hoisting, luffing, and jib section extension

$$\begin{aligned} T &= \frac{1}{2} [J_1 + J_2 + m_2(z+L)(z+L-l)] \left(\dot{\alpha}^2 + \omega^2 (\cos \alpha)^2 \right) + \frac{1}{2}m_2 \dot{z}^2 \\ &\quad + \frac{1}{2}J_3 \dot{\beta}^2 + \frac{1}{2}m \left(\dot{x}^2 + \dot{y}^2 + \omega^2 x^2 \right) + \frac{1}{2}J_4 \omega^2, \end{aligned} \quad (7)$$

where J_1 , J_2 are the moments of inertia of the main jib section relative to the axis of rotation and the jib extension section relative to the axis of its lower edge; J_3 is the moment of inertia of the drive of the load hoisting mechanism, reduced to the axis of rotation of the drive drum; J_4 is the moment of inertia of the crane turntable relative to its own rotation axis; m_2 , m are, respectively, the mass of the extendable jib section and the load; L , l are lengths of the main and extendable jib sections, respectively; ω is steady-state angular velocity of crane slewing; x , y are horizontal and vertical coordinates of the centre of mass of the load.

The potential energy of the jib system for simultaneous movement of the mechanism for load hoisting, luffing, and jib section extension during a steady-state crane slewing is as follows

$$\Pi = \frac{1}{2}c [\beta r - (z - z_0) - (u_0 - u)n]^2 + (m_1 y_1 + m_2 y_2 + m y)g, \quad (8)$$

where c , r are rope stiffness coefficient and radius of the drive drum of the load hoisting mechanism; n – multiplicity of a single pulley block of a load hoisting mechanism; g – acceleration of free fall; m_1 – mass of the jib's root section; y_1 , y_2 are vertical coordinates of the mass centres of root and extension sections of the jib, respectively; u_0 , z_0 are initial coordinates of the length of the flexible load suspension and the displacement of the jib extension section.

The vertical coordinates of the load mass centres of the root and extension sections of the jib are determined by the following formulae:

$$y = (z + L) \sin \alpha - u \cos \nu, \quad (9)$$

$$y_1 = \frac{1}{2}L \sin \alpha, \quad y_2 = \left(z + L - \frac{l}{2}\right) \sin \alpha, \quad (10)$$

where ν is the angular coordinate of the vertical deviation of a flexible load suspension, which is defined as follows

$$\nu = \frac{[x - (L + z) \cos \alpha]}{u}. \quad (11)$$

The potential energy function of the jib system is found after substituting expressions (9) and (10) into dependence (8)

$$\begin{aligned} \Pi = & \frac{1}{2}c [\beta r - (z - z_0) - (u_0 - u)n]^2 \\ & + \left[\frac{m_1 L}{2} + m_2 \left(z + L - \frac{l}{2} \right) + m(z + L) \right] g \sin \alpha - mgu \cos \nu. \end{aligned} \quad (12)$$

The derivatives of kinetic (7) and potential (12) functions are given in the Appendix A [26].

The system of differential equations for the simultaneous motion of the mechanism for load hoisting, luffing, and jib section extension during steady crane slewing was obtained by substituting expressions (A2), (A4) and (A11)–(A15) into

system (2):

$$\begin{aligned}
 & [J_1 + J_2 + m_2(z + L)(z + L - l)] \left(\ddot{\alpha} + \omega^2 \sin \alpha \cos \alpha \right) \\
 & + 2m_2 \dot{\alpha} \dot{z} \left(z + L - \frac{l}{2} \right) + m \ddot{y}(z + L) \cos \alpha \\
 & = M_1 - \left[\frac{m_1 L}{2} + m_2 \left(z + L - \frac{l}{2} \right) + m(z + L) \right] g \cos \alpha \\
 & - mg(L + z) \nu \sin \alpha, \\
 J_2 \beta & = M_3 - cr [\beta r - z + z_0 - (u_0 - u)n], \\
 m_2 \ddot{z} - m_2 \left(z + L - \frac{l}{2} \right) & \left[\dot{\alpha}^2 + (\omega \cos \alpha)^2 \right] + m \ddot{y} \sin \alpha \\
 & = F_2 + c [\beta r - z + z_0 - (u_0 - u)n] \\
 & - [(m_2 + m) \sin \alpha - m \nu \cos \alpha] g, \\
 m \ddot{y} & = cn [\beta r - z + z_0 - (u_0 - u)n] - mg(1 + \nu^2), \\
 \ddot{x} - x\omega^2 & = -g\nu.
 \end{aligned} \tag{13}$$

The second derivative with respect to time of the vertical coordinate of the load mass centre that appears the system of differential equations (13), can be expressed by the following formula, taking into account the assumptions made

$$\ddot{y} = [\ddot{\alpha}(z + L) + 2\dot{\alpha}\dot{z}] \cos \alpha + [\ddot{z} - \dot{\alpha}^2(z + L)] \sin \alpha - \ddot{u}. \tag{14}$$

From expression (11) and the last equation of system (13), we obtain

$$-\frac{\ddot{x} - x\omega^2}{g} = \frac{x - (L + z) \cos \alpha}{u}.$$

The angular coordinate of the jib slewing is found from the above equation

$$\begin{aligned}
 \cos \alpha & = \frac{gx + u(\ddot{x} - x\omega^2)}{g(L + z)}, \quad \sin \alpha = \sqrt{1 - \left[\frac{gx + u(\ddot{x} - x\omega^2)}{g(L + z)} \right]^2}, \\
 \alpha & = \arccos \frac{gx + u(\ddot{x} - x\omega^2)}{g(L + z)}.
 \end{aligned} \tag{15}$$

As a result of differentiation by time of the last expression (15), the angular velocity and acceleration of the jib slewing are found:

$$\begin{aligned}
 \dot{\alpha} & = -\frac{1}{g(L + z)^2 \sin \alpha} \\
 & \times \left[(L + z) \left(g\dot{x} + \dot{u}(\ddot{x} - x\omega^2) + u(\ddot{x} - \dot{x}\omega^2) \right) - \dot{z} \left(gx + u(\ddot{x} - x\omega^2) \right) \right], \tag{16}
 \end{aligned}$$

$$\ddot{\alpha} = -\frac{1}{g(L+z)^3 \sin \alpha} \left[\left\{ \left[g\ddot{x} + u \left(x^{IV} - \ddot{x}\omega^2 \right) + 2\dot{u} \left(\ddot{x} - \dot{x}\omega^2 \right) + \ddot{u} \left(\ddot{x} - x\omega^2 \right) \right] \right. \right. \\ \times (z+L) - \ddot{z} \left[gx + u \left(\ddot{x} - x\omega^2 \right) \right] \left. \right\} (z+L) \sin \alpha - [2\dot{z} \sin \alpha + \dot{\alpha}(z+L) \cos \alpha] \\ \times \left\{ \left[g\dot{x} + u \left(\ddot{x} - \dot{x}\omega^2 \right) + \dot{u} \left(\ddot{x} - x\omega^2 \right) \right] (z+L) - \dot{z} \left[gx + u \left(\ddot{x} - x\omega^2 \right) \right] \right\} \right]. \quad (17)$$

The expression for the angular coordinate of the drum's rotation of the load hoisting mechanism is found from the penultimate equation of system (13)

$$\beta = \frac{1}{r} \left[\frac{m}{cn} \left(g + \ddot{y} + \frac{(\ddot{x} - x\omega^2)^2}{g} \right) + (u_0 - u)n + z - z_0 \right]. \quad (18)$$

After differentiating expression (18) by time, the expressions for the angular velocity and acceleration of the drive drum of the load hoisting mechanism are found

$$\dot{\beta} = \frac{1}{r} \left[\frac{m}{cn} \left(\frac{2(\ddot{x} - x\omega^2)(\ddot{x} - \dot{x}\omega^2)}{g} + \ddot{y} \right) - \dot{u}n + \dot{z} \right], \quad (19)$$

$$\ddot{\beta} = \frac{1}{r} \left[\frac{m}{cn} \left(2 \frac{(\ddot{x} - x\omega^2)^2 + (\ddot{x} - x\omega^2)(x^{IV} - \ddot{x}\omega^2)}{g} + y^{IV} \right) - \ddot{u}n + \ddot{z} \right]. \quad (20)$$

The driving forces of the drive mechanisms are determined from the system of equations (13):

$$M_1 = \left[J_1 + J_2 + m_2(z+L) \left(a + L - \frac{l}{2} \right) \right] \left(\ddot{\alpha} + \omega^2 \sin \alpha \cos \alpha \right) \\ + 2m_2 \dot{\alpha} \dot{z} \left(z + L - \frac{l}{2} \right) + m \ddot{y} (z+L) \cos \alpha \\ + \left[\frac{m_1 L}{2} + m_2 \left(z + L - \frac{l}{2} \right) + m(z+L) \right] \\ \times g \cos \alpha - m \left(\ddot{x} - x\omega^2 \right) (L+z) \sin \alpha, \quad (21)$$

$$M_3 = J_3 \ddot{\beta} + cr [\beta r - z + z_0 - (u_0 - u)n], \quad (22)$$

$$F_2 = m_2 \ddot{z} - m_2 \left(z + L - \frac{l}{2} \right) [\dot{\alpha}^2 + (\omega \cos \alpha)^2] + m \ddot{y} \sin \alpha \\ - c [\beta r - z + z_0 - (u_0 - u)n] + (m_2 + m)g \sin \alpha \\ + m \left(\ddot{x} - x\omega^2 \right) \cos \alpha. \quad (23)$$

The obtained dependencies of the driving forces (torques) are reduced to three generalised coordinates: the linear coordinate of the jib section extension – z , the length of the flexible load suspension – u , and the change load reach – x .

Dynamic loads in the structural elements of the crane and its drive mechanisms that lead to oscillations of the load on the flexible suspension, increase significantly. The reason is the simultaneous start the mechanisms for load hoisting, luffing, and jib section extension during the steady slewing of the crane. In addition, the energy losses in the drives, which negatively impact on the structural elements of the jib system of the crane, are increasing. For this reason, there is a need to optimize the modes of simultaneous start-up of the mechanisms for load hoisting, luffing, and jib section extension during a steady slewing of the crane, which would minimize the driving forces of the drives. Therefore, as a criterion for optimizing the mode of simultaneous start-up of mechanisms, we use a complex dimensionless criterion in the form of the RMS value for the start-up time of the combined action of the driving forces of the load hoisting, luffing, and jib section extension at a steady slewing of the crane. The selected criterion is presented in the following form

$$K_{ck} = \left[\frac{1}{t_1} \int_0^{t_1} \left\{ \delta_1 \left(\frac{M_1}{M_{1n}} \right)^2 + \delta_2 \left(\frac{M_3}{M_{3n}} \right)^2 + (1 - \delta_1 - \delta_2) \left(\frac{F_2}{F_{2n}} \right)^2 \right\} dt \right]^{\frac{1}{2}} \rightarrow \min, \quad (24)$$

where t_1 is the duration of simultaneous start-up of the mechanism for load hoisting, luffing, and jib section extension during a steady crane slewing; δ_1, δ_2 weighting coefficients that reflect the influence of the driving moments of the mechanisms, respectively, jib slewing and the load; M_{1n}, M_{3n} are the rated values of the driving torques of the drives of the mechanisms for jib slewing and the load; F_{2n} is the rated driving force of the drive for extending the jib section.

Criterion (24) is an integral function that must be minimized under the following boundary conditions of starting the mechanisms for jib slewing, lengthening its section, and load hoisting at a steady crane slewing:

$$\begin{aligned} t = 0 : \quad \alpha = \alpha_0, \quad \dot{\alpha} = 0, \quad \beta = \beta_0 = \frac{mg}{cnr}, \\ \dot{\beta} = 0, \quad z = z_0, \quad \dot{z} = 0, \quad u = u_0, \quad \dot{u} = 0, \quad x = x_0, \quad \dot{x} = 0, \end{aligned} \quad (25)$$

$$\begin{aligned} t = t_1 : \quad \alpha = \alpha_1 = \alpha_0 + \frac{\omega_1 t_1}{2}, \quad \dot{\alpha} = \omega_1, \quad \beta = \beta_1 = \frac{mg}{cnr} + \frac{\omega_3 t}{2}, \\ \dot{\beta} = \omega_3, \quad z = z_1 = z_0 + \frac{v_2 t_1}{2}, \quad \dot{z} = v_2, \\ u = u_1 = u_0 + \frac{v_4 t_1}{2}, \quad \dot{u} = v_4, \quad x = x_1 = x_0 + \frac{v_1 t_1}{2}, \quad \dot{x} = v_1 \end{aligned} \quad (26)$$

and constraints on the driving torques M_1 , M_3 and forces F_2 of the drives of the mechanisms for load hoisting, luffing, and jib section extension, respectively:

$$M_{1 \min} \leq M_1 \leq M_{1 \max}, \quad (27)$$

$$F_{2 \min} \leq F_2 \leq F_{2 \max}, \quad (28)$$

$$M_{3 \min} \leq M_3 \leq M_{3 \max}, \quad (29)$$

$$P_{1 \min} \leq P_1 \leq P_{1 \max}, \quad (30)$$

$$P_{2 \min} \leq P_2 \leq P_{2 \max}, \quad (31)$$

$$P_{3 \min} \leq P_3 \leq P_{3 \max}, \quad (32)$$

where ω_1 and ω_3 are the steady angular velocities of the jib slewing and the drive drum of the load hoisting mechanism, respectively; α_0 , x_0 , z_0 , u_0 are initial coordinates of the jib slewing, the load extension, the jib extension section and the length of the flexible load suspension; v_1 , v_2 , v_4 are steady velocities of the change in the extension, the jib section extension and the change in the length of the flexible load suspension; $M_{1 \min}$, $M_{1 \max}$ are the minimum and maximum permissible values of the drive torque of the luffing mechanism; $M_{3 \min}$, $M_{3 \max}$ are the minimum and maximum permissible values of the drive torque of the load hoisting mechanism; $F_{2 \min}$, $F_{2 \max}$ are minimum and maximum permissible values of the driving force of the jib section extension hydraulic cylinder; $P_{1 \min}$, $P_{1 \max}$ are the minimum and maximum permissible values of the drive power of the jib slewing mechanism; $P_{2 \min}$, $P_{2 \max}$ are minimum and maximum permissible power values of the jib section extension hydraulic cylinder; $P_{3 \min}$, $P_{3 \max}$ are minimum and maximum permissible values of the drive power of the load hoisting mechanism.

The modes of simultaneous start-up of the mechanisms for load hoisting, luffing, and jib section extension during a steady crane slewing, which provide the minimum value of criterion (24) and satisfy the boundary conditions (25) and (26), as well as the constraints on the driving forces (27)–(29) and powers (30)–(32) of the drives, must be determined in the given optimization problem (24)–(32).

The optimization problem is a constrained nonlinear one, the solution of which leads to significant difficulties. Therefore, we will reduce this problem to an unconstrained optimization problem. Taking into account the constraints of the driving forces (27)–(29) and the power (30)–(32) of the drives in the optimization problem, we formulate a special dimensionless generalised criterion that incorporates an integral criterion (24) and the constraints, which will be the subject to minimization

$$Cr = K_{ck} + \delta_p \sum_{i=1}^{12} p_i \rightarrow \min, \quad (33)$$

where δ_p is the penalty coefficient, which reflects the importance of fulfilling the constraints (27)–(32). To ensure the constraints (27)–(32), we apply penalty functions given in [26] (Appendix B).

The penalty coefficient $\delta_P = 10^8$ was chosen to ensure that the optimization algorithm first finds a solution that satisfies all constraints (27)–(32) and that the generalized criterion (33) is insensitive to the integral term K_{ck} . Once the constraints have been satisfied, the penalty coefficient is set to zero, i.e., $\delta_P = 0$, and the search then proceeds to find solutions that minimize the integral criterion (24).

From expression (24) it can be seen that the terms of the generalised criterion (33) are dimensionless quantities. The first term of expression (33) is criterion (24), and the second term provides the constraints (27)–(32).

In [26] (Appendix C), boundary conditions (25) and (26) are given, which are reduced to generalised coordinates.

As a result of the calculations, we obtain the following boundary conditions for the motion of the jib system during simultaneous slewing of the jib, extension of its section and hoisting of the load with the crane in steady slewing:

$$\begin{aligned} t = 0 : z = z_0, \dot{z} = 0, \ddot{z} = 0, \dddot{z} = 0, z^{IV} = 0, u = u_0, \dot{u} = 0, \ddot{u} = 0, \\ \ddot{u} = 0, u^{IV} = 0, x = x_0, \dot{x} = 0, \ddot{x} = 0, \dddot{x} = 0, x^{IV} = 0, \end{aligned} \quad (34)$$

$$\begin{aligned} t = t_1 : z = z_0 + \frac{v_2 t_1}{2}, \dot{z} = v_2, \ddot{z} = 0, \dddot{z} = 0, z^{IV} = 0, \\ u = u_0 + \frac{v_4 t_1}{2}, \dot{u} = v_4, \ddot{u} = 0, \dddot{u} = 0, u^{IV} = 0, \\ x = x_1 = x_0 + \frac{v_1 t_1}{2}, \dot{x} = v_1, \ddot{x} = 0, \dddot{x} = 0, x^{IV} = 0. \end{aligned} \quad (35)$$

Here v_1, v_4 are the average steady-state velocities of change in jib extension and flexible load suspension length.

The differential equations of motion (13), the integral functional (24) together with the constraints (27)–(32), expressions (15)–(23) and boundary conditions (41) and (42) represent an optimization problem. In this problem, one must determine the laws of motion of the linear coordinates of the jib extension z , the change in the load reach x and the length of its suspension u , which minimise criterion (24) and ensure constraints (27)–(32) and boundary conditions (34), (35).

3. Optimization problem solving

The optimization problem is nonlinear, so an approximate method is used to solve it. In this task, the solutions are represented by unknown functions $z(t)$, $u(t)$ and $x(t)$ in the form of polynomials with two terms:

$$z(t) = z_1(t) + z_2(t), \quad (36)$$

$$u(t) = u_1(t) + u_2(t), \quad (37)$$

$$x(t) = x_1(t) + x_2(t). \quad (38)$$

In expressions (36)–(38), the first terms $z_1(t)$, $u_1(t)$ and $x_1(t)$ are selected polynomials that satisfy boundary conditions (34), (35). The second ones, $z_2(t)$, $u_2(t)$ and $x_2(t)$ are the polynomials that contain unknown coefficients and satisfy the following zero boundary conditions:

$$\begin{aligned} z_2(0) = \dot{z}_2(0) = \ddot{z}_2(0) = \dddot{z}_2(0) = z_2^{IV}(0) = z_2(t_1) = \dot{z}_2(t_1) \\ = \ddot{z}_2(t_1) = \dddot{z}_2(t_1) = z_2^{IV}(t_1) = 0, \end{aligned} \quad (39)$$

$$\begin{aligned} u_2(0) = \dot{u}_2(0) = \ddot{u}_2(0) = \dddot{u}_2(0) = u_2^{IV}(0) = u_2(t_1) = \dot{u}_2(t_1) \\ = \ddot{u}_2(t_1) = \dddot{u}_2(t_1) = u_2^{IV}(t_1) = 0, \end{aligned} \quad (40)$$

$$\begin{aligned} x_2(0) = \dot{x}_2(0) = \ddot{x}_2(0) = \dddot{x}_2(0) = x_2^{IV}(0) = x_2(t_1) = \dot{x}_2(t_1) \\ = \ddot{x}_2(t_1) = \dddot{x}_2(t_1) = x_2^{IV}(t_1) = 0. \end{aligned} \quad (41)$$

The functions $u_1(t)$, $z_1(t)$, $x_1(t)$ are represented by ninth-order polynomials as they make it possible to ensure boundary conditions (34) and (35):

$$u_1(t) = \sum_{i=0}^9 A_i t^i, \quad (42)$$

$$z_1(t) = \sum_{j=0}^9 B_j t^j, \quad (43)$$

$$x_1(t) = \sum_{k=0}^9 C_k t^k, \quad (44)$$

where A_i ($i = 0, \dots, 9$), B_j ($j = 0, \dots, 9$), C_k ($k = 0, \dots, 9$) are unknown coefficients determined from the boundary conditions of motion.

In boundary conditions (34) and (35), the time derivatives of functions $u(t)$, $z(t)$ and $x(t)$ are of the fourth order, so they are defined as functions of expressions (42)–(44):

$$\begin{aligned} \dot{u}_1(t) = A_1 + 2A_2t + 3A_3t^2 + 4A_4t^3 + 5A_5t^4 + 6A_6t^5 + 7A_7t^6 \\ + 8A_8t^7 + 9A_9t^8, \end{aligned} \quad (45)$$

$$\begin{aligned} \ddot{u}_1(t) = 2A_2 + 6A_3t + 12A_4t^2 + 20A_5t^3 + 30A_6t^4 \\ + 42A_7t^5 + 56A_8t^6 + 72A_9t^7, \end{aligned} \quad (46)$$

$$\begin{aligned} \dddot{u}_1(t) = 6A_3 + 24A_4t + 60A_5t^2 + 120A_6t^3 \\ + 210A_7t^4 + 336A_8t^5 + 504A_9t^6, \end{aligned} \quad (47)$$

$$\begin{aligned} u_1^{IV}(t) = 24A_4 + 120A_5t + 360A_6t^2 + 840A_7t^3 \\ + 1680A_8t^4 + 3024A_9t^5. \end{aligned} \quad (48)$$

Since the derivatives of the functions $z(t)$ and $x(t)$ are similar in form to the function $u(t)$, so they are transferred to the Appendix D [26].

As a result of substituting boundary conditions (34) and (35) into expressions (42) and (45)–(48), the following is found:

$$A_0 = u_0; \quad A_1 = 0; \quad A_2 = 0; \quad A_3 = 0; \quad A_4 = 0. \quad (49)$$

The constants A_5 – A_9 are determined from the following system of equations:

$$\begin{aligned} A_5 + A_6 t_1 + A_7 t_1^2 + A_8 t_1^3 + A_9 t_1^4 &= \frac{v_4}{2t_1^4}, \\ 5A_5 + 6A_6 t_1 + 7A_7 t_1^2 + 8A_8 t_1^3 + 9A_9 t_1^4 &= \frac{v_4}{t_1^4}, \\ 10A_5 + 15A_6 t_1 + 21A_7 t_1^2 + 28A_8 t_1^3 + 36A_9 t_1^4 &= 0, \\ 10A_5 + 20A_6 t_1 + 35A_7 t_1^2 + 56A_8 t_1^3 + 84A_9 t_1^4 &= 0, \\ 5A_5 + 15A_6 t_1 + 35A_7 t_1^2 + 70A_8 t_1^3 + 126A_9 t_1^4 &= 0. \end{aligned} \quad (50)$$

Since the constant coefficients in the functions $z(t)$ and $x(t)$ are defined similarly as those in expressions (49) and (50), so their definition is also provided in the Appendix E [26].

After substituting constants (49), (E1) and (E3) [22], and those found when solving systems of equations (50), (E2) and (E4), into dependencies (42)–(44), we obtain functions that satisfy the boundary conditions (34) and (35) for the coordinates z , u , x and their derivatives with respect to time (45)–(48), (D1)–(D8) [22].

The polynomials $z_2(t)$, $u_2(t)$ and $x_2(t)$ are represented by the following expressions:

$$z_2(t) = \left(\frac{t}{t_1}\right)^9 \left(\frac{t_1 - t}{t_1}\right)^9 \sum_{k=0}^l D_k \left(\frac{t}{t_1}\right)^k \frac{\omega_1 t_1}{2}, \quad 0 \leq t \leq t_1, \quad (51)$$

$$u_2(t) = \left(\frac{t}{t_1}\right)^9 \left(\frac{t_1 - t}{t_1}\right)^9 \sum_{j=0}^p E_j \left(\frac{t}{t_1}\right)^j \frac{v_2 t_1}{2}, \quad 0 \leq t \leq t_1, \quad (52)$$

$$x_2(t) = \left(\frac{t}{t_1}\right)^9 \left(\frac{t_1 - t}{t_1}\right)^9 \sum_{i=0}^n K_i \left(\frac{t}{t_1}\right)^i \frac{v_1 t_1}{2}, \quad 0 \leq t \leq t_1. \quad (53)$$

where $D_0, D_1, \dots, D_l, E_0, E_1, \dots, E_p, K_0, K_1, \dots, K_n$ are free coefficients, the value of the optimization criterion (24) depends on which;

$x_2(t) = \left(\frac{t}{t_1}\right)^9 \left(\frac{t_1 - t}{t_1}\right)^9 \sum_{i=0}^n K_i \left(\frac{t}{t_1}\right)^i \frac{v_1 t_1}{2}$, $0 \leq t \leq t_1$, $\left(\frac{t}{t_1}\right)^9 \left(\frac{t_1 - t}{t_1}\right)^9$ are the multipliers that ensure the fulfilment of zero boundary conditions, respectively,

(39)–(41) for arbitrary values of coefficients $D_0, D_1, \dots, D_l, E_0, E_1, \dots, E_p, K_0, K_1, \dots, K_n$. These coefficients are free and make it possible to minimise the value of the optimization criterion (24).

The functions $z(t)$, $u(t)$, and $x(t)$, which include the free coefficients $D_0, D_1, \dots, D_l, E_0, E_1, \dots, E_p, K_0, K_1, \dots, K_n$ are obtained after substituting expressions (42)–(48), (D1), $\dots$, (D8) [22], (51)–(53) in dependencies (36)–(38) taking into account the boundary conditions (34)–(35).

The angular coordinates of the jib and the drum of the load hoisting mechanism, and their time derivatives, which also depend on the free coefficients $D_0, D_1, \dots, D_l, E_0, E_1, \dots, E_p, K_0, K_1, \dots, K_n$ are found from expressions (15)–(20), taking into account expressions (34)–(53). The optimization criterion (24), which depends on the driving forces (21)–(23), also becomes a function of the free coefficients $D_0, D_1, \dots, D_l, E_0, E_1, \dots, E_p, K_0, K_1, \dots, K_n$. Thus, the approximate solution of the optimization problem (13), (24)–(32) is reduced to minimising criterion (24) as a function of many variables of free coefficients $D_0, D_1, \dots, D_l, E_0, E_1, \dots, E_p, K_0, K_1, \dots, K_n$.

The VCP-PSO metaheuristic method [21], modified for solving problems of this class, is used for the numerical solution of the optimization problem.

In this problem, the generalised optimization criterion (33), (B1) [22], taking into account expression (24) and conditions (34) and (35), is represented by a function that depends on 12 free coefficients.

$$C_r = C_r(D_0, D_1, D_2, D_3, E_0, E_1, E_2, E_3, K_0, K_1, K_2, K_3). \quad (54)$$

Calculations of optimal modes for simultaneous start-up of the mechanism for load hoisting, luffing, and jib section extension at steady crane slewing based on the criterion of the RMS value of the total action of the driving forces of the drives (24) and constraints on the driving forces (27)–(29) and power (30)–(32) while ensuring boundary conditions (34) and (35) were performed for the following jib system parameter values: $m = 4500$ kg, $m_1 = 1800$ kg, $m_2 = 900$ kg, $J_1 = 48600$ kg m², $J_2 = 12675$ kg m², $J_3 = 1183$ kg m², $c = 1.25 \cdot 10^6$ Nm/rad, $l = 6.5$ m, $L = 9.0$ m, $v_1 = 0.473$ m/s, $\omega_1 = -0.0396$ rad/s, $v_4 = -0.2$ m/s, $\omega_3 = 2.885$ rad/s, $u_0 = 6.0$ m, $n = 4$, $t_1 = 3.0$ s, $g = 9.81$ m/s², $M_{1\min} = 0$, $M_{1\max} = 475000$ Nm, $F_{2\min} = 0$, $F_{2\max} = 41000$ N, $M_{3\min} = 0$, $M_{3\max} = 4000$ Nm, $P_{1\min} = -31000$ W, $P_{1\max} = 0$, $P_{2\min} = 0$, $P_{2\max} = 3000$ W, $P_{3\min} = 0$ W, $P_{3\max} = 20000$ W, $\alpha_0 = 0.5857$ rad, $x_0 = 7.5$ m, $r = 0.208$ m, $P_{st} = 35000$ W, $a = 1.5$ m, $b = 2.1$ m, $\lambda = 0.0872$ rad, $\theta = 0.3189$ rad, $z_0 = 0$, $v_2 = 0.2$ m/s, $\omega = 0.0157$ rad/s, $M_{1n} = 475000$ Nm, $M_{3n} = 5500$ Nm, $F_{2n} = 18500$ N.

The following values of polynomial coefficients were obtained using the VCT-PSO method: $(D_0, D_1, D_2, D_3, E_0, E_1, E_2, E_3, K_0, K_1, K_2, K_3) = (50, -1.82, 10.79, 33.37, 18.30, -6.40, -39.36, 50, -7.97, 39.55, 36.42, -50)$.

4. Results and discussion

In [26] (Appendix F), a plot of the convergence of criterion (54) during its minimisation is provided.

The graphs of the speeds of individual links (Fig. 3), the phase portrait of pendulum oscillations of a load on a flexible suspension (Fig. 4), driving forces and moments (Fig. 5) and power (Fig. 6) of the drive mechanisms illustrate the results obtained. In the graphs shown below, the grey curves correspond to the basic

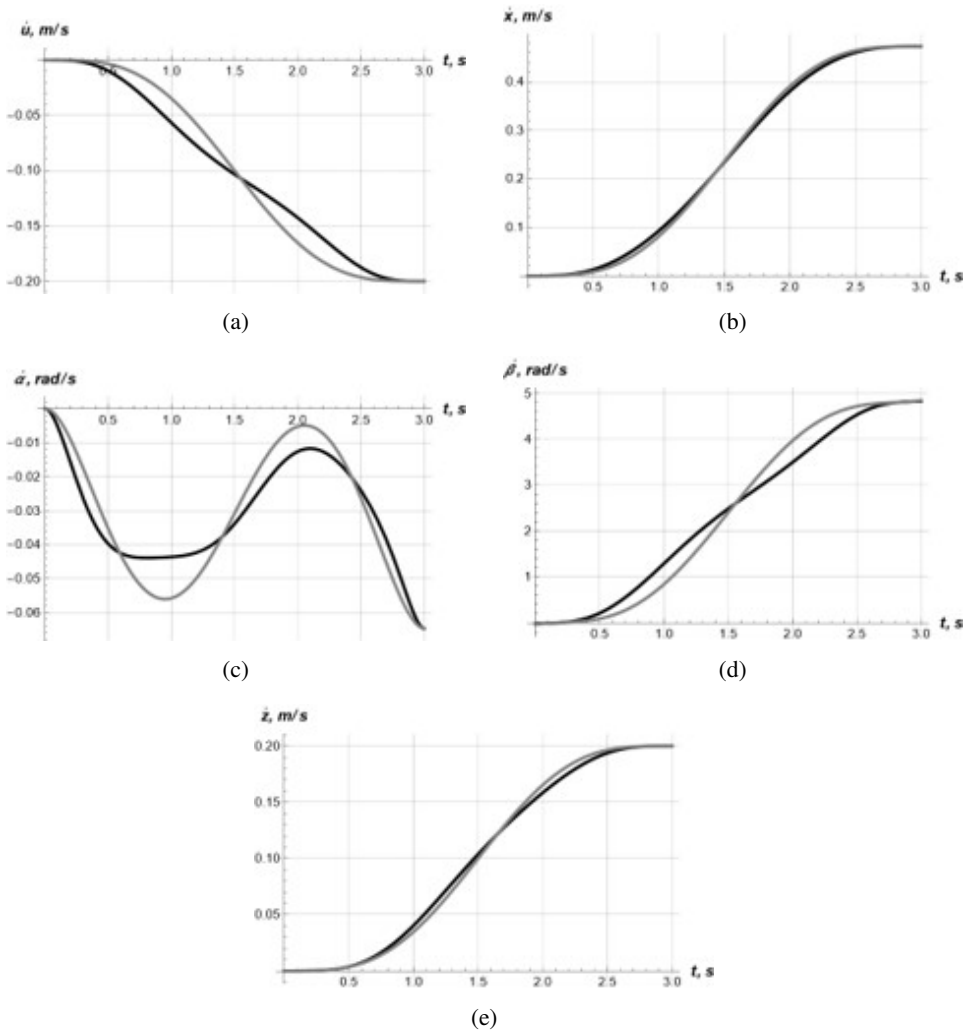


Fig. 3. Plots showing the variation in: (a) the rate of change of the flexible suspension length; (b) the rate of change of the load outreach; (c) the angular velocity of jib slewing; (d) the angular velocity of the drive drum of the load hoisting mechanism; (e) the velocity of the jib extension section relative to the main jib section

solution of the problem, and the black ones – to the optimal one. The base solution is a solution represented by a polynomial of the same order as the optimal one. It satisfies the boundary conditions of motion but does not satisfy the constraints and does not minimize the optimality criterion.

An analysis was then performed of the obtained graphical dependencies of the kinematic (Figs. 3 and 4), dynamic (Fig. 5) and energy (Fig. 6) characteristics of the jib system during the simultaneous start of the jib slewing mechanisms, extension of its section and load hoisting at a steady crane slewing.

Fig. 3 shows that the rates of change in the outreach and length of the flexible load suspension, the extension of the jib section and the rotation of the drive drum of the load hoisting mechanism change smoothly throughout the entire start-up process, both in the basic and optimal calculations. Moreover, the rates of change in outreach (Fig. 3b) and jib section extension (Fig. 3d) obtained in the basic and optimal calculations are practically the same. However, there are some deviations in the rates of change of the length of the flexible load suspension (Fig. 3a) and the rotation of the drum (Fig. 3d) between the results of these calculations. At the same time, when changing the angular velocity of the jib slewing (Fig. 3c), low-frequency oscillations are observed both in the basic and optimal calculations. At the same time, the amplitude of the jib speed oscillations in the optimal calculation is 40.4% smaller than that in the basic calculation.

The phase portrait of the load oscillations on a flexible suspension (Fig. 4) shows that the oscillations dampen during the start-up process, both in the basic and optimal calculations. At the same time, the deviation of the flexible load suspension from the vertical by 8.3% is smaller in the optimal calculation compared to the basic calculation. The phase portrait of the oscillations shows a slight deviation of 2 mm between the initial and final positions of the load, which is caused by the centrifugal force acting on the load when the crane is turning. This deviation occurs in both the basic and optimal calculations.

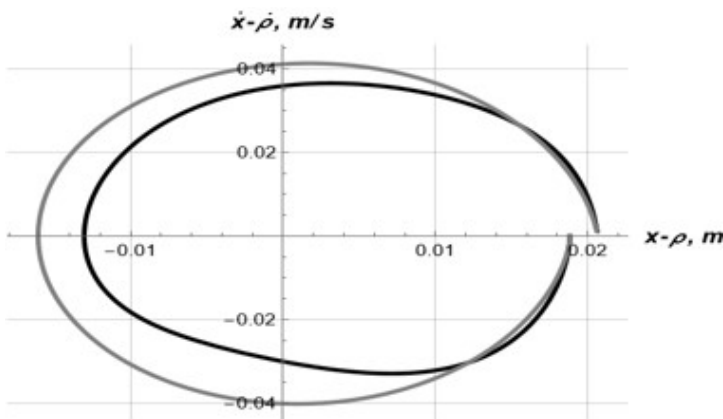


Fig. 4. Phase portrait plots of pendulum oscillations of a load

The driving forces of the drives (Fig. 5) show that they change in oscillatory mode, both in the basic and optimal calculations. At the same time, the maximum values of driving moments in the optimal calculation are 18.2% lower for the load hoisting mechanism and 3.1% lower for the jib slewing mechanism compared to the basic calculation of these mechanisms. The maximum values of the force of the jib section extension mechanism are practically the same for both calculation methods.

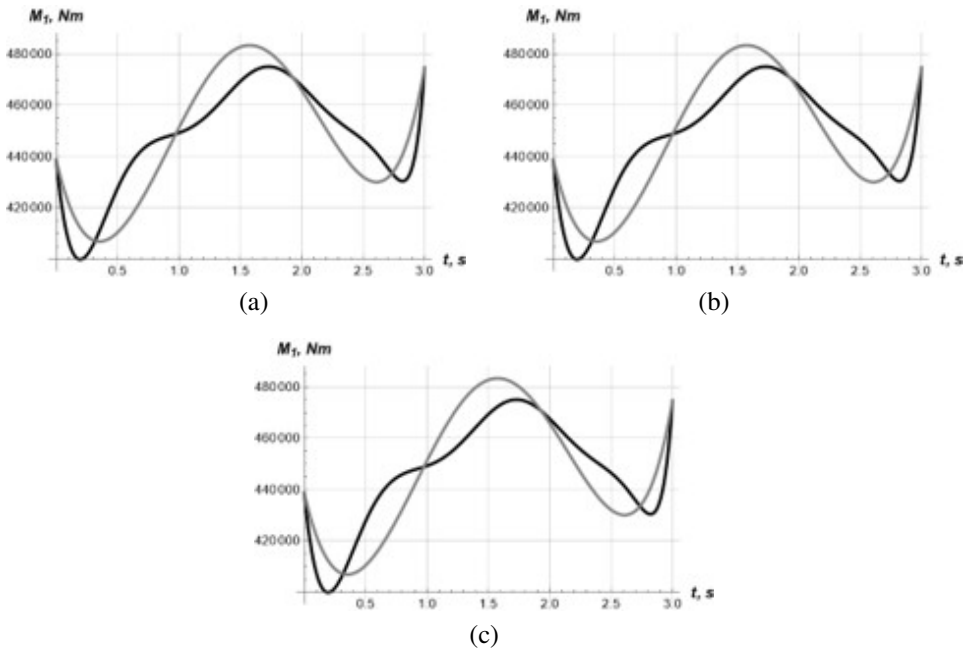


Fig. 5. Plots of combined driving torques and forces of mechanism drives: (a) luffing; (b) jib section extension; (c) load hoisting

The power of the jib slewing drive (Fig. 6a) varies in an oscillatory mode in both the optimal and basic calculations. The amplitude of power oscillations in the optimal calculation is 34.8% lower than the amplitude in the basic calculation. The dependencies of the jib extension drive power (Fig. 6b) are quite close in value and have a smooth character of change, both in the base and optimal calculations. The power of the load hoisting mechanism drive (Fig. 6c), determined by both calculation methods, changes quite smoothly. At the same time, the maximum power value determined by the optimal calculation is 5.6% lower than in the basic calculation.

The presented work is essentially similar to [23], which solves the optimization problem of simultaneous movement of two mechanisms for crane slewing and load hoisting during the start-up process with restrictions on the driving torques of the drives of the mechanisms under consideration. The presented article considers a

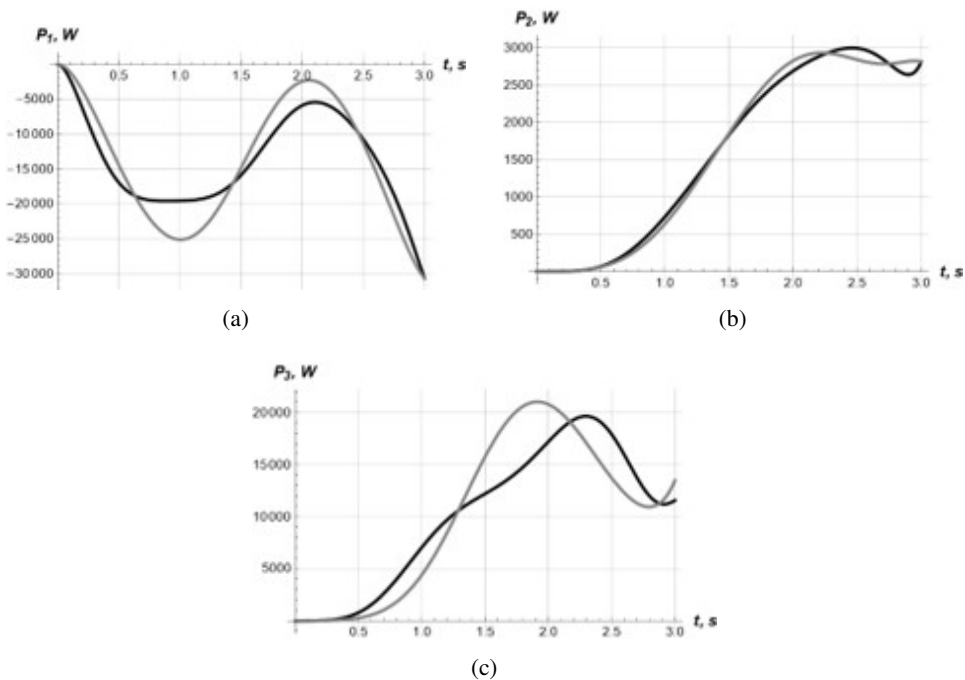


Fig. 6. Power plots of drive mechanisms: (a) jib slewing; (b) load hoisting; (c) jib section extension

more complex jib system design, where the jib consists of a root and extendable sections. For such a jib system, the optimization problem of simultaneous movement of four mechanisms is solved, taking into account luffing, load hoisting, jib section extension and crane rotation with restrictions on the driving torques and power of the drives. In this case, the first three mechanisms perform simultaneous start-up processes, while the fourth mechanism operates in a steady state. The article also proposes a method for determining the kinetic energy of the jib system during its spatial movement.

This paper also presents the root-mean-square and maximum values of the key performance indicators of the crane jib system, obtained for the baseline and optimal cases, as shown in Table 1.

The table above shows that, in almost all respects, the optimal calculation yields better results than the baseline calculation.

The present study is essentially similar to [1], in which the optimization problem of the simultaneous motion of two mechanisms – the crane slewing mechanism and the load hoisting mechanism – is solved during the start-up phase, subject to constraints on the driving torques of the drives of the considered mechanisms.

The present paper considers a more complex jib system configuration, in which the jib consists of a fixed and an extendable section. For such a jib system, the optimization problem of the simultaneous motion of four mechanisms has been

Table 1. Comparison of the results of the basic and optimal calculations

Parameter	Unit	Value corresponding to the solution	
		base	optimal
Root mean square indicators			
Pendulum oscillation angle of the rope with load	rad	0.0134	0.0121
Driving moment M_1	kNm	447	447
Driving force F_2	kN	17	17
Driving moment M_3	kNm	4.48	4.34
Power of luffing mechanism P_1	kW	16.3	15.2
Power of jib section extension mechanism P_2	kW	1.99	1.97
Power of load hoisting mechanism P_3	kW	12.6	12.1
Maximum indicators			
Pendulum oscillation angle of the rope with load	rad	0.0211	0.0206
Driving moment M_1	kNm	483	475
Driving force F_2	kN	18.7	18.4
Driving moment M_3	kNm	6.56	5.46
Power of luffing mechanism P_1	kW	30.7	30.7
Power of jib section extension mechanism P_2	kW	2.93	2.93
Power of load hoisting mechanism P_3	kW	21.0	19.8

solved: luffing, load hoisting, jib section extension, and crane slewing, subject to constraints on the driving torques and power of the drives.

In this case, the first three mechanisms perform a simultaneous start-up processes, while the fourth mechanism operates in a steady-state regime after achieving the optimal rotation mode, when there are no load oscillations on the flexible suspension in the plane of rotation and the angular velocity is constant.

The paper also proposes a method for determining the kinetic energy of the jib system during spatial motion.

5. Conclusions

Research on the simultaneous motion of the mechanism for load hoisting, luffing, and jib section extension during steady crane slewing was conducted to increase the productivity of the jib crane and reduce dynamic loads in its elements. The optimization problem, which includes a mathematical model of the dynamics of

simultaneous motion of mechanisms, optimization criteria, constraints on driving forces and energy consumption, and boundary conditions for the movement of mechanisms, was set for such a movement of mechanisms. The mathematical model of the dynamics of simultaneous motion of mechanisms is a system of five second-order nonlinear differential equations. A complex dimensionless criterion is used as the optimization criterion, which is a linear convolution of the RMS values of the driving forces of the drive mechanisms. The constraints here are the maximum and minimum values of the driving forces (torques) of the drives and their power. The boundary conditions of motion are the initial and final values of the generalised coordinates and their velocities, which are reduced to three coordinates and their derivatives with respect to time up to the fourth order inclusive at the beginning and end of the motion.

The optimization problem is constrained and non-linear, so an approximate method is used to solve it, and the problem is reduced to an unconstrained one. A specially formed dimensionless criterion, which includes the integral criterion and constraints, was introduced for this purpose. Unknown functions in the form of polynomials with two terms are represented in the following optimization problem solution. The first terms are selected polynomials that provide boundary conditions and represent the basic solution to the problem. The second ones are polynomials that contain unknown coefficients and correspond to zero boundary conditions and ensure minimisation of the formed dimensionless integral criterion. The VCP-PSO metaheuristic method, modified for solving problems of this class, is used for the numerical solution of the optimization problem.

As a result of the optimization, the modes of simultaneous start-up of the jib slewing drives, extension of load hoisting, luffing, and jib section extension were determined. The obtained optimal motion modes minimise the driving forces of the drives with the established restrictions on the maximum values of power and energy characteristics. The optimal start-up mode, compared to that obtained with basic calculation, provides better results in almost all indicators without imposing constraints or minimising the optimization criterion. For example, it made it possible to reduce the maximum values of the driving torque by 18.2% and the power of the hoisting mechanism drive by 5.6%, as well as the deviation from the vertical of the flexible load suspension by 8.3% compared to the basic calculation option.

Two calculation options were used to optimize the modes of simultaneous start-up of the mechanism for load hoisting, luffing, and jib section extension at a steady crane slewing. The basic and optimal options were compared with each other and it was shown that the optimal calculation method provides better results for most of the indicators, while some indicators of the basic and optimal options are almost the same. The basic calculation option provided only the boundary conditions for the movement of the jib system mechanisms, and their optimization coefficients were zero. The optimal calculation method, in addition to the boundary conditions, also provided constraints on the driving forces and power of the drives

and made it possible to determine the optimal modes for the simultaneous start-up of the mechanisms. Therefore, when selecting the movement modes of crane mechanisms, preference should be given to the optimal calculation method, and the basic method should be used for approximate preliminary calculations.

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