

Simulation-Driven Analysis and Modelling of Infrastructural Construction Works Performance

Michał Juszczyk^{*1,3}, Michał Pyzalski^{2,3}, Dariusz Sala², Leonas Ustinovichius³, Czesław Miedziałowski⁴

¹ Faculty of Civil Engineering, Cracow University of Technology, Poland

² AGH University of Krakow, Poland

³ Faculty of Civil Engineering, Vilnius Gediminas Technical University, Lithuania

⁴ Faculty of Civil Engineering and Environmental Sciences, Bialystok University of Technology, Poland

Abstract. The assessment of performance in infrastructural construction works is essential for improving efficiency and managing uncertainty in project delivery. This study proposes and empirically validates a simulation-driven framework for probabilistic modelling of construction duration based on statistically fitted productivity distributions. Sewerage construction works are analysed as a representative case of linear infrastructural projects characterised by repetitive production cycles and measurable daily outputs. Field data on daily work output, crew size, and working hours were statistically examined to identify variability patterns and dependency structures. After outlier assessment and goodness-of-fit testing at a significance level of $\alpha = 0.05$, the Pearson Type III distribution provided the most consistent representation of daily productivity. The research tests the hypothesis that empirically fitted probability distributions of daily work output constitute a statistically valid basis for Monte Carlo simulation of project duration and deadline-compliance probabilities. Monte Carlo simulations performed for a 1,000 m work section (1,000–10,000 runs) produced stable and convergent results, yielding mean and median completion times of approximately 50.6 and 51 days, respectively. The probability of completion within 50 days was estimated at approximately 0.48, increasing to 0.75 within 51 days. The results confirm that distribution-based stochastic modelling provides a reliable and transferable framework for probabilistic assessment of performance in linear infrastructural construction works under uncertainty.

Key words: infrastructural construction works performance; statistical distribution; Monte Carlo simulation; simulation-based modelling; sewerage construction projects; construction works efficiency and duration.

1. INTRODUCTION

Infrastructure systems underpin economic activity, public health, and environmental protection. As networks age and urbanisation intensifies, improving the reliability of planning and delivery of infrastructural construction projects becomes increasingly important. Linear infrastructure works are characterised by repetitive production processes and measurable daily outputs; however, their execution is subject to substantial variability arising from geological, environmental, organisational, and operational conditions. This variability directly affects productivity and, consequently, project duration and deadline compliance. Despite advances in construction simulation and risk modelling, operational productivity is still frequently represented using assumed parameters rather than statistically validated empirical distributions.

Within the broad spectrum of infrastructural works, sewerage construction projects constitute a technically demanding and representative category of linear construction processes. Their execution involves repetitive daily production cycles, quantifiable work outputs, and sensitivity to environmental and organisational conditions, making them a suitable empirical basis for investigating stochastic performance modelling in linear infrastructure. This study addresses the probabilistic estimation of

project duration under stochastic daily productivity. Let Q denote the required work quantity and X_i the daily performance of construction works treated as random variables. The objective is to determine the distribution of total completion time T and to estimate the probability $P(T \leq D)$ of meeting a specified deadline D . Such probabilistic assessment enables risk-informed planning and more realistic scheduling decisions compared with deterministic duration estimates.

The research hypothesis is formulated as follows:

H1: Daily productivity in linear infrastructural construction works can be adequately represented by a continuous probability distribution fitted to empirical field data, and such representation provides a statistically stable basis for Monte Carlo simulation of project duration and deadline-compliance probabilities.

To verify this hypothesis, a simulation-driven methodological framework is developed and applied to field data collected from sewerage construction works treated as a representative case of linear infrastructural projects. The framework integrates data preprocessing, statistical distribution fitting with goodness-of-fit testing, and Monte Carlo simulation of duration outcomes.

The contribution of this study is threefold. First, it provides empirical validation of distribution-based modelling of daily productivity in infrastructural construction. Second, it formalises the transformation of field productivity data into a probabilistic

*e-mail: michal.juszczyk@pk.edu.pl

duration model. Third, it demonstrates the stability and practical applicability of the approach for deadline-risk assessment in linear infrastructure projects.

2. STATE-OF-THE-ART

The literature relevant to this study may be grouped into three complementary domains: management and risk modelling in sewerage construction projects; empirical research on construction productivity and performance determinants, and statistical and simulation-based approaches to modelling construction performance under uncertainty.

Research on sewerage infrastructure has primarily addressed optimisation, prioritisation, financial risk, and governance mechanisms. Budget-constrained optimisation and dynamic programming models have been developed to improve investment allocation and sequencing [1]-[2]. Probabilistic cost-risk approaches have enhanced financial forecasting reliability in water and sewerage projects, including improved parameter estimation techniques [3]-[4]. Organisational aspects such as public-private partnership governance and principal-agent relationships have also been examined [5]-[6]. Simulation-based analyses have explored delay propagation using system dynamics models [7], while environmental and lifecycle cost considerations have been incorporated into sustainability-oriented infrastructure frameworks [8]. Empirical evidence further indicates that construction technologies significantly influence time-cost relationships in sewerage works [9]. Although these studies strengthen strategic and financial decision-making, they rarely address statistically validated stochastic modelling of operational productivity.

Construction productivity has been widely analysed through systematic reviews and empirical investigations. Research identifies managerial, organisational, technological, and environmental determinants influencing performance [10]. This perspective is further supported by a study examining the relative significance of site- and management-related parameters affecting daily construction labour productivity [11]. Monitoring methods include time studies, earned value techniques, and digital tracking systems [12], while quantitative analyses assess non-value-adding time proportions in construction processes [13]. Multi-criteria decision methods such as AHP have been used to prioritise productivity drivers [14], and workforce-related factors including training and motivation have been empirically assessed [15]. Scheduling research has examined rate-driven and due-date-driven models [16], lean and Industry 4.0 strategies [17], duration estimation methods [18], and PERT-based probabilistic task modelling [19]. Additional studies have analysed productivity impacts of change orders [20] and environmental conditions [21]-[22]. Despite this extensive body of knowledge, productivity research typically focuses on average performance levels and determinants rather than modelling the full probability distribution of empirically observed daily outputs for stochastic duration simulation.

Probabilistic modelling in construction builds upon applied statistics and stochastic simulation theory [23]-[25]. Monte Carlo simulation is widely used in cost and schedule risk analysis; however, assumed distributions such as triangular forms are frequently adopted without formal statistical validation against

empirical data [26]. Approaches incorporating subjective correlations into probabilistic estimation have also been proposed [27]. Simulation has long supported operational analysis in construction, including visual interactive environments [28], activity-based resource models [29], and optimisation-oriented simulations for repetitive scheduling [30]. Productivity variability has been modelled probabilistically in earthmoving operations [31], and system-level frameworks have assessed uncertainty effects on construction performance [32]. Recent developments integrate artificial neural networks with Monte Carlo simulation [33] and introduce Bayesian probabilistic approaches [34].

In linear infrastructure contexts, stochastic extensions of linear scheduling methods incorporate productivity variability and discrete-event simulation [35], whereas many advanced linear scheduling models remain predominantly deterministic [36]. Contemporary schedule risk modelling increasingly combines data-driven techniques with Monte Carlo simulation [37], and robustness-oriented sampling methods such as Markov Chain Monte Carlo have been proposed [38]. Nevertheless, classical CPM-PERT approaches enhanced with Monte Carlo simulation often rely on assumed three-point estimates rather than empirically fitted productivity distributions [39].

Despite substantial progress in simulation-based construction modelling, the literature review reveals three methodological gaps:

- Monte Carlo applications frequently rely on assumed parametric distributions rather than statistically validated goodness-of-fit testing of observed productivity data;
- empirical productivity research often remains disconnected from probabilistic duration modelling frameworks;
- integrated approaches explicitly linking statistical data preprocessing (including outlier assessment), distribution fitting, hypothesis testing, and Monte Carlo-based performance evaluation for sewerage construction works remain limited.

There is therefore a need for a structured methodology that derives probability distributions directly from empirical daily productivity data, validates them statistically, and integrates the validated stochastic model into Monte Carlo-based duration and deadline-risk evaluation.

Accordingly, the present study addresses the following research questions:

RQ1: How can empirical productivity data from sewerage construction works be statistically modelled and validated for use as simulation inputs?

RQ2: Which probability distribution provides the most appropriate representation of observed daily work output under goodness-of-fit criteria?

RQ3: How can Monte Carlo simulation based on statistically fitted distributions support probabilistic performance assessment?

RQ4: How sensitive are the resulting performance estimates to modelling assumptions and data preprocessing decisions?

3. METHODOLOGY

The study adopts a quantitative simulation-driven research design aimed at transforming empirical observations of construction productivity into a statistically validated stochastic duration model. The methodological framework integrates classical statistical analysis with probabilistic modelling and Monte Carlo simulation

techniques widely applied in engineering research. The conceptual workflow consists of successive sequential stages:

- (i) empirical data collection,
- (ii) statistical preprocessing and exploratory analysis,
- (iii) probability distribution fitting and goodness-of-fit validation,
- (iv) stochastic duration modelling and Monte Carlo simulation.

The procedure ensures traceability from raw field observations to probabilistic estimation of project completion time.

The statistical procedures applied in the preprocessing and distribution-fitting stages are grounded in established methods of engineering statistics and probability theory [23]-[24], [40]. These include descriptive analysis, outlier detection using the interquartile range method, correlation assessment, maximum likelihood estimation, and goodness-of-fit testing.

The stochastic modelling and simulation stages rely on Monte Carlo methods, which provide a well-recognised framework for analysing uncertainty in engineering systems and construction processes [42]-[44]. Monte Carlo simulation enables the generation of repeated productivity scenarios and the derivation of empirical probability distributions of project duration under explicitly modelled randomness.

By combining empirically fitted productivity distributions with stochastic aggregation of daily outputs, the framework ensures methodological traceability from raw field observations to probabilistic estimates of completion time. This structure directly supports the research objective of deriving statistically validated deadline-compliance probabilities for linear infrastructure projects.

3.1. Empirical Data Collection

Data were collected over an eight-week observation period from a selected section of a sewerage construction project. The project involved the installation of underground sewerage infrastructure as part of a broader urban development scheme in Małopolska voivodship, Poland. The aim of the data collection was to record performance-related parameters. The analysed works involved installation of a PVC-U sewer pipeline (DN 200) in mechanically excavated trenches supported with a box-type shoring system. Excavation was performed in easily excavatable soils with trench depths up to 2.5 m and groundwater below trench bottom level. The workforce consisted of an excavator operator and a small crew. (Details regarding the site location, project name, and contractor have been withheld at the request of the data provider for confidentiality reasons. Raw data used in this research were collected for [45].) The dataset comprised daily observations of performance-related variables:

DWO – daily work output in meters of completed sewerage line [m];

NCM – number of crew members present (number of workers not including the excavator operator);

NWH – actual number of working hours (i.e. shift duration).

In the modelling framework, the recorded variables play different methodological roles. The variable DWO represents the primary performance indicator describing the realised productivity of the construction works and therefore constitutes the main stochastic variable used in subsequent probabilistic modelling and simulation. In contrast, NCM and NWH were treated as auxiliary explanatory variables describing organisational conditions. These variables were initially considered as potential predictors of daily

performance and could therefore have been incorporated into a multivariate modelling framework. However, their final role in the simulation model depended on the results of the statistical preprocessing and dependency analysis presented in the following subsection. Consequently, the outlier detection procedure was applied primarily to the DWO variable, as it represents the central stochastic input for the probabilistic modelling.

3.2. Statistical Preprocessing and Exploratory Analysis

Prior to probabilistic modelling, the empirical dataset was subjected to statistical preprocessing to ensure consistency, robustness, and suitability for distribution fitting.

Outlier analysis was performed for the primary performance variable (daily work output, DWO) using the interquartile range (IQR) method. This ensured that extreme values did not distort the subsequent analyses. The IQR-based approach is a non-parametric and distribution-independent technique widely applied in exploratory data analysis [23], [41]. It identifies extreme observations located outside the interval:

$$|Q_1 - 1.5 \text{ IQR}, Q_3 + 1.5 \text{ IQR}|$$

where Q_1 and Q_3 denote the first and third quartiles, and:

$$\text{IQR} = Q_3 - Q_1$$

The purpose of outlier detection in this context was to prevent extreme observations from exerting disproportionate influence on parameter estimation and goodness-of-fit testing. In construction productivity datasets, unusually low or high daily outputs may result from incidental operational disruptions rather than reflecting the intrinsic stochastic behaviour of the production process. Eliminating such values improves the stability of subsequent probabilistic modelling while preserving genuine variability.

Descriptive statistics including typical measures (compare e.g. [23]-[24], [41]), were computed to characterise the empirical distributions of the observed variables. Measures of central tendency (mean, median), dispersion (standard deviation, range), and distribution shape were evaluated to assess variability patterns and to support subsequent distribution fitting.

Exploratory graphical analysis, including histograms and empirical cumulative distribution function (ECDF), was conducted to identify preliminary distributional characteristics.

To explore potential dependencies between the variables DWO, NCM, and NWH, linear correlation analysis was conducted using Pearson's correlation coefficient.

The objective of this step was to assess whether multivariate modelling would be justified. If explanatory variables exhibit substantial predictive power, productivity may be modelled conditionally. Conversely, weak or moderate associations suggest that productivity variability should be treated as predominantly stochastic.

Based on the observed strength of correlations and explanatory power, a modelling decision was taken regarding whether to adopt a multivariate or univariate stochastic representation of productivity. This decision directly informs the structure of the simulation model developed in subsequent subsections.

This preprocessing stage addresses RQ1 by establishing the statistical basis for modelling daily productivity as a stochastic variable suitable for probabilistic simulation.

3.3. Probability distribution modelling

To represent stochastic nature of construction works performance, several continuous probability distributions commonly applied in construction engineering and reliability analysis were considered: Lognormal - Eq. (1), Gamma - Eq. (2), Beta - Eq. (3) and (4), Triangular - Eq. (5), and Pearson Type III - Eq. (6). Equations (1)-(6) are given below:

$$f(x; \delta; \mu) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \quad (1)$$

for $x > 0$ where: σ – standard deviation, μ – average, (both σ and μ for the distribution of the logarithms);

$$f(x; k; \theta) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}} \quad (2)$$

for $x > 0$ where: Γ – gamma Euler's function, k – shape parameter, θ – scale parameter;

$$f(x; \alpha; \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} \quad (3)$$

for $0 < x < 1$ where: Γ – gamma Euler's function, α – shape parameter, β – shape parameter, $\alpha, \beta > 0$ and for

$$x' \in (a; b) \rightarrow x = \frac{x'-a}{b-a}; \quad (4)$$

$$f(x; a; b; c) = \begin{cases} 0 & \text{for } x < a, \\ \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x \leq c, \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c \leq x \leq b, \\ 0 & \text{for } x > b \end{cases} \quad (5)$$

where: a – minimum, b – maximum, c – modal value;

$$f(x; \alpha; \beta; \mu) = \frac{1}{\beta\Gamma(\alpha)} \left(\frac{x-\mu}{\beta}\right)^{\alpha-1} \exp\left(-\frac{x-\mu}{\beta}\right) \quad (6)$$

where: Γ – gamma Euler's function, α – shape parameter, β – scale parameter, μ – location parameter.

Parameter estimation was performed using maximum likelihood estimation. Let $f(x|\theta)$ denote a candidate probability density function with parameter vector θ . The parameter estimates were obtained by maximising the log-likelihood function:

$$\hat{\theta} = \arg \max_{\theta} \sum_{i=1}^n \ln f(X_i|\theta) \quad (7)$$

Goodness-of-fit was assessed primarily with the Kolmogorov–Smirnov (KS) statistic, which measures the maximum distance between the empirical cumulative distribution function (ECDF) of the observed data and the cumulative distribution function (CDF) of the theoretical model. The fitted distributions were also compared using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), where lower values indicate

a better trade-off between goodness of fit and model complexity. In addition, quantile–quantile (Q–Q) and probability–probability (P–P) plots were used to visually assess the goodness of fit. Q–Q plots compare empirical and theoretical quantiles, highlighting deviations in the distribution tails, while P–P plots compare cumulative probabilities, making discrepancies in the central range more apparent. (The concepts and fundamentals of distribution fitting can be found in statistical literature, such as [23]–[24], [41]). The distribution providing the best statistical agreement with empirical data was selected for stochastic simulation. This stage addresses RQ2.

3.4. Stochastic model and Monte Carlo simulation

Let the random variable X denote the daily work performance. The total required work volume is denoted by Q . The project duration T is then defined as the smallest integer n such that the cumulative sum of simulated daily performances:

$$T = \min\{n \in N: \sum_{i=1}^n X_i \geq Q\}, \quad (8)$$

where X_i are independent and identically distributed realisations of X and θ denotes the set of fitted distribution parameters. Monte Carlo simulation was applied to generate repeated daily work performance sequences. For each simulation run j daily outputs were sampled from the fitted distribution and aggregated until cumulative output reached Q . The resulting completion time T_j was recorded. After R simulation runs, the empirical distribution of T was obtained. The probability of completing the works within a specified deadline D was estimated as:

$$\hat{P}(T \leq D) = \frac{1}{R} \sum_{j=1}^R 1_{\{T_j \leq D\}} \quad (9)$$

where $1_{\{T_j \leq D\}}$ is the indicator function that equals 1 if $T_j \leq D$ and 0 otherwise. Simulation stability was evaluated by comparing results for increasing numbers of runs. (The Monte Carlo simulation approach is presented broadly in the related literature e.g. [42]–[44].)

This stage addresses RQ3, while robustness assessment contributes to RQ4.

4. DATA PREPROCESSING AND ANALYSIS RESULTS

This section presents the empirical results obtained by applying the statistical preprocessing and analysis procedures described in Section 3. The analysis includes outlier detection, descriptive statistical evaluation, correlation analysis, and probability distribution fitting for the daily work output variable.

4.1. Outlier detection and elimination results

Outlier detection was conducted for the primary performance variable DWO, using the *IQR* method described in Section 3.2. The initial dataset contained 40 observations and the DWO which initially ranged from 1.0 m to 28.0 m. The calculated quartile values for DWO Q_1 , Q_3 , and *IQR* were:

$$Q_1 = 16.75, \quad Q_3 = 22.25, \quad IQR = 5.5.$$

The lower and upper bounds for outlier detection were calculated respectively:

$$Q_1 - 1.5 IQR = 8.5; \quad Q_3 + 1.5 IQR = 30.50$$

Three observations (1 m, 5 m, and 6 m) fell below the lower bound and were therefore classified as outliers. These values correspond to atypically low production levels likely associated with operational interruptions rather than normal variability of the construction process. In accordance with the preprocessing procedure described in Section 3.2, these observations were removed from the dataset.



Fig.1. Boxplot illustrating outlier detection for the DWO (daily work output) variable

Figure 1 presents the boxplot illustrating the elimination of outliers based on the analysis of the daily work output variable. The excluded outlier values are highlighted with a red ellipse. After preprocessing the dataset consisted of 37 observations, with DWO values ranging from 12 to 28 m.

4.2. Descriptive statistics.

Descriptive statistics were computed for the dataset including 37 observations of the three observed variables: DWO, NCM, and NWH. The descriptive statistics are presented together with the corresponding frequency distributions of the analysed variables. For the DWO variable, the data are shown both in a chart and in a table. In the case of NCM and NWH due to its simplicity, the data is presented in tabular form only. Figure 2 shows the frequency distribution for the DWO variable. Table 1 presents the same data in tabular form, including observed frequencies (as shown in Figure 3) and cumulative frequencies. In addition, the table includes relative frequencies and cumulative relative frequencies, both expressed as percentages. Tables 2 and 3 present similar data for the NCM and NWH variables, respectively.

Table 4 presents the descriptive statistics calculated for the final dataset, covering all three variables. The statistics include the minimum (Min.), mean (Mean), and maximum (Max.) values, as well as standard deviations (Std. Dev.) and medians (Med.). Based on the descriptive statistics DWO showed the greatest variability, ranging from 12.0 m to 28.0 m, with a mean of

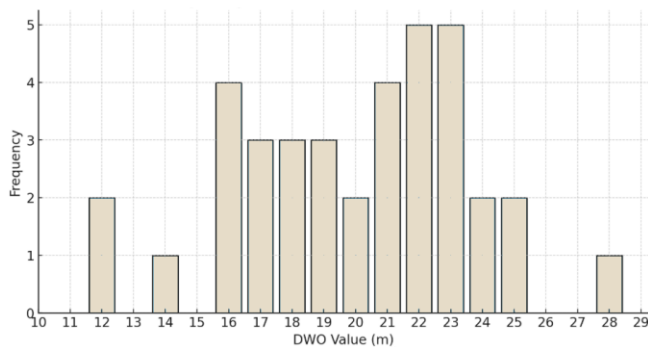


Fig.2. Chart illustrating frequency distribution for the DWO (daily work output) variable.

19.97 m, a median of 21.0 m, and a relatively high standard deviation of 3.67 m.

TABLE 1. Frequency distribution for the DWO variable

Observed values [m]	Observed frequency	Accum. observed frequency	Observed relative frequency [%]	Accum. observed relative frequency [%]
12	2	2	5.41%	5.41%
13	0	2	0.00%	5.41%
14	1	3	2.70%	8.11%
15	0	3	0.00%	8.11%
16	4	7	10.81%	18.92%
17	3	10	8.11%	27.03%
18	3	13	8.11%	35.14%
19	3	16	8.11%	43.24%
20	2	18	5.41%	48.65%
21	4	22	10.81%	59.46%
22	5	27	13.51%	72.97%
23	5	32	13.51%	86.49%
24	2	34	5.41%	91.89%
25	2	36	5.41%	97.30%
26	0	36	0.00%	97.30%
27	0	36	0.00%	97.30%
28	1	37	2.70%	100.00%

TABLE 2. Frequency distribution for the NCM variable

Observed values [number of workers]	Observed frequency	Accum. observed frequency	Observed relative frequency [%]	Accum. observed relative frequency [%]
3	4	4	10.8%	10.8%
4	24	28	64.9%	75.7%
5	9	37	24.3%	100.0%

TABLE 3. Frequency distribution for the NWH variable

Observed values [hours]	Observed frequency	Accum. observed frequency	Observed relative frequency [%]	Accum. observed relative frequency [%]
8	2	2	5.4%	5.4%
9	1	3	2.7%	8.1%
10	34	37	91.9%	100.0%

TABLE 4. Descriptive statistics computed for final dataset of variables

Variable	Descriptive statistics				
	Min.	Mean.	Max.	St. Dev.	Med.
DWO	12.0	19.97	28.0	3.67	21.0
NCM	3	4.14	5	0.59	4.0
NWH	8.0	9.86	10.0	0.48	10.0

In contrast, the NCM varied only slightly, between 3 and 5, averaging 4.14, with a low standard deviation of 0.59. The NWH was the most stable, ranging narrowly from 8.0 h to 10.0 h, with a mean of 9.86 h and a very low standard deviation of 0.48. The low variability observed in the NCM and NWH values indicates that these parameters remain relatively stable across the observed dataset. These results indicate that for the observed dataset fluctuations in DWO are not primarily driven by variations in NCM or NWH, suggesting the presence of additional stochastic influences related to site conditions or operational factors.

4.3. Correlation analysis

To additionally examine potential dependencies between the observed variables, linear correlation analysis was performed using Pearson's correlation coefficient. Table 5 presents the correlation matrix based on Pearson's correlation coefficients.

TABLE 5. Correlation matrix for the final dataset of variables

Variable	DWO	NCM	NWH
DWO	1.000	0.506	0.187
NCM		1.000	-0.131
NWH			1.000

The correlation analysis showed a moderate positive relationship between DWO and NCM with a coefficient

$$r = 0.506,$$

however the coefficient of determination:

$$R^2 \approx 0.256,$$

indicates that NCM explains only approximately 26% of the observed variability in DWO.

$$r = 0.187,$$

while the correlation between NCM and NWH was weakly negative:

$$r = -0.131.$$

Given the limited explanatory power of these relationships and the relatively low variability observed in NCM and NWH, the development of a multivariate dependency model was not considered justified.

Consequently, DWO was treated as a stochastic variable and used as the primary input for probabilistic modelling and simulation. A simulation approach based on the distribution of DWO enables the modelling of work performance while capturing the actual variability observed in production rates, without relying on weak explanatory relationships that would limit predictive accuracy.

4.4. Distribution fitting results

Following the preprocessing and exploratory analysis, probability distribution fitting was performed for the DWO variable.

Five continuous distributions commonly applied in engineering modelling defined by Eq. (1) - (6) were considered (see 3.3). The parameters of the candidate distributions were estimated using maximum likelihood estimation. The adequacy of the fitted models was evaluated primarily using the Kolmogorov-Smirnov (KS)

goodness-of-fit statistic and associated p-values. The null hypothesis that the empirical distribution does not differ significantly from the theoretical distribution was tested at the significance level of $\alpha = 0.05$.

Table 6 presents the parameters of the fitted distributions, together with the KS statistic values and corresponding p-values, in order to reflect testing the null-hypothesis. The table also presents the AIC and BIC values computed for the distributions. The distributions are ordered according to their goodness of fit, from best to worst.

TABLE 6. Distribution fitting results for the DWO variable

Distribution (Equation)	Parameters	KS statistic values	p-value	AIC	BIC
Pearson Type III Eq. (6)	$\alpha = -0.346$, $\beta = 3.625$, $\mu = 19.973$	0.1045	0.7751	205.57	210.40
Beta Eq. (3)-(4)	$\alpha = 10.732$, $\beta = 6.505$, $x' \in <0.155, 28.000>$	0.1066	0.7549	207.46	213.91
Triangular Eq. (5)	$a = 10.29$, $b = 28.61$, $c = 22.99$	0.1245	0.5716	204.99	209.83
Lognormal Eq. (1)	$\sigma = 1.381e-05$, $\mu = 12.476$	0.1252	0.5652	207.76	210.99
Gamma Eq. (2)	$k = 302.973$, $\theta = 0.211$	0.1328	0.4901	206.18	209.40

The results show that the Pearson Type III and Beta distributions provide the best fit to the empirical data, as reflected by the lowest KS statistics (0.1045 and 0.1066, respectively) and the highest p-values (0.7751 and 0.7549). A comparison of the AIC and BIC values indicates a consistent ranking of all candidate distributions. The differences in AIC values are small, whereas the differences in BIC values are slightly more pronounced. (Although the triangular distribution yields the lowest AIC and BIC values, this result does not affect the selection of the Pearson Type III and Beta distributions for further investigation.)

To complement the statistical goodness-of-fit evaluation, graphical diagnostics were performed using Q-Q and P-P plots for the Pearson Type III and Beta distributions. Figures 3 and 4 present Q-Q and P-P plots respectively.

The Q-Q plots confirm the findings of the KS tests for the two distributions. The Pearson Type III distribution shows a better alignment across the entire range, particularly at the upper tail, whereas the Beta distribution exhibits deviations. These results suggest that Pearson Type III provides a more consistent fit.

The P-P plots illustrate the agreement between the empirical and theoretical cumulative probabilities. The Pearson Type III distribution demonstrates a closer alignment with the 45° reference line, indicating a more consistent fit across the full range of values. The Beta distribution shows larger deviations in the central part of the distribution, confirming that it provides a weaker fit than the Pearson Type III distribution.

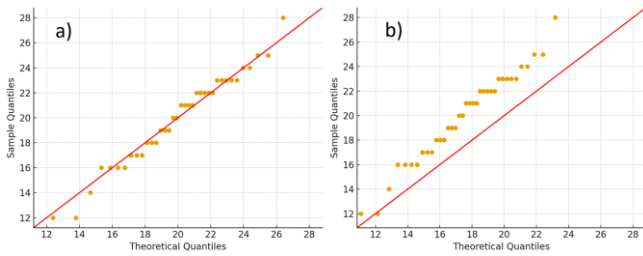


Fig.3. Q-Q plots for fitted distributions: a) for Pearson type III, b) for Beta distribution.

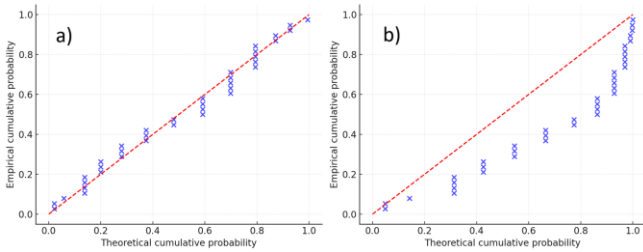


Fig.4. P-P plots for fitted distributions: a) for Pearson type III, b) for Beta distribution.

The overall assessment, including the comparison of KS statistics, p-values, AIC and BIC values, as well as Q-Q and P-P plots, indicates that the Pearson Type III distribution provides the most reliable fit to the empirical data. Consequently, this distribution was adopted as the basis for the simulations conducted in the subsequent phase of the research.

4.5. Empirical specification of the simulation input model

The analyses presented in the preceding subsections enabled the specification of the empirical input model used in the simulation stage of the study. The dataset was first preprocessed through outlier detection using the IQR method which resulted in a final dataset of 37 observations. Descriptive statistical analysis provided ground for a decision to use the DWO as the primary stochastic variable representing the variability of the construction works performance. Probability distribution fitting demonstrated that the Pearson Type III distribution provides the most consistent statistical representation of the empirical productivity data. This distribution was therefore adopted as the probabilistic model. The fitted distribution of the DWO variable constitutes the stochastic input used in the Monte Carlo simulations described in the following section. By sampling daily productivity values from this empirically validated distribution, the simulation model captures the variability observed in real construction operations and enables probabilistic estimation of project duration and deadline compliance.

5. RESULTS

Based on the empirical simulation model defined in Section 4.5, Monte Carlo simulations were performed to analyse the stochastic variability of project duration. The simulations were conducted for a total work volume of $Q = 1,000$ m, corresponding to the analysed section of sewerage construction works. To assess the stability of the simulation results, three different numbers of simulation runs R were considered: $R_1 = 1,000$, $R_2 = 5,000$, $R_3 = 10,000$.

Figure 5 illustrates the simulated probability distributions of completion time obtained for the three simulation scenarios, while Table 7 summarises the corresponding descriptive statistics.

TABLE 7. Descriptive statistics for the simulated distributions of work duration [days]

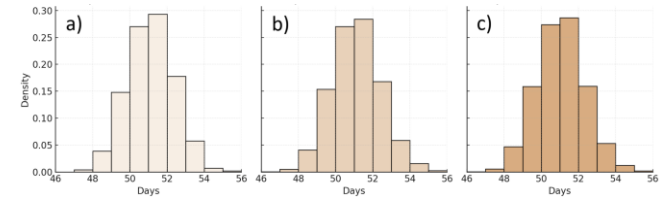


Fig.5. Simulated distributions of work duration for: a) 1,000 runs; b) 5,000 runs; c) 10,000 runs.

Number of simulation runs	Descriptive statistics				
	Min.	Mean.	Max.	St. Dev.	Med.
$R_1 = 1,000$	47	50.629	54	1.293	51.0
$R_2 = 5,000$	46	50.623	55	1.341	51.0
$R_3 = 10,000$	46	50.553	55	1.329	51.0

The results indicate a high level of consistency across all simulations. Based on the analysis of Figure 5. and Table 7.:

- the mean completion time stabilised at approximately 50.6 days,
- the median remained close to 51 days in each case,
- the simulated completion times ranged between approximately 46 and 55 days, with standard deviations of about 1.3 days - the spread of results was narrow,
- increasing the number of runs produced slightly smoother histogram shapes but did not significantly change the central tendency or dispersion of the results.

The convergence of the simulated distributions confirms the stability of the stochastic duration model. Even with 1,000 iterations the resulting distribution already approximates the final solution well, while larger numbers of runs primarily improve the smoothness of the empirical distribution.

The cumulative distributions of completion time are presented in Figure 6. These distributions enable estimation of the probability

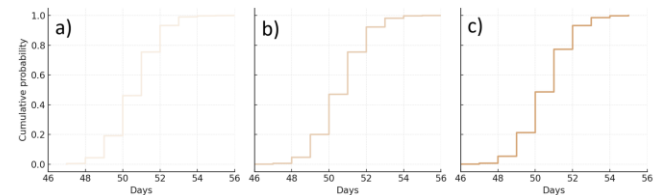


Fig.6. Cumulative distributions of work duration for: a) 1,000 runs; b) 5,000 runs; c) 10,000 runs.

of completing the analysed work volume within specified deadlines.

The estimated probabilities for deadlines between 48 and 52 days - see Eq. (8) and (9) - are summarised in Table 8.

The results in Table 8 demonstrate that the probability of completing the 1,000 m section within 50 days is approximately 0.48, while the probability increases to about 0.75 for completion

within 51 days. The results demonstrate a high level of consistency between the simulated distributions indicating that increasing the number of runs mainly smooths random variability without significantly affecting the probabilistic characteristics of project duration. The consistency of the probability estimates across simulations with different numbers of iterations further confirms the robustness of the simulation framework.

TABLE 8. Probabilities of completing given work volume within a specified deadline D .

Number of simulation runs	$P(T \leq 48)$	$P(T \leq 49)$	$P(T \leq 50)$	$P(T \leq 51)$	$P(T \leq 52)$
$R_1 = 1,000$	0.043	0.191	0.461	0.754	0.932
$R_2 = 5,000$	0.046	0.200	0.470	0.754	0.922
$R_3 = 10,000$	0.054	0.212	0.486	0.773	0.932

The obtained results form the basis for the interpretation and contextualisation presented in the following discussion section.

6. DISCUSSION

This section interprets the simulation results in relation to the research hypothesis and the research questions formulated in Sections 1 and 2.

The results obtained in this study confirm that stochastic modelling based on empirically fitted productivity distributions provides a consistent and statistically reliable framework for analysing the performance of linear infrastructural construction works. The empirical dataset describing daily work output demonstrated moderate variability that could not be sufficiently explained by deterministic organisational parameters such as crew size or working hours. As shown in the correlation analysis, these variables exhibited limited explanatory power, indicating that productivity fluctuations are largely influenced by additional operational and environmental factors inherent to construction processes.

From a methodological perspective, the study demonstrates that empirical productivity data can be successfully transformed into a stochastic input model suitable for simulation-based performance analysis. The distribution fitting procedure identified the Pearson Type III distribution as the most appropriate statistical representation of daily work output, as confirmed by the Kolmogorov–Smirnov goodness-of-fit test and graphical diagnostics. This finding supports the research hypothesis (H1) that daily productivity in linear infrastructure works can be adequately represented by a continuous probability distribution fitted to empirical observations.

The Monte Carlo simulations conducted using the fitted distribution produced stable and convergent results across different numbers of iterations. The simulated completion time distributions exhibited very similar descriptive statistics for 1,000, 5,000, and 10,000 simulation runs, indicating that the probabilistic duration model is robust with respect to the number of simulation iterations. The resulting duration estimates, with mean values of approximately 50.6 days and median values close to 51 days, demonstrate that stochastic modelling enables realistic estimation of project completion time and deadline-compliance probabilities.

Compared with previous studies on productivity and time modelling in construction [30], [34], the present analysis indicates that relatively simple probabilistic models, when properly fitted to empirical data, can provide reliable performance estimates without requiring more complex hybrid modelling approaches. The simulated completion times obtained in this study exhibit a relatively narrow range (approximately 46–55 days), while the cumulative probability distributions demonstrate stable convergence across simulation runs. These results suggest that the variability of daily production rates, although stochastic in nature, remains within statistically predictable bounds when represented through empirically fitted productivity distributions.

The obtained results are consistent with previous research demonstrating the usefulness of Monte Carlo simulation for modelling uncertainty in construction processes. However, many earlier studies rely on assumed parametric distributions such as triangular distributions derived from expert judgement rather than from empirical productivity measurements. The present study shows that fitting probability distributions directly to observed field data may provide a more objective and statistically justified basis for stochastic performance modelling. In this context, the proposed framework contributes to bridging the methodological gap between empirical productivity research and probabilistic schedule simulation identified in the literature review.

The preprocessing stage, including outlier detection using the interquartile range method, played an important role in ensuring the stability of the statistical modelling. The excluded observations corresponded to exceptionally low daily outputs likely caused by temporary operational interruptions rather than representing the normal stochastic behaviour of the construction process. Retaining such values would increase the skewness and dispersion of the fitted distribution and could lead to unrealistically wide simulated duration ranges. Consequently, their removal improves the representativeness of the stochastic productivity model while preserving the underlying variability of the construction process.

From a practical perspective, the proposed modelling framework may support more realistic planning and risk-informed scheduling in linear infrastructural construction projects. By deriving stochastic productivity distributions directly from empirical field data, project planners can estimate not only expected completion times but also the probability of meeting specific deadlines. Such probabilistic information may assist decision-makers in evaluating schedule buffers, assessing deadline risks, and comparing alternative planning scenarios under uncertainty. The approach is particularly relevant for repetitive linear construction processes, where measurable daily outputs provide a natural basis for stochastic performance modelling.

Despite these contributions, several limitations should be acknowledged. The empirical dataset was obtained from a single sewerage construction project executed under specific geological and organisational conditions. Therefore, the fitted productivity distribution should not be interpreted as universally representative for all infrastructure projects. Moreover, the stochastic model assumes independence between daily productivity observations, whereas real construction processes may exhibit temporal dependencies related to weather conditions, workforce learning effects, or operational constraints. Future research should therefore

explore multi-factor stochastic models incorporating environmental variables, technological parameters, and potential serial correlations in productivity data.

Overall, the findings demonstrate that statistically validated productivity distributions combined with Monte Carlo simulation provide an effective methodological framework for probabilistic performance analysis in linear infrastructural construction projects. The approach enables more realistic estimation of project duration under uncertainty and may support improved planning, scheduling, and risk-informed decision-making in infrastructure project management.

7. CONCLUSIONS

The results of the study lead to the following conclusions:

1. The Pearson Type III distribution provided the best fit to the empirical data describing daily work output, confirming that daily productivity in sewerage construction works can be effectively represented using a continuous probability distribution fitted to observed field data.
2. Monte Carlo simulations based on the fitted productivity distribution produced stable and convergent results, with mean and median project durations stabilising at approximately 50.6 and 51 days, respectively.
3. Increasing the number of simulation runs from 1 000 to 10 000 did not significantly alter the results, indicating that the stochastic duration model exhibits strong robustness and computational stability.
4. The probabilistic analysis enables estimation of deadline-compliance likelihoods. For the analysed 1 000 m section, the probability of completion within 50 days was estimated at approximately 0.48, increasing to about 0.75 for completion within 51 days.
5. The proposed modelling framework demonstrates that empirically fitted productivity distributions can serve as reliable stochastic inputs for simulation-based duration modelling in linear infrastructure projects.

Overall, the results confirm the research hypothesis that empirically fitted probability distributions of daily work output provide a statistically valid basis for Monte Carlo simulation of project duration. By integrating empirical productivity data with stochastic modelling, the proposed approach contributes to improved performance analysis and supports more realistic planning and risk-informed decision-making in infrastructure construction projects.

Despite the demonstrated applicability of the proposed framework, several limitations should be acknowledged. The empirical dataset was derived from a single sewerage construction project, which limits the generalisability of the findings. In addition, the model assumes independence of daily productivity, whereas real processes may exhibit temporal dependencies and be influenced by environmental and operational factors. Future research should therefore focus on developing multi-factor stochastic models that incorporate these effects.

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