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Theoretical analysis on bio-magnetic Casson hybrid nanofluid blood flow through a stenotic artery with a heat source/sink

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Abstract

The current study focuses on numerical analysis of magnetohydrodynamic Casson hybrid nanofluid blood flow carrying hybrid nanoparticles through a porous stenotic artery. In order to improve thermal conductivity and flow management in the presence of heat source/sink, gold-silver nanoparticles are suspended in a base fluid blood. The governing highly non-linear differential equations for momentum and energy were solved employing the MATLAB's bvp4c solver, ensuring high accuracy and stability. The effects of various physical parameters including magnetic parameter, permeability, curvature, Casson parameter, Eckert number and heat source/sink parameter are examined on velocity, temperature, skin friction, and Nusselt number profiles. Results indicate that higher magnetic field strength and permeability declined velocity due to Lorentz and frictional resistance, while increases in curvature and Casson parameter improve flow rates. Temperature distribution is notably elevated by magnetic forces, viscous dissipation and internal heating, with hybrid nanoparticles contributing to enhanced thermal energy. The correctness of the model is validated by its strong agreement with previously reported results. These results offer important new information for biological uses such as tailored medication delivery, heat-assisted therapies and blood flow regulation in diseased arteries, offering a reliable computational framework for future research involving nanoparticle-assisted magnetohydrodynamic flow through porous biological channels.

Keywords: Blood/gold-silver nanoparticles; Bvp4c method; Stenotic artery; Heat source/sink; Casson fluid

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1. Introduction

Human physiological fluids serve essential purposes in lubrication, protection and transportation. Blood is the most intricate physiological ionic fluid that conveys essential components, including organic compounds, nutrients and oxygen to cells, while simultaneously removing waste products from metabolism and carbon dioxide from those cells, constituting 7% of the human body weight [1]. Among many components that make up blood are enzymes, hormones, sugar glucose, ionised mineral substances, carbon dioxide, cellular membranes, haemoglobin and proteins (albumin, globulin and fibrinogen). It shields the body from mechanical and microbiological damage and controls the body's overall temperature, osmotic pressure and pH [2]. Blood is a non-Newtonian fluid. Examining non-Newtonian fluids in connection to extending surfaces has received attention in rec-

Nomenclature

C_f – skin friction
C_p – specific heat capacity, J/(kg K)
(ρC_p) – effective heat capacity, J/(m^3 K)
Ec – Eckert number
$f'(\eta)$ – dimensionless velocity
k – thermal conductivity, W/(m K)
K – porosity parameter
K^* – permeability of porous medium, m^2
L_0 – length, m
M – magnetic parameter
Nu – Nusselt number
Pr – Prandtl number
Q – heat source parameter
r – radial coordinate, m
R – width of unblocked region, m
R_0 – artery radius, m
$Re_r^{1/2}$ – local Reynolds number
t – time, s
T – temperature, K
u, υ – velocity component, m/s
x – axial coordinate, m

Greek symbols

β – Casson parameter
γ – curvature parameter
η – similarity index
θ – dimensionless temperature
μ – viscosity, kg/(m s)
ν – kinematic viscosity, m^2/s
ρ – density, kg/m^3
σ – electrical conductivity, S/m
ϕ_1 – nanoparticle volume fraction
χ – dimensionless maximum height of the stenosis
ψ – stream function, m^2/s

Subscripts and Superscripts

f – fluid
hnf – hybrid nanofluid
w – wall
$(\cdot)'$, …, $(\cdot)'''$ – first- and higher-order differentiation with respect to η

Abbreviations and Acronyms

HNF – hybrid nanofluid
MHD – magnetohydrodynamics

ent decades. The unique and diverse applications of these fluids have garnered significant interest, including their roles in ink-jet printing, polymer manipulation and geological flow systems, among others.

The Casson fluid model is a well-known non-Newtonian rheological model that was created to explain the flow behaviour of viscoplastic fluids with shear-thinning and yield stress properties. It was originally introduced by Casson to characterise the flow of printing inks and has since been extended to model a variety of complex fluids. Human blood, honey, tomato sauce and jelly exemplify the Casson fluid. Numerous academics are concentrating on the examination of Casson fluid behaviour due to its substantial applications. Casson fluid flow behaviour in an unstable boundary layer across an extending surface was studied in [3]. A quantitative study of Casson fluid dynamics on flat and cylindrical surfaces, taking into account elements like chemical reactions and Joule heating, was carried out in [4] using computational techniques to assess the impact of several parameters on heat and mass transfer rates. In [5], the peristaltic flow of a Casson hybrid nanofluid was examined in a wavy-walled channel, deliberating the influence of induced magnetic fields, heat generation and entropy production. The analysis highlights the impact of the Casson fluid parameter on flow impedance and thermal transport, as well as the use of hybrid nanoparticles in enhancing thermal conductivity and heat transfer. Mehdi et al. [6] focused on entropy generation in magnetohydrodynamic Casson fluid flow with the effects of thermal radiation in a porous micro-annulus, and the study provides valuable insights into energy efficiency and thermal management in engineering systems. A numerical simulation of mass and heat movement in hydromagnetic Casson fluid flow under sinusoidal boundary conditions was conducted in [7]. The study offers a thorough examination of how different physical characteristics affect the flow behaviour.

Unusual local intravascular endothelial development in many blood vessel locations, such as the abdominal, cerebral, carotid, thoracic and femoral arteries, can exacerbate arterial stenosis [8]. One of the most harmful outcomes of arterial stenosis is increased resistance and reduced blood flow to the particular vascular bed that the artery supplies. The study of stenotic hemodynamics in narrowed arteries is crucial because it has many applications in cardiovascular diseases such as atherosclerosis, aneurysms, angina or heart attacks, which are among the leading causes of death worldwide. Cardiovascular diseases, which commonly arise in the coronary arteries, are associated with fatty material accumulation inside the arterial wall lumen. Numerous theoretical and experimental studies have examined the specific causes of stenosis formation and how it affects blood flow properties [9].

Coughing, chest pain, fatigue, palpitations, fainting, breathing difficulties, weakness and dizziness are some of the symptoms of artery stenosis. Arteriosclerosis is the name given to the conditions that arise from a partial blockage that impairs blood circulation and poses serious health risks, such as strokes, cerebral ischemia and heart ischemia [10,11]. The blood vessel's permeability is a crucial characteristic of actual arteries. This characteristic enables oxygen and other essential nutrients to diffuse into the blood flow as it flows through the channel. Hemodynamic transport with artery wall permeability has been the subject of numerous computer studies in recent years. Adopting a cylindrical artery model balances computational simplicity with physiological relevance. It captures key hemodynamic features, that is, velocity profiles – WSS (wall shear stress) – pressure gradients and facilitates efficient simulation of complex flow with nanoparticles and magnetic fields. Pincombe and Mazumdar [12] investigated the impact of post-stenotic dilatation on the blood flow pattern; the non-Newtonian character of the blood was covered in their study. Additionally, Pincombe et al. [13] discussed the same result in relation to coronary arteries that have post-stenotic dilatation. Wong et al. [14] later provided a theoretical study, in which they computed the blood flow resistance in a damaged artery. From the analysis of Akbar et al. [15], exact solutions for the unsteady pressure-driven peristaltic transport in regular and irregular channels with temperature-dependent nanofluid viscosity were obtained. A comprehensive investigation on hemodynamic flow in a composite artery with a porous wall was carried out in [16]. In the investigation, blood was considered a Newtonian fluid. Moreover, Singh et al. [17] conducted a theoretical investigation of magnetohydrodynamic blood flow through a porous artery with stenosis, Tripathi [18] performed a theoretical study of blood flow in tilted arteries affected by an obliquely applied magnetic field, and Eldesoky [19] investigated unsteady pulsatile magnetohydrodynamic blood flow through a porous medium with slip boundary conditions and body acceleration effects. Furthermore, several studies have reported the effects of permeability and numerous stenoses.

The magnetic field is important because it helps to regulate blood flow during surgery or thermal therapy. Therefore, the importance of heat transfer through a cylindrical artery using the Casson liquid and gold nanoparticles (GNPs) was investigated [20]. Using blood Au nanoparticles and micropolar nanoliquid from a curved surface that shrinks and stretches, a new study on radiative flow was carried out by Nisar et al. [21]. Akbar et al. [22] investigated the magnetic force effects and differently shaped nanoparticles in diverging tapering arteries with stenoses via a blood flow model. There has not been any research done on using metallic nanoparticles of different shapes with water as the base fluid. A radially symmetric but axially non-symmetric stenosis is used to depict the blood flow. A further significant contribution of the present work was the examination of the symmetric distribution of wall shear stress associated with resistive impedance, as well as the enhancement of these quantities with increasing stenosis severity. Surgical site infections are reduced when silver nanoparticles are applied to surgically implanted catheters. In the emerging discipline of nanomedicine, it may also serve as a thrombolytic and anticoagulant [23,24]. It also possesses anti-inflammatory, anti-fungal, anti-permeability, anti-angiogenic and drug-carrying qualities. Hence, hybrid nanofluid (HNF) produced from blood with suspended Au and Ag nanoparticles is considered a highly potent and sophisticated chemical instrument for the diagnosis and management of illnesses like HIV (human immunodeficiency virus), cancer, heart issues and other infections. Heat transmission using Ag-Au/blood hybrid nanofluid flow via an artery stenosis was investigated in [25]. A 3D numerical model making use of blood as the base fluid was investigated in [26] for tracking velocity, temperature and pressure in a pathologically dilated artery with Ag-Au. Zaman et al. [27] studied the effect of MHD (magnetohydrodynamics) on power law flow through an angled artery. In a channel subjected to pressure fluctuations and an external magnetic field, Eldesoky et al. [28] examined the MHD flow and peristaltic heat transfer of a suspended viscous fluid.

The influence of heat transfer on MHD flow of micropolar fluids across porous media was shown by [29]. Rehman et al. [30] studied magnetised and non-magnetised Casson fluid flow around a cylinder, finding reduced flow momentum under a magnetic field. This offers insights into the behaviour of non-Newtonian fluids when magnetic fields are present. Jena et al. [31] looked at how an inclined magnetic field could interact with nanofluid to improve its heat transfer properties. Pattnaik et al. [32] looked at how velocity slip affected the MHD flow of a micropolar fluid across a stretching surface, presenting alterations in velocity due to suction and enhanced rate coefficients. This type of flow can occur in arteries that have been stenosed, or narrowed, due to plaque buildup. A higher value of the Brownian motion parameter enhances the fluid temperature, according to a quantitative analysis of the magnetohydrodynamic blood flow via a permeable artery with radiative influence by [33]. Some noteworthy results in nanofluid flow issues with MHD impact are presented in the following papers: Ali et al. [34] presented magnetohydrodynamic flow of a nanofluid past a moving slender needle, considering nanoparticle aggregation and viscous dissipation effects. In another work, Ali et al. [35] investigated magnetohydrodynamic flow of a tangent hyperbolic nanofluid over a vertically stretched surface using Levenberg-Marquardt backpropagation-based artificial neural networks. Further, Ali et al. [36] examined an unsteady magnetohydrodynamic fluid flow between two parallel plates experiencing uniform oscillations, considering modified Hall current and activation energy effects, and, recently, utilised artificial neural networks to model MHD flow of a tangent hyperbolic nanofluid containing motile microorganisms over a vertically stretched slender surface [37]. Jalali et al. [38] investigated the simulation of pulsatile blood flow for magnetic drug targeting (MDT), focusing on drug particle dispersion through symmetric and asymmetric stenosed vessels. The research of Akbar et al. [39] uses the finite element method (FEM) to produce numerical solutions and artificial neural networks (ANN) to enhance visualisation in a computational study of Prandtl-nanofluid flow inside tapered arteries. The nanofluid exhibits enhanced flow and heat transfer characteristics relative to pure blood, suggesting its effectiveness in biomedical therapies like precise drug delivery and blood flow simulation.

Recent advances in nanomedicine have introduced the use of hybrid nanofluids-mixtures of two or more different nanoparticles dispersed in a base fluid for enhanced drug delivery, diagnostic imaging and thermal therapy. Among various nanomaterials, silver and gold nanoparticles have emerged as particularly attractive due to their high thermal conductivity, biocompatibility, and antibacterial and plasmonic properties. Gold nanoparticles are extensively used in photothermal cancer therapy and biosensors, while silver nanoparticles are prominent in vascular stents, antibacterial coatings and controlled drug release systems [40,41]. The glaring absence of research on the behaviour of non-Newtonian Casson fluid inside a cylindrical stenosed artery in the previously stated studies reveals a substantial research gap. Additionally, more needs to be done to understand the intricate relationships that exist between blood flow in arteries that are cylindrically stenosed and physical processes like the MHD effect. In the research of blood flow in this complex vascular structure, there is still much to learn about the study of nanoparticles, particularly four distinct types: copper (Cu), aluminium oxide, silver (Ag), and gold (Au) of cylindrical shape. The characteristics and applications of various nanoparticle kinds in medicine and drug delivery vary. By closely examining their blood flow dynamics, particularly in complex geometries like cylindrically stenosed arteries, there may be new developments in targeted medication delivery and medical treatments.

To address the aforementioned shortcomings, the use of cylindrical-shaped silver and gold nanoparticles is considered in our work. These particles have great heat conductivity and a high aspect ratio, among other special thermal characteristics. When cylindrical-shaped silver and gold nanoparticles are added to a base fluid, the base fluid's viscosity, flow characteristics and thermal conductivity are all improved. The study by Tanveer et al. [42] investigated magnetohydrodynamic flow and heat transfer of a silver–aluminium oxide (Ag-Al_2O_3) hybrid nanofluid within a trapezoidal cavity. A Cu-Al_2O_3/water hybrid nanofluid flowing via a wavy enclosure exposed to inclined magnetic fields was investigated by [43].

Research by Zaman et al. [44] on the effect of magnetic force on hybrid nanoliquid embedded with a heat source/sink on blood flowing beyond stenosed arteries shows that a multiplication of hybrid nanoparticles leads to an amplification of the heat transfer rate in biological systems. Modelling the tissue or vascular environment as a porous medium provides an effective framework for capturing the complex interactions between fluid and structural components. Tissues such as the extracellular matrix (ECM), capillary walls and cellular matrices present significant hydraulic resistance to fluid flow. This porous medium approach becomes particularly critical in pathological conditions like atherosclerosis, tumour-induced angiogenesis, or fibrotic tissues, where abnormal cellular growth or plaque deposition severely alters the porosity and permeability of the medium. In atherosclerotic arteries, for instance, the accumulation of lipids and immune cells narrows the vessel lumen and changes the local porosity, leading to elevated shear stress and reduced perfusion. Similarly, in the tumour microenvironment, irregular and leaky vasculature surrounded by dense stromal tissue behaves like a low-permeability porous medium. Modelling such zones helps predict nanoparticle-based drug delivery, solute transport and pressure gradients more accurately [45,46]. Furthermore, the interaction between blood plasma, macromolecules and therapeutic nanoparticles within these porous media dictates the diffusive and convective mass transfer processes. Therefore, porous media models are extensively used to simulate filtration across capillary membranes, interstitial fluid movement and lymphatic clearance, offering a more physiologically realistic perspective [47,48].

Incorporating non-uniform heat sources and sinks in thermal transport models is essential for representing spatially heterogeneous thermal phenomena in biological tissues [49]. Unlike conventional uniform source models, non-uniform terms account for localised metabolic activities, inflammation-induced heating, external therapies like laser ablation, hyperthermia or cryotherapy, and blood perfusion-dependent heat redistribution [50]. For instance, during laser-based photothermal therapy, the absorption of light by target chromophores generates localised heat that decays radially. A spatially varying source term can accurately capture this radial thermal profile [51]. Likewise, in feverish tissues, elevated metabolic heat production is not uniformly distributed but varies with inflammatory cytokine gradients. From a mathematical modelling perspective, these heat source/sink terms are commonly introduced as space-dependent functions (e.g., Gaussian or exponential profiles) in the bioheat equation (Pennes or dual-phase-lag models). Their presence significantly impacts temperature fields, tissue damage thresholds and thermal conductivity profiles, especially under transient or nonlinear physiological conditions. Moreover, entropy generation due to thermal gradients and viscous dissipation becomes pronounced in such scenarios, making it a valuable metric in the thermodynamic optimisation of hyperthermia protocols and heat-sensitive nanoparticle delivery systems [52].

In this work, the movement of blood containing Au-Ag nanoparticles in a Casson hybrid nanofluid through a stenosed artery embedded in a porous medium is examined in the presence of thermal radiation and a heat source/sink. The model incorporates magnetohydrodynamics and a porous medium, which are mainly used in physiologically and biomedically relevant contexts. The study's main focus is on analysing the impacts of several parameters, including the magnetic parameter, radiation parameter and porosity parameter. Numerical solutions are found for the governing nonlinear partial differential equations using the bvp4c method in the MATLAB solver. The present study has applications in various sectors in microcirculatory transport, hyperthermic tumour ablation and nanoparticle-assisted drug targeting, offering potential improvements in clinical diagnostics and personalised therapy planning.

2. Motivation of the proposed model

Cardiovascular diseases involving arterial stenosis require precise modelling of blood flow to improve diagnostic and therapeutic techniques. The effect of magnetohydrodynamics on blood moving through arteries with cosine-shaped stenosis was examined. This study is motivated by the need to bridge that gap by exploring hybrid nanoparticle-enhanced Casson fluid flow through a porous stenotic artery under magnetic and thermal influences. Casson fluid models better represent blood rheology under low shear conditions, accounting for the yield stress exhibited by blood in microcirculatory systems. Gold-silver nanoparticles offer superior heat conduction, while magnetic fields provide control over targeted blood flow modulation. By numerically analysing the impact of key parameters, this work aims to offer insights relevant to biomedical uses such as heat treatments and magnetically guided medication distribution, contributing to more effective and minimally invasive cardiovascular treatment strategies and advancing biomedical technologies reliant on magneto-thermal control of blood flow.

Research questions

1) How bio-magnetic forces (Lorentz force) influence hybrid nanofluid blood flow in a stenotic artery?
2) How Casson rheology modifies velocity, shear stress and temperature distribution?

3) How a heat source/sink affects thermal regulation within a partially occluded artery?
4) How hybrid nanoparticles enhance heat transfer and modify hemodynamic resistance?

3. Mathematical formulation of the problem

1) In the mathematical analysis, the incompressible 2D Casson hybrid nanofluid flow through a stenosed artery is assumed. Moreover, the effects of the magnetic field, porous medium and heat source/sink on the Casson hybrid nanofluid flow through a stenosed artery are examined.
2) Arterial blood is assumed to flow through an incompressible stenosed artery, and the characteristic half-length of the stenosis is taken as $L_0/2$. The cylindrical coordinates are considered (r, θ, x), where the x-axis is taken along the direction of the horizontal artery, while θ and r represent the radial and circumferential directions, respectively.
3) In the current model, the flow is considered along the axial direction x and r is perpendicular to the flow. The temperature T is set to the arterial wall temperature T_w, signifying that the wall is preserved at a constant temperature.
4) In the physical model, blood flow is along the x-axis and the r-axis is taken perpendicular to the blood flow, as presented in Fig. 1; thermophysical properties of hybrid nanofluid are shown in Table 1.
5) As Fig. 1 portrays, blood is moving over a stenosed blood artery with an unobstructed area of $2R_0$; $R(x)$ is the radius of the artery and χ is the maximum height of stenosis.

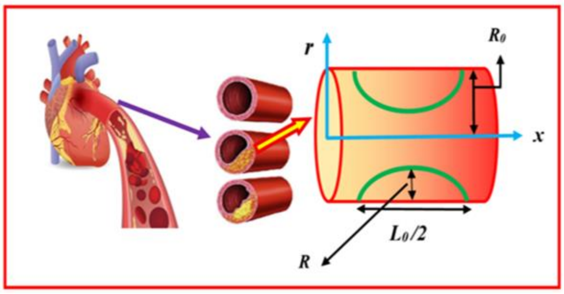


Fig. 1. The artery's physical structure.

Table 1. Thermophysical characteristics of base fluid and nanoparticles (Shahzad et al. [57]).

Property	Unit	Blood	Silver (Ag)	Gold (Au)
ρ	kg/m^3	1063	10 500	19 282
C_p	J/(kg K)	3594	235	129
k	W/(m K)	0.492	429	310
σ	1/(Ω m)	0.667	6.3×10^7	4.1×10^6
Pr	–	21		

A stenosis profile is constructed to represent the stenosed region as follows:

$$\frac{R(x)}{R_0} = \begin{cases} 1 - \frac{\chi}{2}\left[1 + \cos\left(\frac{4\pi x}{L_0}\right)\right], & -\frac{L_0}{4} < x < \frac{L_0}{4} = R_0, \\ 1 & , \text{ otherwise.} \end{cases} \quad (1)$$

The mathematical flow equations are as follows [53−56]:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \quad (2)$$

$$u\left(\frac{\partial}{\partial x} + v\frac{\partial}{\partial r}\right)u = \frac{\mu_{hnf}}{\rho_{hnf}}\left(1 + \frac{1}{\beta}\right)\frac{\partial}{r\partial r}\left(r\frac{\partial u}{\partial r}\right) - \frac{\mu_{hnf}}{\rho_{hnf}K^*}u - \frac{\sigma_{hnf}B^2}{\rho_{hnf}}u, \quad (3)$$

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial r}\right)T = \frac{k_{hnf}}{(\rho C_p)_{hnf}}\frac{\partial}{r\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\sigma_{hnf}B^2u^2}{(\rho C_p)_{hnf}} + \frac{Q_0}{(\rho C_p)_{hnf}}(T - T_\infty), \quad (4)$$

where u and v are the velocity components, T is the temperature, σ_{hnf}, μ_{hnf}, ρ_{hnf} and k_{hnf} are the electrical conductivity, viscosity, density and thermal conductivity of hybrid nanofluids, respectively, β is the Casson parameter, $(\rho C_p)_{hnf}$ is the effective heat capacity, K^* represents the permeability of the porous medium, B is the applied magnetic field strength, and Q_0 is the volumetric flow rate.

The boundary conditions are:

$$\left.\begin{array}{ll} u = u_0,\ v = 0 \text{ and } T = T_w & \text{at } r = R(x), \\ u \to 0, \text{ and } T \to T_\infty & \text{as } r \to \infty \end{array}\right\}, \quad (5)$$

where u_0 is the characteristic velocity.

The thermo-physical nature is determined by:

$$A_1 = \frac{\mu_{hnf}}{\mu_f},\quad A_2 = \frac{\rho_{hnf}}{\rho_f},\quad A_3 = \frac{\sigma_{hnf}}{\sigma_f},\quad A_4 = \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f},\quad A_5 = \frac{k_{hnf}}{k_f}, \quad (6)$$

where A_1, A_2, A_3, A_4, A_5 denote the viscosity ratio parameter, density ratio parameter, electrical conductivity ratio parameter, effective heat capacity ratio and thermal conductivity ratio parameters, respectively.

The appropriate self-similarity modulations are:

$$\left.\begin{array}{l} u = \frac{u_0 x}{L_0}f'(\eta),\quad v = -\frac{R}{r}\sqrt{\frac{u_0 v_f}{L_0}}f(\eta),\quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \\ \eta = \frac{r^2 - R^2}{2R} \end{array}\right\}, \quad (7)$$

where v_f is the kinematic viscosity of the fluid and η is the similarity index.

The modified Eq. (1) can be written as:

$$f = \begin{cases} 1 - \frac{\chi}{2}[1 + \cos(4\pi x)], & -\frac{1}{4} < x < \frac{1}{4}, \\ 1 & , \text{ otherwise.} \end{cases} \quad (8)$$

where $f = \frac{R(x)}{R_0}$, and $\chi = \frac{\lambda}{R_0}$ is the non-dimensional measure of stenosis in the referring artery.

Substituting Eq. (7) into Eqs. (2)−(4), modified dimensionless equations are obtained:

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\left[(1+2\gamma\eta)\left(1+\frac{1}{\beta}\right)f''' + 2\gamma f''\right] + ff'' - f'^2 - \frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}Kf' - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f}Mf' = 0, \tag{9}$$

$$\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}\frac{1}{\Pr}[(1+2\gamma\eta)\theta'' + 2\gamma\theta'] + f\theta' - f'\theta + \frac{\sigma_{hnf}/\sigma_f}{(\rho C_p)_{hnf}/(\rho C_p)_f}M\mathrm{Ec}(f')^2 + \frac{\Pr Q}{(\rho C_p)_{hnf}/(\rho C_p)_f}\ \theta = 0, \tag{10}$$

where:

$$\gamma = \sqrt{\frac{\upsilon_f L_0}{u_0 R^2}}, \quad K = \frac{\mu_f L_0}{\rho_f K^* u_0}, \quad M = \frac{\sigma_f B^2 L_0}{u_0 \rho_f}, \quad \Pr = \frac{k_f}{(\mu C_p)_f}, \quad \mathrm{Ec} = \frac{U_w^2}{c_p(T_w - T_\infty)},$$

are the curvature parameter, porosity parameter, magnetic parameter, Prandtl number, Eckert number, and U_w is the wall velocity.

The dimensionless boundary conditions are:

$$\left.\begin{array}{l} f(\eta)=0,\ \ f'(\eta)=1,\ \ \theta(\eta)=1,\ \ \eta=0, \\ f'(\eta)\to 0,\ \ \theta(\eta)\to 0,\ \ \eta\to\infty \end{array}\right\}. \tag{11}$$

The physical quantities of C_f and Nu are:

$$C_f = \frac{2\tau_w}{\rho_f U_w^2}, \quad \tau_w = \mu_{hnf}\left.\frac{\partial u}{\partial r}\right|_{r=R}, \tag{12}$$

$$\mathrm{Nu} = \frac{x q_w}{k_f(T_w - T_\infty)}, \quad q_w = -k_{hnf}\left.\frac{\partial T}{\partial r}\right|_{r=R}, \tag{13}$$

where τ_w is the wall shear stress and q_w represents the heat flux.

Equations (12)–(13) take the following dimensionless form:

$$\mathrm{Re}_r^{1/2} C_f = A_1 f''(0), \tag{14}$$

$$\mathrm{Re}_r^{1/2}\mathrm{Nu}_r = -A_5\ \theta'(0), \tag{15}$$

where $\mathrm{Re}_r^{1/2}$ represents the local Reynolds number, and Nu_r denotes the local Nusselt number.

4. Numerical methodology

The highly nonlinear coupled ordinary differential equations (ODEs) (9)−(11) are challenging to resolve numerically. A number of methodologies may be employed to solve nonlinear differential equations, including boundary layer movement. Several methods may be used, which involve semi-analytical, analytical, and computational approaches. In order to solve this numerically, the present study used the bvp4c MATLAB package.

Step 1: New unknown variables are introduced into the system of higher-order non-linear ordinary differential equations, following the procedure:

$$f = \xi_1,\ f' = \xi_2,\ f'' = \xi_3,\ f''' = \xi_3', \quad \theta = \xi_4,\ \theta' = \xi_5,\ \theta'' = \xi_5'.$$

Step 2: To solve the higher-order ODEs effectively, this method reduces them to a system of first-order ODEs, to simplify the computation in MATLAB solver by utilising syntax of the bvp4c solver like *sol* = *bvp4c* (*codesys*, *bcfun*, *solinit*):

$$\xi_1' = \xi_2,$$
$$\xi_2' = \xi_3,$$
$$\xi_3' = -\left[\frac{1}{(1+2\gamma\eta)\left(1+\frac{1}{\beta}\right)\frac{A_1}{A_2}}\left[\frac{A_1}{A_2}2\gamma\xi_3 + \xi_1\xi_3 - {\xi_2}^2 - \frac{A_1}{A_2}K\xi_2 - \frac{A_3}{A_2}M\xi_2\right]\right],$$
$$\xi_4{}' = \xi_5$$
$$\xi_5' = -\left[\frac{1}{(1+2\gamma\eta)\frac{A_5}{A_4}\frac{1}{\Pr}}\left[\frac{A_5}{A_4}\frac{1}{Pr}2\gamma\xi_5 + \xi_1\xi_5 - \xi_2\xi_4 + \frac{A_3}{A_4}M\mathrm{Ec}{\xi_2}^2 + \frac{1}{A_4}\Pr Q\xi_4\right]\right],$$

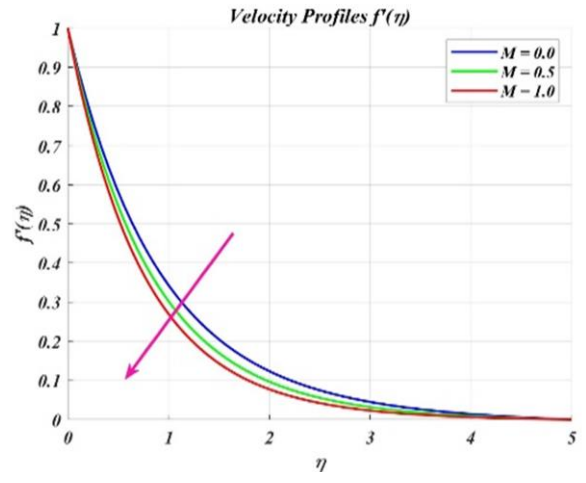
and the boundary conditions are:

$$\left.\begin{array}{l} \xi_1(\eta)=0,\ \ \xi_2(\eta)=1,\ \ \xi_4(\eta)=1\ \ \text{at}\ \eta=0, \\ \xi_2(\eta)\to 0,\ \ \xi_4(\eta)\to 0\ \ \text{as}\ \eta\to\infty \end{array}\right\}.$$

For the numerical computations, a convergence tolerance of 10^{-6} and a step size of 0.01 were employed throughout the analysis. When compared with the shooting approach and the bvp4c MATLAB software, this level of accuracy is much greater.

5. Results and discussion

This section focuses on exploring the numerical implications through the utilisation of physical descriptions. The goal is to quantify the parameters of heat transfer and flow through a stenosis artery by employing the hybrid nanoparticles (Au and Ag), as well as blood as the base fluid. Velocity and temperature



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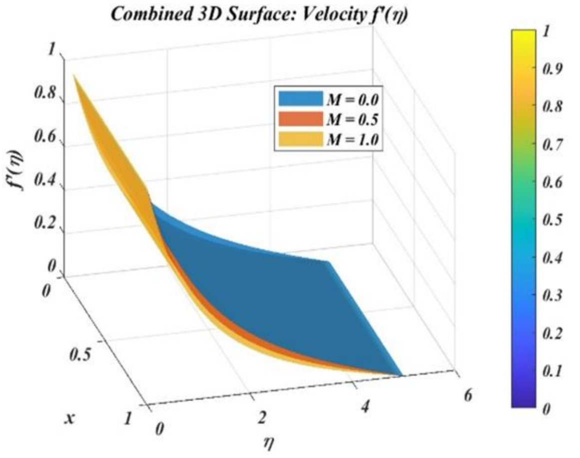
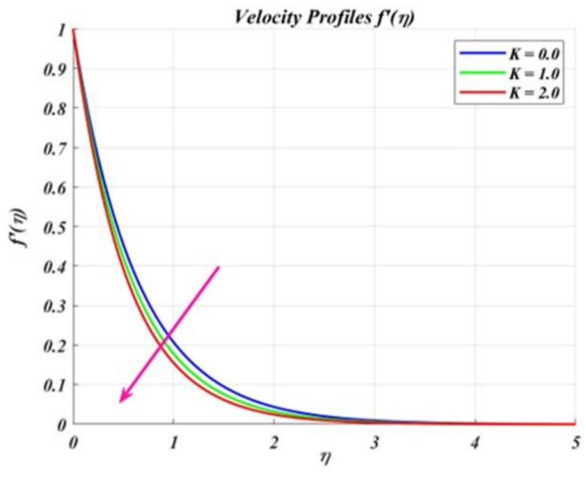


(b)

Fig. 2. Impact of *M* on the velocity profile.



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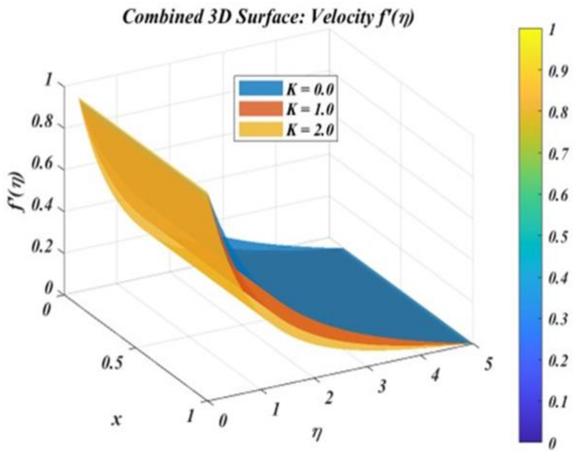


(b)

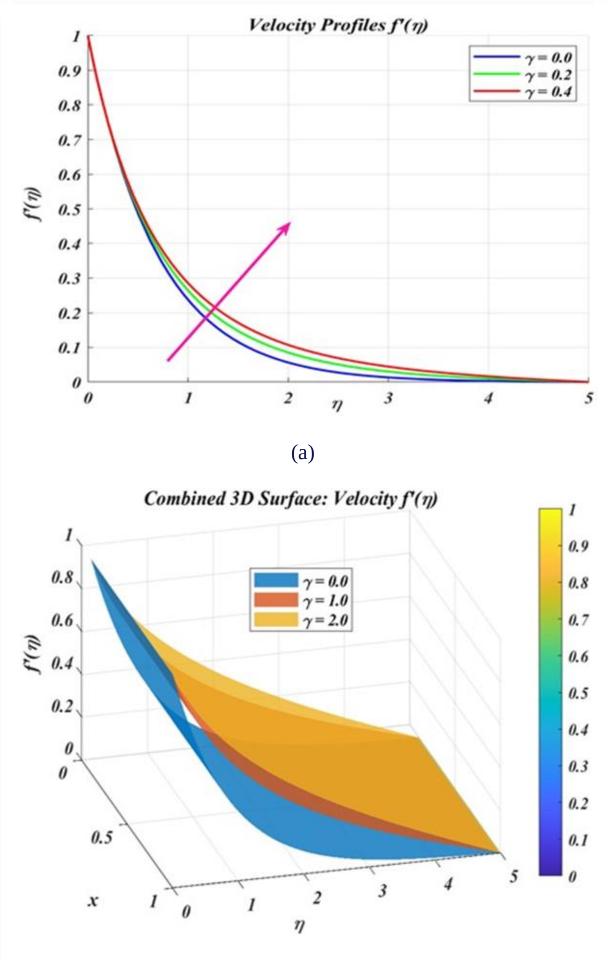
Fig. 3. Impact of *K* on the velocity profile.

graphs reveal various key characteristics. The study used computational methods to find the right answer to the altered highly chaotic coupled equations that described how the Casson nanofluid flowed along a stenotic artery. This research phase made use of the bvp4c procedure in the MATLAB solver, as well as the current examination and a number of significant active parameters. Within these ranges, the figures show the impact of several significant factors: $0.0 < M < 0.5$, $0.0 < K < 0.2$, $0.0 < \gamma < 0.4$, $0.0 < \beta < 1.0$, $0.0< Q < 0.6$, $0.0 < \mathrm{Ec} < 0.2$.

Figure 2 shows a declining trend in the velocity profile as *M* (magnetic field strength) increases. The presence of an electrically charged fluid has been shown to generate Lorentz power, which works against the natural inclination of blood to flow through arteries in the same direction as a magnetic field that is being applied.

The charged and magnetic particles in blood will probably spin when a magnetic field is applied to it because of the magneto force. With increasing resistance, more flow energy is converted into frictional losses, thereby diminishing the fluid's capacity to maintain its velocity. Figure 3 shows a decline in the velocity profile with a higher value of *K* (permeability parameter), as the resistance dissipates more of the flow energy into frictional losses, reducing the fluid's capacity to sustain velocity. Figure 4 illustrates how the curvature parameter (γ) affects the velocity profile; that is, the velocity profile increases as γ values rise. Physically, the blood vessel's curvature changes when the stenosis causes the arterial walls to shrink. Changes in blood circulation and associated patterns may result from this occurrence, which could have significant physiological implications.

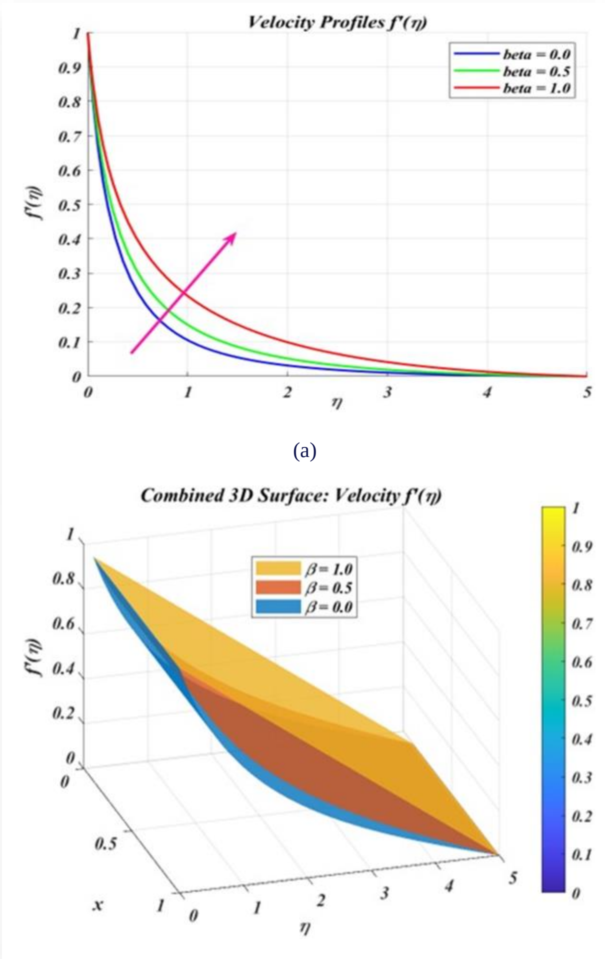
As observed from Fig. 5, the velocity profile increases as the Casson parameter (β) becomes larger; this trend occurs because the fluid's resistance to deformation decreases when its yield stress decreases, which is correlated with a larger β value. Consequently, the fluid exhibits a more Newtonian-like behaviour, allowing it to flow more freely. This effect is particularly noticeable in complex geometries such as stenosed arteries, where reduced internal resistance significantly enhances the fluid motion. It can be seen from Fig. 6 that the magnetic field creates resistance to the fluid's mobility, slowing down the flow. The opposing force from the magnetic field results in internal energy dissipation in the form of heat, a phenomenon known as Joule heating. Moreover, the reduced flow velocity weakens convective heat transfer, allowing thermal energy to accumulate in the



(a)

(b)

Fig. 4. Impact of γ on the velocity profile.



(a)

(b)

Fig. 5. Impact of β on the velocity profile.

fluid. Consequently, stronger magnetic fields contribute to a rise in the temperature profile.

Figure 7 shows that the curvature parameter increase leads to a noticeable rise in the fluid temperature. This behaviour arises due to the geometric effects associated with curved surfaces, where the boundary layer becomes thinner and convective heat loss is reduced. As a result, thermal energy accumulates more effectively near the surface. In addition, the presence of hybrid nanoparticles (Au-Ag) enhances heat conduction, further amplifying the temperature within the curved stenotic artery. Thus, curvature plays a critical role in regulating heat transfer characteristics in biomedical flows. Figure 8 shows a rise in the heat source parameter (Q), which leads to a noticeable enhancement in the temperature profile. This is because positive values of Q stand for the fluid's internal heat generation, which contributes additional thermal energy to the system. As a result, the thermal boundary layer becomes thicker, and the overall temperature distribution increases.

Figure 9 shows that the Eckert number (Ec) enhances viscous dissipation, converting more thermal energy from kinetic energy. This raises the fluid temperature, resulting in a thicker thermal boundary layer.

Figure 10 reveals that the skin friction coefficient decreases with increasing permeability and magnetic field strength. Higher porosity facilitates fluid seepage through the medium, lowering

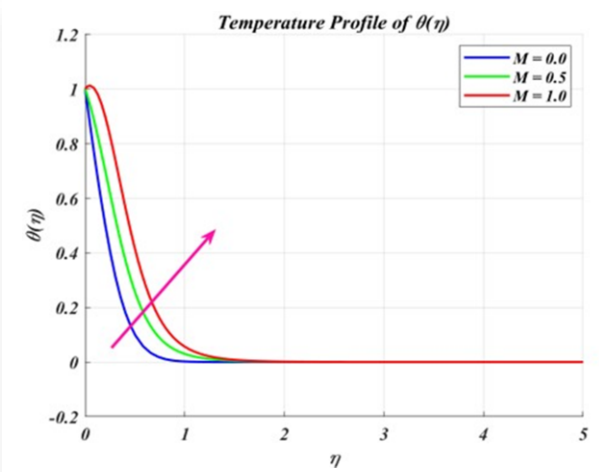


Fig. 6. Impact of M on the temperature profile.

wall shear. Meanwhile, the Lorentz force due to magnetic effects suppresses flow velocity near the surface, further reducing frictional resistance. From Fig. 11, it is seen that a rise in either magnetic parameter or the heat source parameter cuts the Nusselt number (Nu); added internal heating flattens the wall's temperature gradient, while the Lorentz force from a stronger

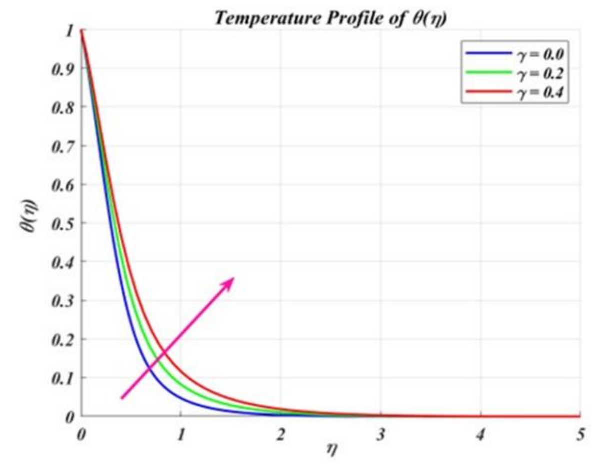


Fig. 7. Impact of γ on the temperature profile.

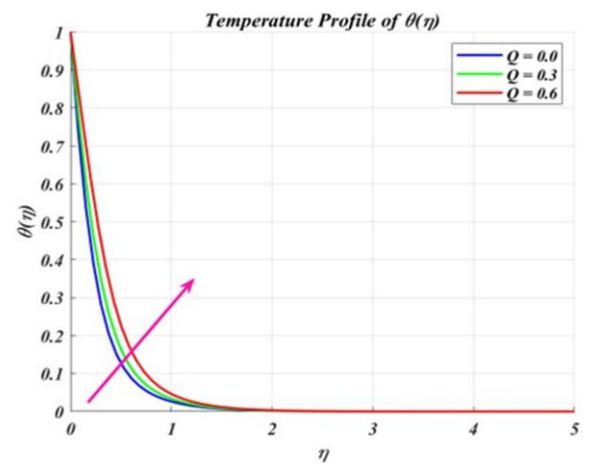


Fig. 8. Impact of Q on the temperature profile.

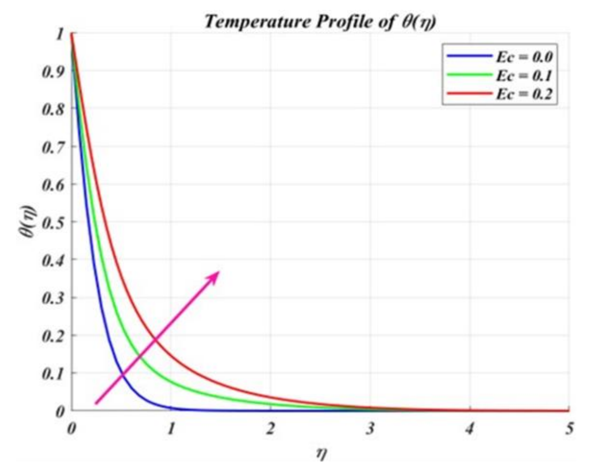


Fig. 9. Impact of Ec on the temperature profile.

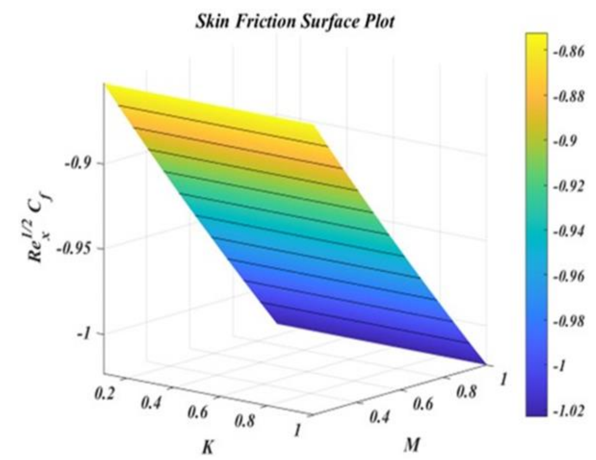


Fig. 10. Impact of K and M on the skin friction profile.

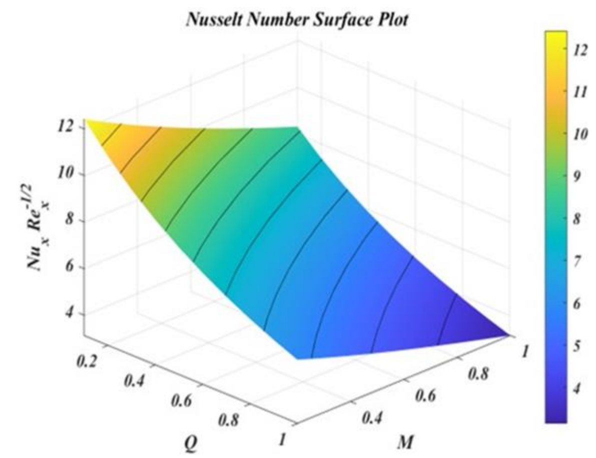


Fig. 11. Impact of Q and M on the Nusselt number profile.

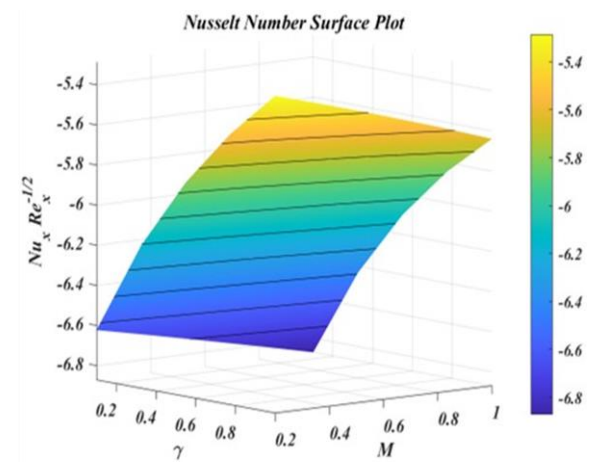


Fig. 12. Impact of γ and M on the Nusselt number profile.

magnetic field slows the fluid. Both effects curb convective heat transfer. Figure 12 shows that as the radius of curvature increases, the Nusselt number also increases, indicating improved heat transfer near the artery wall; this happens because a smoother curvature allows better fluid flow. On the other hand, increasing M lowers the Nusselt number, since the magnetic force slows down the fluid, lessening the heat it can transfer from the wall.

In Fig. 13, it has been observed that the streamline plot for $K = 2.0$ illustrates how the fluid flows through the porous artery.

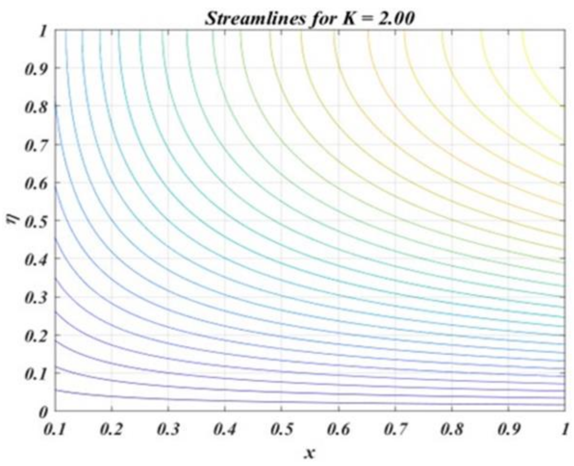


Fig. 13. Streamlines for K = 2.0.

The streamlines are denser near the wall, showing stronger velocity gradients in that region. Farther from the wall, the flow becomes more uniform. This behaviour reflects how moderate porosity influences the blood flow, allowing movement but still offering resistance. Figure 14 shows how the fluid flows under the influence of a magnetic field with M = 1.0; the streamlines

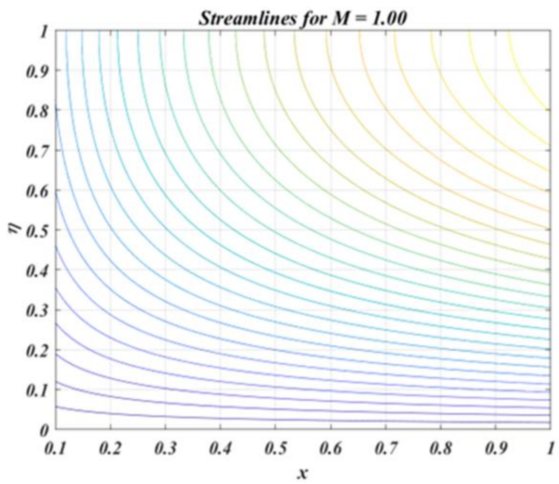


Fig. 14. Streamlines for M = 1.0.

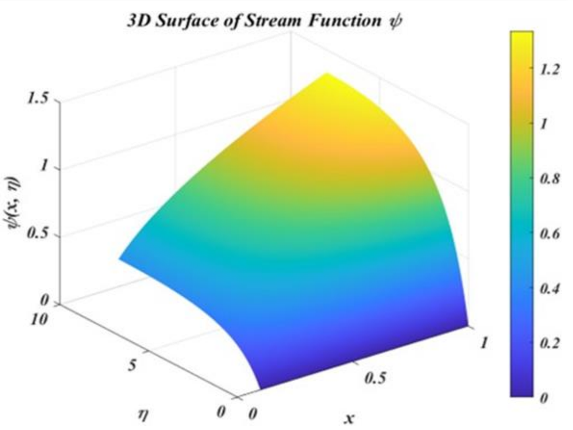


Fig. 15. 3D surface plot of stream function.

are tightly packed near the wall, indicating strong velocity gradients there. The magnetic field slows the flow by opposing motion, especially in the central region, leading to more stable and controlled stream behaviour across the artery. The 3D surface plot in Fig. 15 shows how the stream function changes with position, reflecting the strength and direction of blood flow. The flow starts slowly near the wall and upstream, then becomes stronger as it moves away from the wall and further downstream. This gives a clear picture of how the velocity field evolves through the stenotic artery.

Table 2 confirms the validity of the present numerical method by comparing it with earlier results from Waqas et al. [40]. The strong consensus between both sets of values demonstrates the accuracy of the current model and solution technique.

Table 2. Comparison of results for $C_f \, \mathrm{Re}_r^{1/2}$ for various values of γ and ϕ_1.

Sarwar and Hussein [58]			Present outcomes (bvp4c)
γ	ϕ_1	$C_f\mathrm{Re}_r^{1/2}$	$C_f\mathrm{Re}_r^{1/2}$
0.10	0.01	0.939968	0.939925
0.12	0.01	0.924794	0.924651
0.14	0.01	0.911311	0.911306
0.10	0.05	1.329552	1.329549

6. Conclusions

The numerical investigation of Casson-based hybrid nanofluid flow through a porous, magnetised stenotic artery using the bvp4c MATLAB solver has provided detailed insights into the flow and thermal behaviour of blood with embedded gold-silver nanoparticles. The model revealed critical dependencies of velocity, temperature, wall shear, and heat transfer on parameters such as the magnetic parameter, permeability and curvature parameter, contributing valuable knowledge towards biomedical fluid dynamics and therapeutic control in stenotic conditions:

1) A rise in magnetic field intensity leads to reduced blood velocity due to the opposing Lorentz force. Simultaneously, it raises the fluid temperature, as magnetic resistance contributes to internal heat build-up (Joule heating) and weakens the transport of heat by convection.
2) As the permeability of the artery increases, fluid velocity and wall shear stress decrease; this is attributed to higher flow resistance within the porous structure, which dissipates energy more effectively.
3) Higher curvature values result in increased velocity and temperature. This is due to geometric changes in the artery that affect both flow distribution and heat transfer behaviour, especially in narrowed (stenotic) regions.
4) Larger values of the Casson parameter increase flow velocity by reducing the fluid's yield stress. This change enables the fluid to behave more like a Newtonian fluid, thereby improving its ability to move through irregular arterial structures.
5) An increase in internal heat source strengthens the temperature field; this causes thickening of the thermal boundary layer and decreases the Nusselt number due to reduced temperature gradient.

6) Higher Eckert numbers increase the temperature of the nanofluid by converting kinetic energy into thermal energy. This enhances viscous dissipation and leads to greater heat accumulation in the fluid.

This work shows a comprehensive theoretical framework for analysing bio-magnetic Casson hybrid nanofluid blood flow through a stenosed artery with internal heat generation or absorption. The results demonstrate that:

1. Casson rheology significantly influences velocity and shear stress distribution, reinforcing the need for non-Newtonian modelling in stenotic arteries.
2. Magnetic fields reduce flow velocity but enhance temperature due to Joule heating, revealing both therapeutic potential and safety considerations.
3. Heat source/sink effects strongly modulate temperature, suggesting applications in hyperthermia and inflammatory heat regulation.
4. Hybrid nanoparticles, especially of cylindrical shape, improve thermal conductivity and heat transport but must be carefully controlled to avoid excessive thermal loads.
5. The combined effects of rheology, stenosis geometry, magnetic field, and nanoparticle-enhanced thermal conduction create complex hemodynamic behaviours relevant to cardiovascular pathology and advanced biomedical engineering.

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