

The simulations of transients in a shell transformer with movable magnetic shunt

Bronisław TOMCZUK¹ , Dawid WAJNERT¹ , and Dawid WEBER²

¹ Department of Electrical Engineering and Mechatronics, Opole University of Technology, Opole, Poland

² Technology Center, Hitachi Energy, Lodz, Poland

Abstract. This paper proposes a hybrid field-circuit model of a single-phase shell-type transformer that incorporates a three-dimensional analysis of the magnetic field in a transformer with beveled edges of the magnetic shunt. The finite element method (FEM) was employed, and the expressions for computing the differential and integral parameters of the magnetic field were provided. For various positions of the magnetic shunt, the self- and mutual inductances of the transformer windings were calculated. Their values were stored in matrix form, serving as input data for the transformer circuit model. The differential equations governing the operation of a transformer with a movable magnetic shunt were formulated using Lagrangian methods and applied to the T-type equivalent circuit of the transformer. This simulation model was experimentally validated, with measured and simulated time-course currents showing close agreement. Furthermore, three electric arc models were incorporated to assess the proposed method under nonlinear loading. The results show that the suggested mathematical model accurately predicts the transformer transients across diverse operating scenarios. Moreover, the presented simulation offers potential for broader application to other electromagnetic objects.

Keywords: simulation of voltage and current transients; field-circuit models; finite element analysis; electric arc models; transformer with magnetic shunt.

1. INTRODUCTION

Mathematical models play a significant role in numerous scientific fields [1–3]. They enable simulation and thus understanding, optimization, and prediction under various conditions of phenomena whose complexity precludes direct study. It is almost impossible to develop a mathematical model of a transformer incorporating all physical phenomena. Furthermore, the computational time required to solve such a model would make computer simulations of little practical use. Mathematical models are often simplified by making assumptions and omitting phenomena to improve clarity [4], and despite these simplifications, they remain valuable tools in many applications. In the design and analysis of transformer operation, numerical simulations are widely used to analyze the electromagnetic field distributions [4–6]. Commercial software is available that includes two-dimensional and three-dimensional methods, among which the finite element method (FEM) is the most commonly employed [7]. Understanding the electromagnetic field distribution within a transformer enables the determination of its key parameters already at the design stage of the structure [8, 9]. It further provides a basis for assessing how changes to the transformer design affect these parameters [4, 10].

While numerical calculations of the electromagnetic field distribution can provide accurate results, they are always subject

to convergence errors. Analytical solutions, on the other hand, are often more challenging to obtain and require simplifying assumptions about geometry, as well as the introduction of correction factors to account for these assumptions [11].

This field-circuit model enables the calculation of current and voltage time waveforms in both dynamic and steady states of transformer operation by combining electromagnetic field equations and circuit differential equations into a system. During each calculation step, magnetic field equations are solved to obtain the couplings that generate the electromotive force, while circuit current and voltage equations determine the currents that generate the magnetic field in the next step of the field part calculations. This field-circuit analysis enables reliable simulations, even with highly nonlinear loads, without requiring expensive prototypes. Examples of solutions for field and field-circuit models of transformers are provided in [12, 13], among others.

The proposed approach differs from traditional field-circuit models in that it does not solve the field equations during circuit simulation. Instead, the field parameters are precomputed once and used directly within the circuit model. This approach can significantly reduce the computational time required to simulate complex electromagnetic systems, such as DC–DC converters, because the most time-consuming calculations are field simulations that need to be performed only once [14].

In this paper, the proposed model is analyzed using the example of a transformer with a magnetic shunt inserted into its window, as shown in Fig. 1. The studied object was selected due to the significant nonlinearity of the leakage inductance caused

*e-mail: d.wajnert@po.edu.pl

Manuscript submitted 2025-12-02, revised 2026-03-23, initially accepted for publication 2025-04-08, published in July 2026.

by the movable magnetic shunt [4] and the nonlinearity of the load, which is an electric arc.

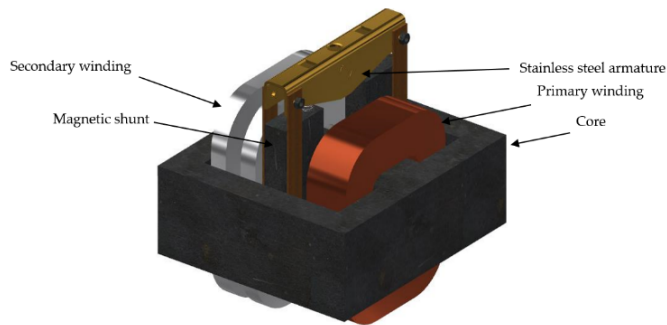


Fig. 1. Three-dimensional outline of the transformer used to verify the proposed mathematical model

The main dimensions (in millimeters) of the magnetic core and shunt are given in Figs. 2 and 3.

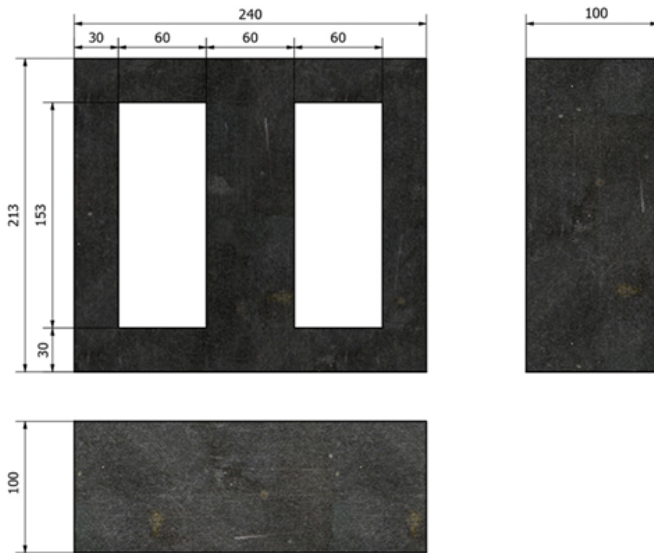


Fig. 2. Dimensions of the magnetic core

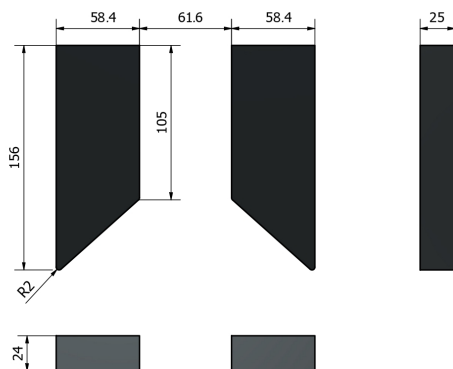


Fig. 3. Dimensions of the magnetic shunt

The parameters for the analyzed transformer are listed in Table 1. The winding resistances were determined analytically [8] and are equal to $R_1 = 198 \text{ m}\Omega$ and $R_2 = 12 \text{ m}\Omega$, respectively.

Table 1
Parameters of the analyzed transformer

Parameter	Value
Rated voltage, V_N	400 [V]
Maximal current of the primary winding, $I_{1 \max}$	40 [A]
Maximal current of the secondary winding, $I_{2 \max}$	225 [A]
Apparent power, S	16 [kVA]
Primary winding resistance, R_1	0.198 [Ω]
Secondary winding resistance, R_2	0.012 [Ω]
Turn number of the primary winding, N_1	180
Turn number of the secondary winding, N_2	34
Material of the magnetic core	M530 – 50A

The magnetic shunt is inserted exactly in the middle of the transformer window. Figures 4 and 5 present the main dimensions (in millimeters) of the primary and secondary windings.

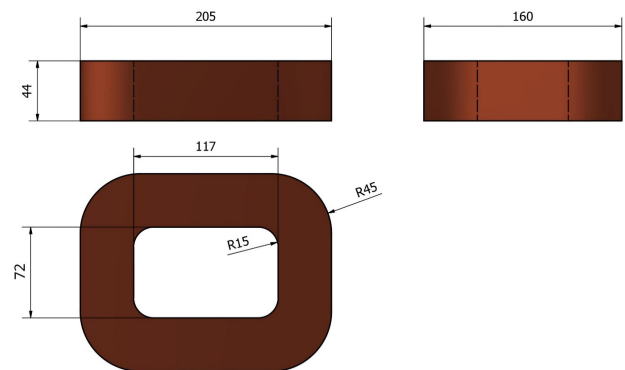


Fig. 4. Dimensions of the primary winding

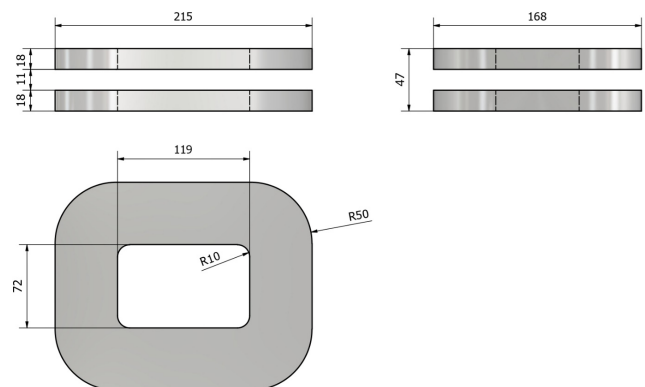


Fig. 5. Dimensions of the secondary winding

The dynamic leakage inductances of the windings and the mutual inductance were determined based on the analysis of the electromagnetic field distributions. Dynamic inductance values were obtained over the entire B–H curve, enabling more accurate simulation of transient waveforms in the circuit model.

The equations of a single-phase transformer were derived using the Lagrange method, and these equations were subsequently used to construct a circuit model capable of simulating the current and voltage waveforms of the transformer. The position of the magnetic shunt and the supply voltage of the primary winding can be incorporated into the model. The proposed model was verified by comparing the measured and calculated currents in the short-circuit state of the transformer. The model was also evaluated under nonlinear loads, such as an electric arc, with three different electric arc models implemented and analyzed.

2. ELECTROMAGNETIC FIELD ANALYSIS

There are many methods for analyzing electromagnetic field distributions, including the Finite Difference Method (FDM), the Finite Element Method (FEM), and the Integral Equation Method (IEM) as the most popular ones [15, 16]. Electromagnetic field calculations performed in this research consider the three-dimensional nature of the field, which is implemented in the commercial Maxwell 3D software, where FEM is used. The plane of symmetry [7], in the analyzed transformer geometry, is presented in Fig. 6.

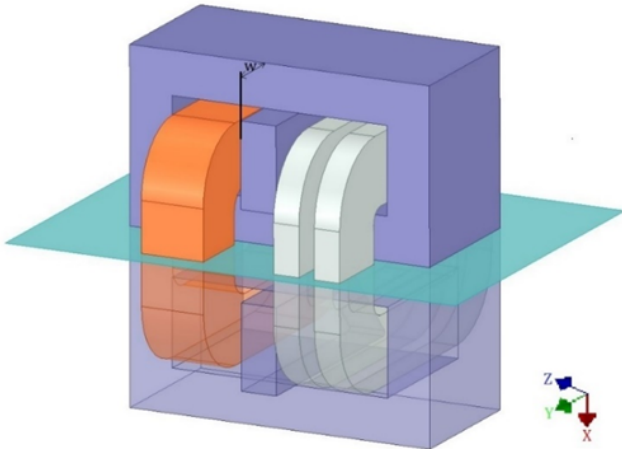


Fig. 6. The symmetry plane in the analyzed transformer geometry

Knowledge of the magnetic field distribution enables the computation of key integral and differential field parameters. The static leakage inductance L_{rs} can be determined by equation (1), using the leakage energy W stored in the nonmagnetic regions (equation (2)) with magnetic permeability μ , where V is the volume surrounding the magnetic core, and B represents the magnetic flux density in this region:

$$L_{rs} = \frac{2W}{i^2}, \quad (1)$$

$$W = \frac{1}{2\mu} \left(\int_V B^2 dV \right), \quad (2)$$

The dynamic leakage inductance L_{rd} in a linear medium can be determined using the expression provided in equation (3) [17, 18]:

$$L_{rd} = L_{rs} + I \frac{\partial L_{rs}}{\partial i}, \quad (3)$$

where I is the winding current.

A magnetic shunt inserted into the transformer window divides the flux produced by the primary winding into two parts. One part closes through the magnetic shunt and is not associated with the secondary winding, while the other bypasses the shunt and passes through the secondary winding.

Given the above, the dynamic mutual inductance $L_{\mu d}$ was determined using (4) by assuming that only the primary winding is excited by current and simultaneously determining the value Ψ of the magnetic flux associated with the secondary winding as defined in (5) [17, 19]:

$$L_{\mu d} = \frac{\partial \Psi}{\partial i}, \quad (4)$$

$$\Psi = \int_S \vec{B} \cdot d\vec{S}, \quad (5)$$

In the transformer short-circuit state, and in the absence of a magnetic shunt, i.e., when the shunt is completely removed, the flux generated by the primary winding is entirely compensated in the core by the flux of the secondary winding. Introducing a shunt into the transformer window allows part of the flux to flow through it. Figure 7 shows the division of the simulated flux into two parts, one closing its path through the magnetic shunt and the other associated with the shorted secondary winding. The amount of flux flowing through the shunt depends on the ratio of the equivalent reluctance of the path associated with the shunt (magnetic shunt and two air gaps) and the path containing the secondary winding. The value of the resultant reluctance of the magnetic circuit path (branch) with the shunt is affected not

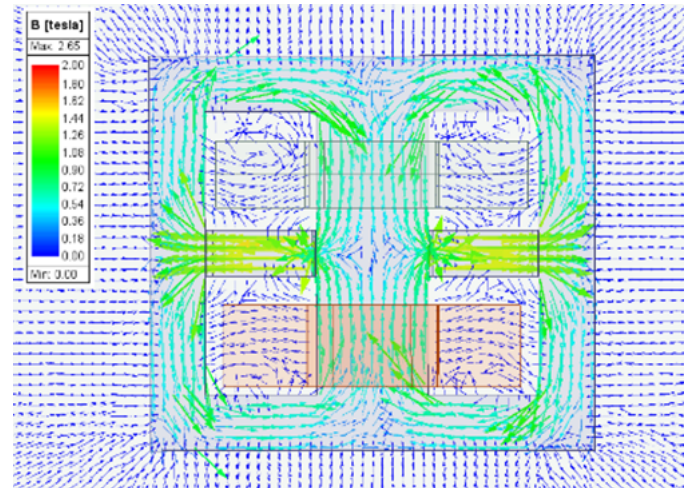


Fig. 7. Direction of the magnetic flux in the analyzed transformer

only by the position of the shunt, but also by the intensity of the current in the transformer windings. Larger current values cause the magnetic shunt to saturate, which forms part of the gap–shunt–gap branch in the transformer magnetic circuit. This results in an increase in the equivalent reluctance of this branch and therefore reduces the flux flowing through it. This also results in less energy stored in the gap between the core and the shunt. Both the change in the position of the shunt and the current intensity translate into a nonlinear characteristic of the leakage inductance and the mutual inductance [4, 10].

The magnetic field distribution was analyzed assuming an ampere-turn balance between the primary and secondary windings. The excitation current of the primary winding varied from 1A to 41A at various magnetic shunt positions, adjusted from 0 mm to 180 mm.

Using the magnetic field distribution defined by (2) and (3), the static and dynamic leakage inductances associated with the primary winding were determined, as illustrated in Figs. 8 and 9. Assuming the equality of the ampere-turns of the transformer coils, the dynamic leakage inductance associated with the secondary winding is $(N_2/N_1)^2$ times smaller. Figure 10 presents the histogram of the calculated dynamic mutual inductance values, referred to the transformer primary side, obtained from the simulation results.

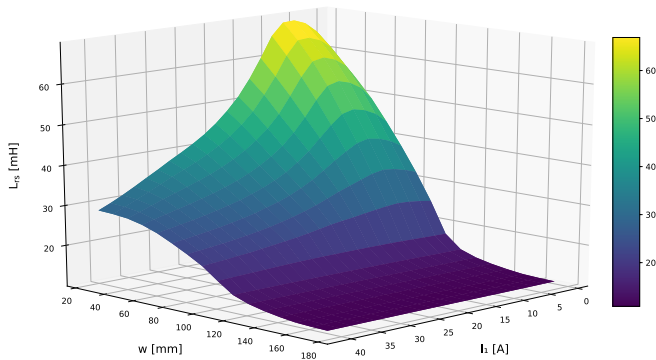


Fig. 8. Calculated static leakage inductance of the transformer primary winding

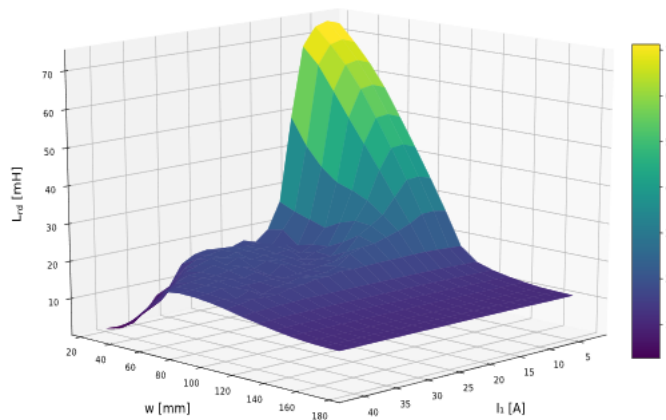


Fig. 9. Calculated dynamic leakage inductance of the transformer secondary winding

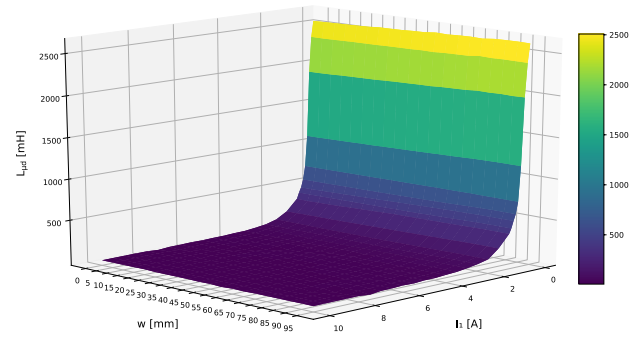


Fig. 10. Characteristic of the dynamic mutual inductance versus current excitation and the magnetic shunt extraction from the transformer window

Measurement verification of the field model was performed by comparing the calculated and measured leakage reactance, as shown in Fig. 11.

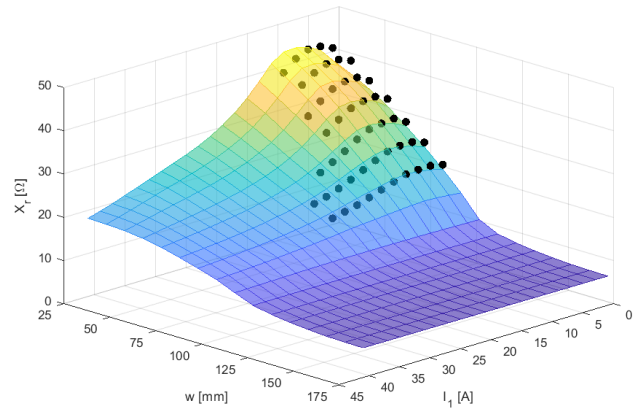


Fig. 11. Comparison between calculated and measured leakage reactance

3. FIELD-CIRCUIT MODEL FOR SIMULATION OF TRANSIENTS IN A TRANSFORMER WITH MOVABLE MAGNETIC SHUNT

The differential equations describing the operation of a single-phase transformer with a movable magnetic shunt were derived using the Lagrange formalism (see (6)), based on the T-shape equivalent circuit of a two-winding transformer, as shown in Fig. 12 [20]. This approach significantly simplifies the analysis,

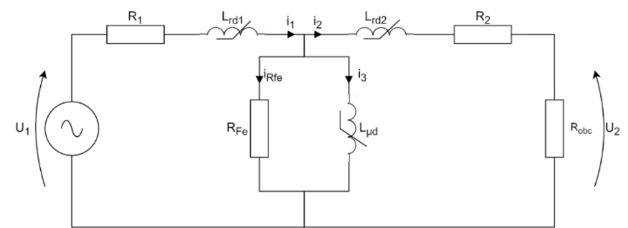


Fig. 12. T-type transformer equivalent circuit loaded with resistance R_{load}

as the equivalent circuit assumes magnetic decoupling of the coils. Thus, the inductance of the individual coils will be affected only by the current flowing through the given coil [21, 22]:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_j} \right) - \frac{\partial \mathcal{L}}{\partial q_j} + \frac{\partial P}{\partial q_j} = f_j. \quad (6)$$

In the case of a transformer with a movable magnetic shunt, the value of the magnetic flux is a function dependent on both the current intensity and the position of the magnetic shunt relative to the transformer window. In the following equations, “ w ” denotes the retraction of the shunt from the transformer window. The generalized variables are the currents associated with the circuit branches. Equations (7), (8), and (9) describe the kinetic energy, potential energy, and heat losses, respectively, due to resistances in the equivalent circuit [21, 22]:

$$E_k = \int_0^{\dot{q}_1} \Psi_1(\tilde{q}_1, w) d\dot{q}_1 + \int_0^{\dot{q}_2} \Psi_2(\tilde{q}_2, w) d\dot{q}_2 + \int_0^{\dot{q}_3} \Psi_3(\tilde{q}_3, w) d\dot{q}_3, \quad (7)$$

$$E_P = 0, \quad (8)$$

$$E_R = \frac{1}{2} R_1 \dot{q}_1^2 + \frac{1}{2} \dot{q}_2^2 (R_2 + R_{obc}) + \frac{1}{2} R_{Fe} (\dot{q}_1 - \dot{q}_2 - \dot{q}_3)^2. \quad (9)$$

After calculating the time derivatives (denoted by the dots above their letters) in the preceding expressions, we obtain the second-order derivatives system (10):

$$\begin{aligned} \ddot{q}_1 &= \frac{U_1 - R_1 \dot{q}_1 - R_{Fe} (\dot{q}_1 - \dot{q}_2 - \dot{q}_3)}{L_{rd1} (\dot{q}_1)}, \\ \ddot{q}_2 &= \frac{-\dot{q}_2 (R_2 + R_{obc}) + R_{Fe} (\dot{q}_1 - \dot{q}_2 - \dot{q}_3)}{L_{rd2} (\dot{q}_2)}, \\ \ddot{q}_3 &= \frac{R_{Fe} (\dot{q}_1 - \dot{q}_2 - \dot{q}_3)}{L_{\mu d} (\dot{q}_3)}. \end{aligned} \quad (10)$$

Due to an extremely slow movable shunt, the velocity $\partial w / \partial t$ was assumed to be zero. During the simulation, the derivatives of the magnetic flux Ψ_j with respect to the shunt position w were not considered. It reduces the term described by (11) to zero:

$$\frac{\partial}{\partial w} (\Psi_j(\dot{q}_j, w)) \frac{\partial w}{\partial t}. \quad (11)$$

Dynamic leakage inductances, determined from magnetic field analysis, are represented in matrix form and are written in bold font in (10).

Figure 13 shows the proposed model created in the Simulink package of MATLAB. The “Load” block contains four load variants, the three arc models described in the next chapter, and a constant resistive load.

Based on the presented simulation model, the influence of the shunt position and the supply voltage on the output current I_2 is shown in Fig. 14. At the rated voltage, the output current varies from 50 A to 120 A.

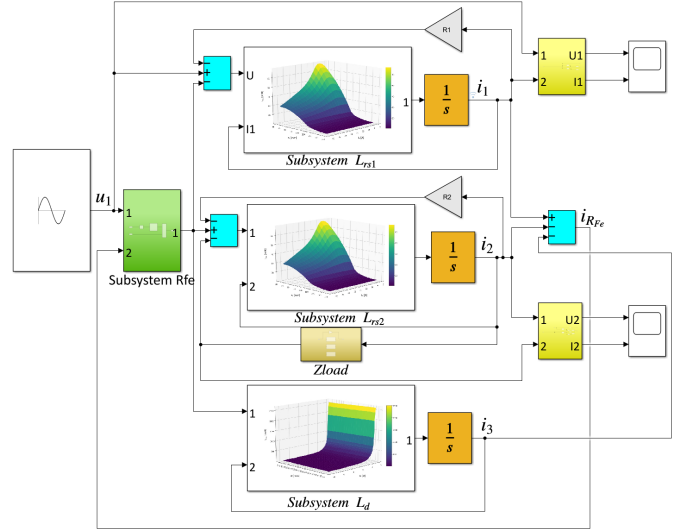


Fig. 13. Circuit model of the analyzed transformer within MATLAB/Simulink

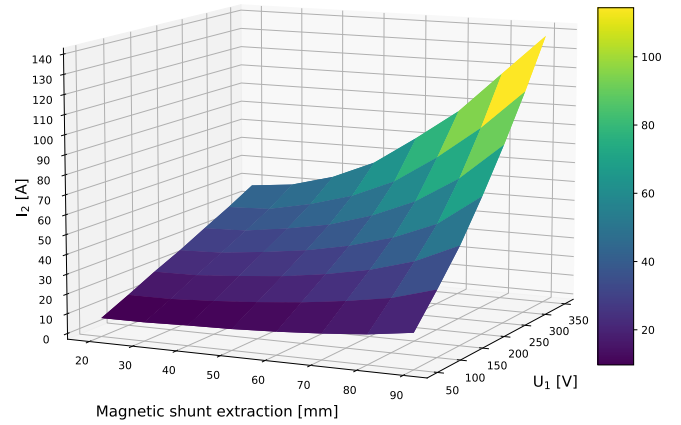


Fig. 14. Transformer secondary winding current I_2 as a function of the magnetic shunt position relative to the transformer window and the voltage supply U_1

4. EXPERIMENTAL VERIFICATION

The simulation of transients enables the evaluation of key parameters, such as transient-state duration, the initial peak of the current wave, and the stabilized RMS values of voltage and current. The accurate simulation of all these parameters is complex; thus, a reasonably close estimation of them helps in selecting transformer protection settings or implementing the above parameters. The model proposed in this paper was validated by comparing the simulated waveforms of current and voltage with measurements obtained from a real object. The comparisons were made for the short-circuit condition of the transformer. Figure 15 shows example waveforms of the primary-side voltage and current, as well as the secondary-side current, for one position of the magnetic shunt, with an RMS supply voltage of 350 V.

Good agreement between the measured and simulated results was observed for the transient state, as well as for the RMS cur-

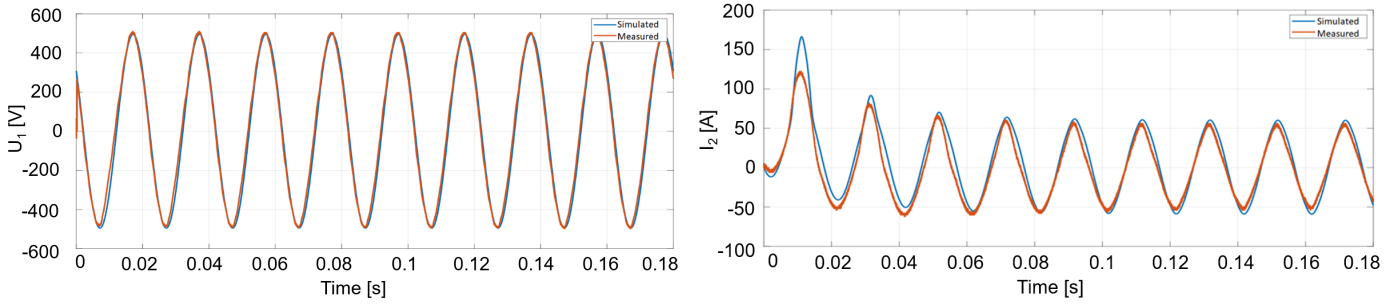


Fig. 15. Waveforms with a $U_1 = 350$ V supply voltage and the magnetic shunt extraction $w = 25$ mm from the transformer window

rent on both the primary and secondary sides of the transformer in the steady state. The simulated peak inrush current is about 30% higher than the measured value, which, given the many simplifications in the model, is considered acceptable.

The results presented in Fig. 15 were experimentally validated using the laboratory setup shown in Fig. 16. The experimental arrangement comprised an autotransformer, the transformer under investigation (Fig. 17), and measuring instruments, including a voltmeter, ammeters, and a wattmeter.

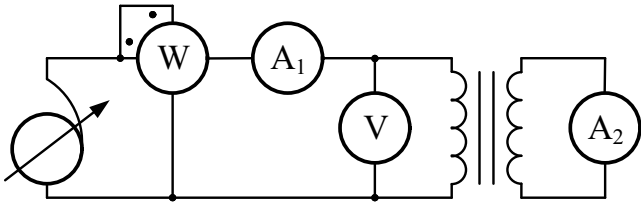


Fig. 16. The laboratory setup

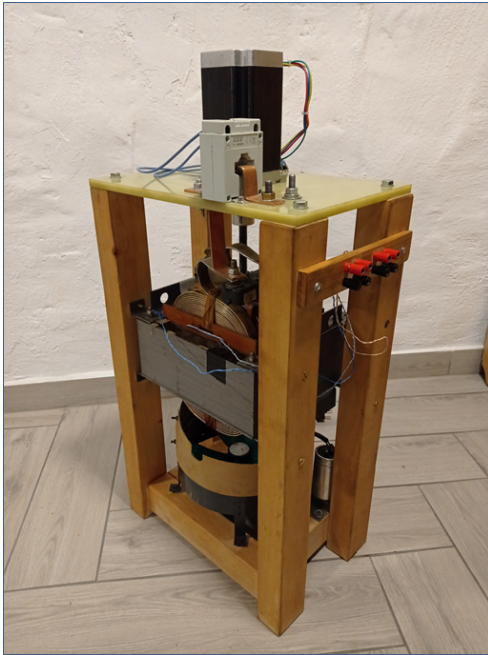


Fig. 17. Photo of the laboratory stand with the investigated transformer

5. TRANSIENT SIMULATIONS

To evaluate how effectively the proposed hybrid field-circuit model, developed in this study, simulates dynamic waveforms, the electric arc simulation models were used as the load.

Over the years, numerous models of electric arcs have been developed. The most widely accepted models for theoretical analysis are those proposed by Cassie in (12) and Mayr in (13). This article also analyzed the Hadedank model in (14), which was derived by serially connecting the elements from the Cassie and Mayr models [23]:

$$\frac{dg}{dt} = \frac{1}{\tau} \left(\frac{i_{\text{arc}} u_{\text{arc}}}{U_c^2} - g \right), \quad (12)$$

where g – arc electrical conductivity [S], τ – time constant [s], u_{arc} – instantaneous voltage drop across the arc [V], i_{arc} – instantaneous current flowing through the arc [A], U_c – arc voltage (constant parameter) [V].

$$\frac{dg}{dt} = \frac{1}{\tau} \left(\frac{g^2 u_{\text{arc}}^2}{P_0} - g \right), \quad (13)$$

where P_0 – power delivered by the arc [W].

In a stable electric arc, the following expression holds $P_0 = i_{\text{arc}} u_{\text{arc}}$, where the product $i_{\text{arc}} u_{\text{arc}}$ represents the instantaneous power of the electric arc. By comparing the models described by equations (12) and (13), it is possible to identify the conditions under which the constant parameters of the models must be determined. For a range of small values of arc current, P_0 takes a continuous value (as in Mayr's model). In contrast, in the region of large current values, the relation $P_0 = g U_c$ applies (Cassie's model) [24].

$$\begin{aligned} \frac{dg_C}{dt} &= \frac{1}{\tau_C} \left(\frac{x}{x} - g_C \right), \\ \frac{dg_M}{dt} &= \frac{1}{\tau_M} \left(\frac{x}{x} - g_M \right), \\ \frac{1}{g} &= \frac{1}{g_M} + \frac{1}{g_C}. \end{aligned} \quad (14)$$

Figures 18 and 19 show the time waveforms of voltages, currents, and electric arc impedance for two different positions of the magnetic shunt. The waveforms are plotted for the three electric arc models presented above. To increase the clarity of

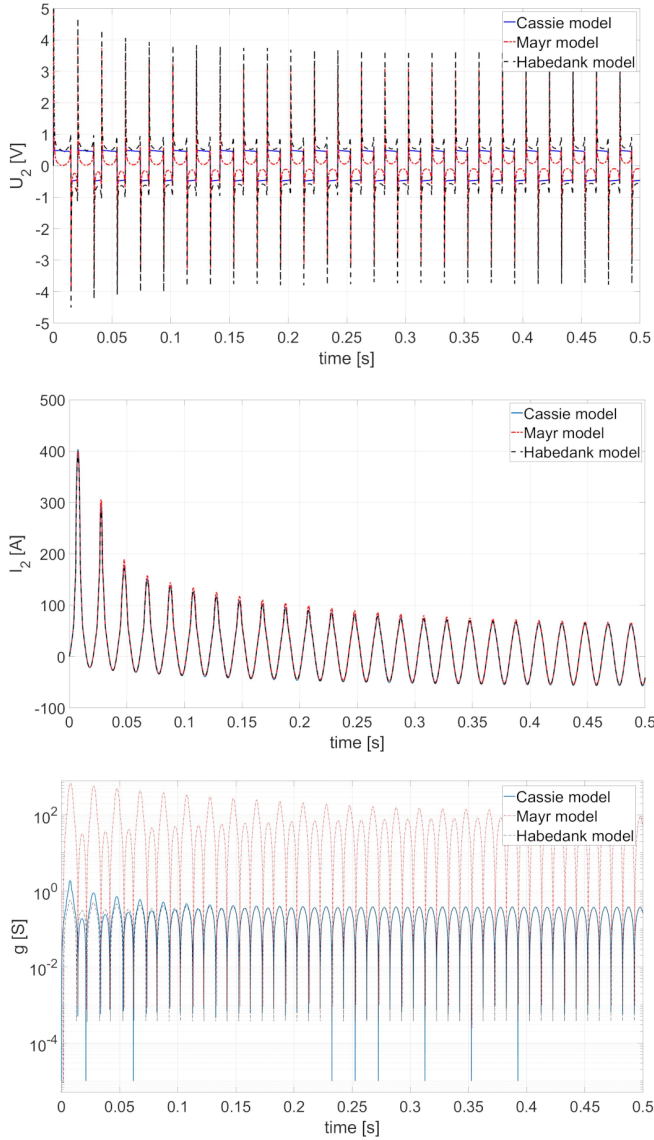


Fig. 18. Simulation of the voltage and current intensity waves and impedance of an electric arc at the supply voltage $U_1 = 350$ V and the magnetic shunt extraction $w = 25$ mm

the impedance values, the impedance scale is shown on a logarithmic axis. There is good agreement in the time waveforms of voltages and currents across simulations for various arc models.

6. SUMMARY

Determining the transformer integral parameters from magnetic field distribution analysis enables simulation of dynamic waveforms using the hybrid field-circuit model proposed in this paper, which accounts for magnetic shunt position changes and transformer voltage supply. Aside from its usefulness, such an approach has advantages: it is simple and easy to interpret, although simulations may be affected by modeling errors. Good agreement between the measured time waveforms and the simulated values for a resistive load, obtained during the measurement verification, confirmed the correctness of the model de-

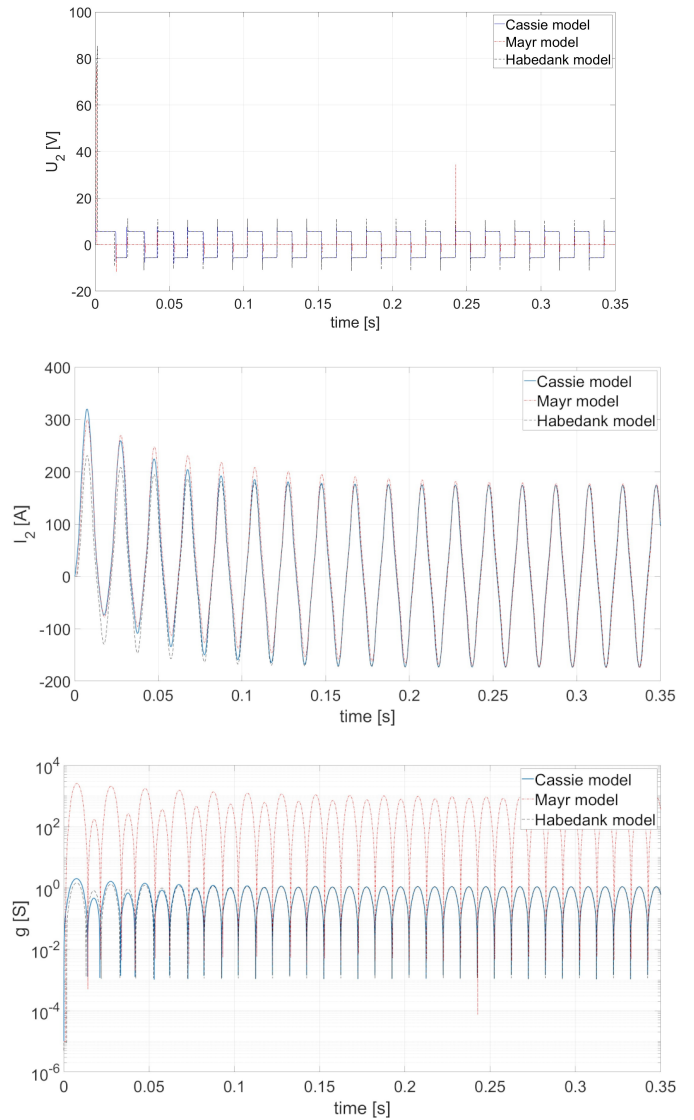


Fig. 19. Simulation of time courses and impedance of an electric arc at a supply voltage U_1 equal to 350 V and a shunt extraction from the transformer window equal to 85 mm

veloped in this paper. The proposed hybrid field-circuit model, therefore, enables accurate simulation of dynamic voltages and currents for both linear and nonlinear loads, including electric arcs. It was observed that nonlinear loads result in a higher peak inrush current and a longer duration of transient states than in a simulation with a constant load. This paper also confirms that the magnetic shunt enables accurate operation over a wide range for both static and nonlinear loads.

The difference between the simulated and measured peak inrush current values shown in Fig. 14 suggests that additional phenomena should be included in the mathematical model. However, it is almost impossible to create a transformer model that accounts for all physical phenomena, as transformer analysis spans numerous fields of science, including electromagnetism, thermodynamics, mechanics, and acoustics. In addition, analysis of such a model would be highly time-consuming, mak-

ing a fully comprehensive model almost impractical. Therefore, mathematical models are created under certain assumptions, omitting some phenomena. Considering hysteresis loops and eddy currents during simulation can improve the accuracy of the proposed mathematical model, especially for transformers with a core and a magnetic shunt made of non-oriented steel.

The advantage of the proposed model is a significant reduction in the simulation time when the system load changes. The analysis of the time-stepping field-circuit model of the transformer studied in the paper for each load condition requires multi-hour simulations. In contrast, the proposed hybrid model allows for obtaining results within a few seconds. It is worth noting that the magnetic shunt position relative to the transformer window significantly alters the object parameters, requiring further simulations.

The rapid development of the electrical power grid, driven by the diversification of power sources, has led to the incorporation of numerous new power electronic devices into the system. Considering this, the proposed model could be effectively used for simulating a complex electrical grid, where transformers are merely one of an ever-growing number of components.

REFERENCES

- [1] E. Palmgren and T. Rasa, "Modelling roles of mathematics in physics," *Sci. Edu.*, vol. 33, pp. 365–382, 2022, doi: [10.1007/s11191-022-00393-5](https://doi.org/10.1007/s11191-022-00393-5).
- [2] Y. Liu, R. Wu, and A. Yang, "Research on medical problems based on mathematical models," *Mathematics*, vol. 11, no. 13, p. 2842, 2023, doi: [10.3390/math11132842](https://doi.org/10.3390/math11132842).
- [3] V. Mokhov, S. Aliukov, A. Alabugin, and K. Osintsev, "A review of mathematical models of macroeconomics, microeconomics, and government regulation of the economy," *Mathematics*, vol. 11, no. 14, p. 3246, 2023, doi: [10.3390/math11143246](https://doi.org/10.3390/math11143246).
- [4] B. Tomczuk and D. Weber, "Effect of magnetic shunts on shell-type transformers characteristics," *Energies*, vol. 16, no. 19, p. 6814, 2023, doi: [10.3390/en16196814](https://doi.org/10.3390/en16196814).
- [5] K. Dawood, M.A. Cinar, B. Alboyaci, and O. Sonmez, "A new method for the calculation of leakage reactance in power transformers," *J. Electr. Eng. Technol.*, vol. 12, pp. 1883–1890, 2017, doi: [10.5370/JEET.2017.12.5.1883](https://doi.org/10.5370/JEET.2017.12.5.1883).
- [6] E. Mechkov, R. Tzeneva, V. Mateev, and I. Yatchev, "Thermal analysis using 3D FEM model of oil-immersed distribution transformer," in *Proc. 19th Int. Symp. Electrical Apparatus and Technologies (SIELA)*, Bourgas, Bulgaria, May 2016.
- [7] ANSYS Maxwell 3D User's Guide, ANSYS Inc. [Online]. Available: www.ansys.com. [Accessed: Jan. 7, 2024].
- [8] S.V. Kulkarni and S.A. Khaparde, *Transformer Engineering: Design, Technology, and Diagnostics*, 2nd ed. Boca Raton, FL, USA: CRC Press, 2012.
- [9] B. Tomczuk, "Analysis of 3-D magnetic fields in high leakage reactance transformers," *IEEE Trans. Magn.*, vol. 30, pp. 2734–2738, 1994.
- [10] B. Tomczuk, D. Weber, and D. Wajnert, "Shunt regulation analysis of the short-circuit inductance based on 3D magnetic field in high reactance transformer with movable shunt," *Prz. Elektrotechn.*, vol. 95, pp. 31–34, 2019.
- [11] B.S. Alves, P. Kuo-Peng, and P. Dular, "Contribution to power transformers leakage reactance calculation using analytical approach," *Int. J. Electr. Power Energy Syst.*, vol. 105, pp. 470–477, 2019.
- [12] G.B. Kumbhar and S.V. Kulkarni, "A directly coupled field-circuit model of a transformer to study surge phenomena and for frequency response analysis," in *Proc. 18th National Power Systems Conf. (NPSC)*, Guwahati, India, 2014, pp. 1–5, doi: [10.1109/NPSC.2014.7103838](https://doi.org/10.1109/NPSC.2014.7103838).
- [13] B. Tomczuk, D. Koteras, J. Zimon, and A. Waindok, "Calculation of the transient currents in transformers using field-circuits methods," *Prz. Elektrotechn.*, vol. 87, pp. 126–130, 2011.
- [14] A. Rashwan, A.I.M. Ali, and T. Senjyu, "Current stress minimization for isolated dual active bridge DC–DC converter," *Sci. Rep.*, vol. 12, p. 16980, 2022, doi: [10.1038/s41598-022-21359-1](https://doi.org/10.1038/s41598-022-21359-1).
- [15] J.K. Sykulski, *Computational Magnetism*. London, U.K.: Chapman & Hall, 1995.
- [16] B. Tomczuk and D. Koteras, "Magnetic flux distribution in amorphous modular transformers," *J. Magn. Magn. Mater.*, vol. 323, pp. 1611–1615, 2011.
- [17] H.F. Farahani, M. Zare, S. Mohammad, P. Razi, and A. Khodakarami, "Finite element analysis of leakage inductance of 3-phase shell-type and core-type transformers," *Res. J. Appl. Sci. Eng. Technol.*, vol. 4, pp. 1721–1728, 2012.
- [18] B. Bandelier and F. Rioux-Damidaou, "Mixed finite element method for magnetostatics in $\mathbb{R}^3$," *IEEE Trans. Magn.*, vol. 34, pp. 2473–2476, 1998.
- [19] K.J. Binns, P.J. Lawrenson, and C.W. Trowbridge, *The Analytical and Numerical Solution of Electric and Magnetic Fields*. New York, NY, USA: Wiley, 1993.
- [20] A. Nogueira, "Calculation of power transformers equivalent circuit parameters using numerical field solutions," *Int. J. Res. Rev. Appl. Sci.*, vol. 17, no. 1, pp. 113–121, 2013.
- [21] A. Hemeida *et al.*, "Magnetic Equivalent Circuit and Lagrange Interpolation Function Modeling of Induction Machines Under Broken Bar Faults," *IEEE Trans. Magn.*, vol. 59, no. 12, pp. 1–5, 2023, doi: [10.1109/TMAG.2023.3306207](https://doi.org/10.1109/TMAG.2023.3306207).
- [22] Z. Bielek, D. Bielek, and V. Biolková, "Lagrangian for circuits with higher-order elements," *Entropy*, vol. 21, no. 11, p. 1059, 2019, doi: [10.3390/e21111059](https://doi.org/10.3390/e21111059).
- [23] H. Haraldsson, Y.A. Tesfahunegn, M. Tangstad, and G. Sævarsdóttir, "Modelling of electric arcs for industrial applications: A review," in *Proc. INFACON XVI*, 2021. doi: [10.2139/ssrn.3927158](https://doi.org/10.2139/ssrn.3927158).
- [24] X. Li, C. Pan, D. Luo, and Y. Sun, "Series DC arc simulation of photovoltaic system based on Habedank model," *Energies*, vol. 13, no. 6, p. 1416, 2020, doi: [10.3390/en13061416](https://doi.org/10.3390/en13061416).