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Disturbance compensation in minimum operator based sliding mode control

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Abstract. This paper proposes an advanced disturbance-compensated sliding mode control framework based on a minimum-operator reaching law for discrete-time uncertain systems. By integrating a difference-based disturbance compensation mechanism with a finite-time minima-based reaching strategy, the original matched disturbance is effectively transformed into its increment, which is typically much smaller in magnitude. As a result, the proposed approach significantly reduces the quasi-sliding mode band width, while preserving rapid finite-time convergence of the switching function. Rigorous analysis establishes explicit settling-time bounds and guarantees finite-time convergence of system trajectories to a compact invariant set determined by the disturbance variation. Simulation studies on a perturbed mechatronic actuator demonstrate improved disturbance compensation, reduced steady-state error, and suppressed chattering compared to conventional discrete-time sliding mode controllers.

Keywords: sliding mode control; disturbance compensation; minimum operator; discrete-time systems; reaching law

1. INTRODUCTION

Sliding mode control (SMC) has long been recognized as a powerful robust control strategy due to its inherent insensitivity to matched uncertainties and external disturbances once trajectories reach a prescribed sliding manifold. Since the pioneering invariance conditions introduced by Draženović [1] and the formalization of variable structure systems by Utkin [2], SMC has become a fundamental tool in nonlinear control. Comprehensive treatments and extensions can be found in [3, 4].

With the widespread implementation of digital controllers, increasing attention has shifted toward discrete-time sliding mode control (DSMC). Unlike continuous-time SMC, exact sliding motion cannot be achieved in discrete systems due to sampling effects, leading instead to quasi-sliding modes (QSM), where trajectories oscillate within a bounded neighborhood of the sliding manifold [5]. Early developments on discrete sliding modes and their stability properties were reported in [6, 7], establishing reaching conditions and characterizing invariant sets in sampled-data systems.

A significant advancement in DSMC design was the reaching-law approach, introduced by Gao and Hung [8] and later generalized in [9]. In this framework, the switching function evolves according to a prescribed difference equation, allowing explicit shaping of transient behavior. Although classical reaching laws ensure convergence to a sliding band, they typically suffer from persistent chattering and limited steady-state accuracy [10]. Several improvements have been proposed, including modified reaching laws, fast output sampling techniques, and enhanced switching strategies [11–14].

Robust disturbance rejection remains a central challenge in DSMC. Uncertainties and external perturbations enlarge the quasi-sliding mode band and degrade tracking precision. To address this issue, disturbance-compensated reaching laws have been introduced. In [15], the authors proposed disturbance compensators for discrete-time sliding mode control that reconstruct the disturbance using the deviation between nominal and actual switching dynamics. Under slowly varying disturbances, their method effectively replaces the disturbance by its increment, significantly reducing the quasi-sliding-mode domain.

Parallel to disturbance compensation strategies, finite-time stability concepts have gained increasing interest in discrete systems. Finite-time convergence provides faster response and stronger robustness margins compared to asymptotic stability. Recent advances have introduced minimum-operator-based reaching laws, where the minimum function blends linear and nonlinear decay terms to achieve finite-time convergence without excessive control effort. A notable contribution in this direction is presented in [16–19], where a difference-equation-based reaching law incorporating a minimum operator guarantees finite-time convergence in the nominal case and bounded behavior under perturbations.

Despite these advances, existing minima-based reaching laws do not explicitly integrate disturbance compensation mechanisms. Consequently, the ultimate bound of the switching variable remains proportional to the disturbance magnitude itself. In practical applications involving slowly varying uncertainties, this can lead to conservative steady-state performance.

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Motivated by the above analysis, this paper proposes an advanced disturbance compensation strategy for discrete-time sliding mode control. The proposed method integrates a minimum operator-based reaching law with a difference-based disturbance compensation law. By reconstructing the disturbance from measurable switching data, the original disturbance term is replaced by its increment, whose magnitude is typically much smaller. This significantly reduces the quasi-sliding mode band width, which directly results in improved robustness of the closed-loop system, while finite-time convergence is preserved.

The main contributions of this paper are summarized as follows:

a) A novel compensated reaching law combining a minimum operator with disturbance increment estimation for discrete-time uncertain systems.
b) Rigorous finite-time analysis establishing the convergence of the switching function to a disturbance-increment-dependent invariant set, together with an explicit settling-time bound.
c) A constructive control design procedure that realizes the proposed reaching law without requiring explicit disturbance measurement.
d) Demonstration of improved disturbance compensation and reduced quasi-sliding mode band width compared to conventional DSMC approach.

The remainder of the paper is organized as follows. The notations and mathematical preliminaries are presented in Section 2. Section 3 presents the system formulation. Section 4 develops the compensated reaching law and establishes its finite-time properties. Section 5 provides the control synthesis. Section 6 presents simulation results, and Section 7 concludes the paper.

2. MATHEMATICAL PRELIMINARIES

2.1. Notation

We begin by introducing essential mathematical notation and function definitions applied further throughout the paper. The set of real numbers is denoted by $\mathbb{R}$, while $\mathbb{R}^n$ represents the n-dimensional Euclidean space. $\mathbb{R}^+$ refers to the set of real positive values, while the set of nonnegative integers is denoted by $\mathbb{Z}_{\geq 0}$. Vectors and matrices are written in bold italics, while italics are used for scalar variables. The absolute value of $x \in \mathbb{R}$ is denoted as $|x|$. For a matrix $\boldsymbol{A}$, $\boldsymbol{A}^T$ denotes its transpose. The signum function is defined as

$$\operatorname{sgn}(x) = \begin{cases} x/|x| & \text{for} \quad x \neq 0, \\ 0 & \text{for} \quad x = 0. \end{cases}$$

Furthermore, the ceiling operator is denoted by $\lceil\cdot\rceil$ and the minimum operator is $\min(\cdot,\cdot)$. The notation x_k is used to represent the system state at time step k, where $k \in \mathbb{Z}_{\geq 0}$.

2.2. Finite-time stabilization of scalar discrete-time systems

We briefly recall finite-time stability results for scalar discrete-time systems that form the background for the subsequent control design.

Consider the nonlinear discrete-time system

$$y_{k+1} = y_k - \operatorname{sgn}(y_k)\min(|y_k|,\lambda), \qquad y_0 \in \delta \subset \mathbb{R}, \tag{1}$$

where $k \in \mathbb{Z}_{\geq 0}$ and $\lambda > 0$ is a design constant. The minimum operator induces two distinct behaviors: when $|y_k| > \lambda$, the state decreases by a constant amount λ at each step, whereas for $|y_k| \leq \lambda$ the system reaches the origin in a single iteration. Consequently, system (1) exhibits finite-time convergence, with settling time given by $T(y_0) = \left\lceil \frac{|y_0|}{\lambda} \right\rceil$.

3. SYSTEM DESCRIPTION

Consider the following discrete-time uncertain system

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{\Delta A}(\boldsymbol{x}_k) + \boldsymbol{b}u_k + \boldsymbol{f}_k, \tag{2}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$ is the state vector, $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is the nominal system matrix, $\boldsymbol{\Delta A}(\boldsymbol{x}_k) \in \mathbb{R}^n$ represents the matched parametric uncertainty, $\boldsymbol{b} \in \mathbb{R}^n$ is the input vector, $u_k \in \mathbb{R}$ denotes the control input, and $\boldsymbol{f}_k \in \mathbb{R}^n$ represents matched external disturbances vector. The pair $(\boldsymbol{A},\boldsymbol{b})$ is assumed to be controllable. Let us define the sliding variable as

$$s_k = \boldsymbol{c}^T\boldsymbol{x}_k, \tag{3}$$

where $\boldsymbol{c} \in \mathbb{R}^n$ is a constant vector chosen so that $\boldsymbol{c}^T\boldsymbol{b} \neq 0$ and the reduced-order dynamics on the sliding manifold is stable. The initial conditions are denoted with $\boldsymbol{x}_0$ and s_0 for the system states and for the sliding variable, respectively. We define the lumped matched disturbance as

$$d_k = \boldsymbol{c}^T\boldsymbol{\Delta A}(\boldsymbol{x}_k) + \boldsymbol{c}^T\boldsymbol{f}_k. \tag{4}$$

It is assumed that d_k is bounded, i.e., $d_{\min} \leq d_k \leq d_{\max}$, where $d_{\min}$ and $d_{\max}$ are known constants. Furthermore, we define the average disturbance d_{avg} and the maximum impact of the disturbance $D_{\max}$ as follows

$$d_{\text{avg}} = \frac{d_{\min} + d_{\max}}{2}, \qquad D_{\max} = \max\left(|d_{\min}|, |d_{\max}|\right). \tag{5}$$

The goal of this study is to design a control law which ensures the existence of a quasi-sliding mode according to the following definition.

Definition 1. The system is said to exhibit a quasi-sliding mode within an ε-vicinity of the sliding hyperplane $s_k = 0$ iff, there exists a finite $k_0 \geq 0$, such that for any $k \geq k_0$, $|s_k| \leq \varepsilon$, for $\varepsilon > 0$. Parameter ε is referred to as the quasi-sliding mode band radius, resulting in the band of 2ε width.

4. PROPOSED CONTROL STRATEGY

4.1. Disturbance compensation framework

Consider the following minimum-operator-based reaching law with disturbance compensation

$$s_{k+1} = d_k - d_{avg} + s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) - \sum_{i=1}^{k}\left[s_i - s_{d_i}\right], \tag{6}$$

where $a > 0$, $q \in (0,1)$, and d_k denotes a bounded matched disturbance satisfying $|d_k| \leq D_{\max}$. The nominal reaching dynamics are introduced as s_{d_k}, i.e.

$$s_{d_{k+1}} = s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right). \tag{7}$$

The variable $s_{d_{k+1}}$ represents the hypothetical value of s_{k+1} if $d_k = 0$. This value will be further utilized to reconstruct previous disturbance in the proposed compensation scheme. In the absence of disturbances, the reaching law (6) guarantees finite-time convergence of s_k to zero. However, the presence of the disturbance d_k results in convergence only to a neighborhood of the origin. Assume further that the disturbance is slowly varying, namely,

$$\left|d_{k+1} - d_k\right| \leq \delta, \tag{8}$$

where $\delta > 0$ is a known constant, with $\delta < D_{\max}$. From (6) and (7), it follows that

$$s_{k+1} - s_{d_{k+1}} = d_k - d_{\mathrm{avg}} - \sum_{i=1}^{k}\left[s_i - s_{d_i}\right]. \tag{9}$$

Hence, the disturbance at the previous step can be reconstructed with the following formula

$$d_{k-1} = s_k - s_{d_k} + d_{\mathrm{avg}} + \sum_{i=1}^{k-1}\left[s_i - s_{d_i}\right]. \tag{10}$$

Now, in order to analyze system performance with the reaching law (6), we evaluate s_k

$$s_k = s_{k-1} - a\,\mathrm{sgn}(s_{k-1})\min\left(|s_{k-1}|^q, \frac{|s_{k-1}|}{a}\right) + d_{k-1} - d_{\mathrm{avg}} - \sum_{i=1}^{k-1}\left[s_i - s_{d_i}\right]. \tag{11}$$

Using (7) and (11), we get

$$\begin{aligned}\sum_{i=1}^{k}\left[s_i - s_{d_i}\right] &= \sum_{i=1}^{k-1}\left[s_i - s_{d_i}\right] + s_k - s_{k-1} \\ &\quad + a\,\mathrm{sgn}(s_{k-1})\min\left(|s_{k-1}|^q, \frac{|s_{k-1}|}{a}\right) \\ &= d_{k-1} - d_{\mathrm{avg}}.\end{aligned} \tag{12}$$

Substituting (12) into (6) yields the compensated reaching law

$$s_{k+1} = s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) + d_k - d_{k-1}. \tag{13}$$

Equation (13) shows that the original disturbance d_k is replaced by its increment $\Delta d_k = d_k - d_{k-1}$. Using (8), we obtain

$$|\Delta d_k| \leq \delta. \tag{14}$$

Consequently, we can rewrite (13) as

$$s_{k+1} = s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) + \Delta d_k, \tag{15}$$

which implies that the maximum disturbance value, entering the reaching law, is reduced from magnitude $D_{\max}$ to δ. Therefore, with proper parameter selection, there exists a finite integer $k_0 \geq 0$ such that

$$|s_k| \leq \delta, \quad \forall k \geq k_0. \tag{16}$$

In other words, as stated in Definition 1, the system trajectories enter a quasi-sliding mode band, whose radius is determined by the disturbance variation bound δ.

The reaching law (15) combines minimum-operator finite-time convergence with a difference-based disturbance compensator. In contrast to the uncompensated case, where the quasi-sliding mode band width is proportional to $D_{\max}$, the proposed scheme guarantees an ultimate bound proportional to δ, which is typically much smaller. This significantly improves steady-state accuracy.

4.2. Compensated reaching law for discrete-time systems

Consider the compensated reaching law (15), with $a > \delta^{1-q}$, $q \in (0,1)$, and $\Delta d_k = d_k - d_{k-1}$ satisfying $|\Delta d_k| \leq \delta$.

Definition 2. The discrete-time system, governed by the reaching law (15), satisfies the quasi-sliding mode reaching condition in the neighborhood of the sliding manifold $s_k = 0$, iff for any $k \geq 0$ the following condition holds:

$$\frac{|s_k|}{a} > |s_k|^q \Rightarrow |s_{k+1}| < |s_k| - \alpha,$$

where $\alpha \in \mathbb{R}^+$ is an arbitrarily small constant. Moreover, there exists some finite $k_0 \geq 0$, such that for any $k \geq k_0$

$$|s_k| \leq \delta.$$

Using the system dynamics (2) and the sliding variable (3), the control input is designed so that (15) is satisfied. The following theorem characterizes the behavior of the compensated reaching law.

Theorem 1. Consider the compensated reaching law (15) with parameters selected so that $q \in (0,1)$ and $a > \delta^{1-q}$. Then the switching function s_k converges to a bounded neighborhood of the sliding manifold in finite time. In particular, there exists a finite integer $k_0 \geq 0$ such that

$$|s_k| \leq \delta, \qquad \forall k \geq k_0.$$

An upper bound for the settling time is given by

$$k_0 \leq K(s_0) = \left\lceil \frac{|s_0| - \delta}{a^{\frac{1}{1-q}} - \delta} \right\rceil. \tag{17}$$

Proof. The considered compensated reaching law (15) is characterized by two distinct behaviors enforced by the minimum operator. The reaching law characteristic switches at a threshold sliding variable value $|s^*| = a^{\frac{1}{1-q}}$, where $|s_k|^q = \frac{|s_k|}{a}$. Consequently, the reaching law (15) may be rewritten as

$$s_{k+1} = \begin{cases} s_k - a\,\mathrm{sgn}(s_k)|s_k|^q + \Delta d_k & \text{for } |s_k| > a^{\frac{1}{1-q}}, \\ s_k - \mathrm{sgn}(s_k)|s_k| + \Delta d_k & \text{for } |s_k| \le a^{\frac{1}{1-q}}. \end{cases} \tag{18}$$

Now, we consider two possible cases specified in (18).

Case 1: If $|s_k| > |s^*| = a^{\frac{1}{1-q}}$, then

$$s_{k+1} = s_k - a\ \mathrm{sgn}(s_k)|s_k|^q + \Delta d_k.$$

Thus,

$$|s_{k+1}| = |s_k - a\ \mathrm{sgn}(s_k)|s_k|^q + \Delta d_k|.$$

Since $|s_k| > a^{\frac{1}{1-q}}$, it follows that $|s_k| > a|s_k|^q$. This implies

$$|s_{k+1}| \le |s_k| - a|s_k|^q + \delta. \tag{19}$$

Therefore, $|s_{k+1}| < |s_k|$ when

$$a|s_k|^q \ge \delta + \alpha, \tag{20}$$

where α is an arbitrarily small positive constant. Considering that in this zone $|s_k| > |s^*| = a^{\frac{1}{1-q}}$, we substitute s^* into (20) as the minimum sliding variable value. Since by design

$$a > \delta^{1-q}, \tag{21}$$

then

$$|s_{k+1}| < |s_k|.$$

We conclude that the absolute value of the sliding variable decreases monotonically, until it enters the band $|s_k| \le a^{\frac{1}{1-q}}$, when the second term of (18) operates.

Case 2: If, for some $k^* \ge 0$, $|s_{k^*}| \le |s^*| = a^{\frac{1}{1-q}}$, then

$$s_{k^*+1} = s_{k^*} - \mathrm{sgn}(s_{k^*})|s_{k^*}| + \Delta d_k = \Delta d_k.$$

Thus, $|s_{k^*+1}| \le \delta$. Considering further the value of $|s_{k^*+1}|$, with $a > \delta^{1-q}$, yields

$$|s_{k^*+1}| = |\Delta d_k| \le \delta < a^{\frac{1}{1-q}}.$$

Consequently, the second term of (18) acts, $k_0 = k^* + 1$ and

$$|s_k| \le \delta, \qquad \forall k \ge k_0,$$

which ensures the existence of the quasi-sliding mode according to Definition 1.

Taking into account (19), the rate of change of the sliding variable satisfies

$$\Delta s_k = |s_k| - |s_{k+1}| \ge a|s_k|^q - \delta.$$

Considering that $|s_k| > |s^*| = a^{\frac{1}{1-q}}$, $a > \delta^{1-q}$, and (20) holds, we get

$$\Delta s_k > a^{\frac{1}{1-q}} - \delta > 0. \tag{22}$$

From (22) one may see that the minimum rate of change of the sliding variable is positive. Consequently, a finite k_0, such that $|s_k| \le \delta$ for any $k \ge k_0$, exists. Considering the convergence of the sliding variable from the initial value of s_0 to the bound of $\pm\delta$ and the minimum rate of change (22), the settling time is upper bounded as follows

$$k_0 \le K(s_0) = \left\lceil \frac{|s_0| - \delta}{a^{\frac{1}{1-q}} - \delta} \right\rceil,$$

which concludes the proof. □

The convergence speed of the compensated reaching law is governed by the parameters a and q. Increasing a or selecting q closer to one reduces the settling time, whereas value of q closer to zero reduces the convergence rate, increasing $K(s_0)$. Hence, the choice of a and q must account for actuator limits and convergence requirements. On the other hand, the quasi-sliding mode band width depends on the disturbance, i.e., larger disturbance increments δ enlarge the quasi-sliding-mode band.

Remark 1. The above analysis holds for the reaching law $s_{k+1} = s_k - \mathrm{sgn}(s_k)\min(|s_k|, a) + d_k$, where $a > D_{\max}$ and $K(s_0) = \left\lceil \frac{|s_0| - D_{\max}}{a - D_{\max}} \right\rceil$. Modified reaching law for disturbance compensation, given as $s_{k+1} = d_k - d_{\text{avg}} + s_k - \mathrm{sgn}(s_k)\min(|s_k|, a) - \sum_{i=1}^{k}\left[s_i - s_{d_i}\right]$, where $s_{d_{k+1}} = s_k - \mathrm{sgn}(s_k)\min(|s_k|, a)$ and $a > \delta$, guarantees $|s_k| \le \delta$ for any $k \ge k_0$ and $k_0 \le K(s_0) = \left\lceil \frac{|s_0| - \delta}{a - \delta} \right\rceil$, if $s_0 > a$.

5. CONTROL DESIGN

Previous section proposed a novel disturbance compensation scheme, based on a nominal sliding variable trajectory, obtained without any external disturbance impact. Such a virtual sliding variable trajectory may be obtained in a separate calculation block and is next used to reconstruct previous disturbance values. The proposed control scheme indeed consists of a few blocks, working in parallel, that is: the nominal trajectory s_{d_k} generator, the disturbance reconstruction block and the main controller's block employing a minimum operator-based reaching law. A block diagram of the proposed control algorithm is presented in Fig. 1.

The aim of this section is to synthesize a control input u_k such that the sliding variable (3), $s_k = \boldsymbol{c}^T\boldsymbol{x}_k$, evolves according to the proposed compensated reaching law (6), i.e.,

$$s_{k+1} = d_k - d_{avg} + s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) - \sum_{i=1}^{k}\left[s_i - s_{d_i}\right].$$

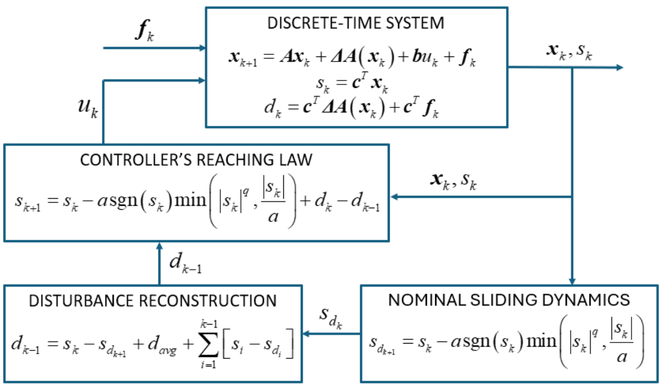

Fig. 1. Block diagram of the proposed control scheme

From the discrete-time plant dynamics (2) and the definition of the sliding variable (3), we obtain

$$s_{k+1} = \boldsymbol{c}^T \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{c}^T \boldsymbol{b} u_k + d_k\,, \tag{23}$$

where d_k, defined in (4), represents the lumped matched disturbance. The control objective is to enforce (6) on the plant dynamics (23). Equating (23) with (6) yields

$$\boldsymbol{c}^T A\boldsymbol{x}_k + \boldsymbol{c}^T \boldsymbol{b} u_k = s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) - d_{\mathrm{avg}} - \sum_{i=1}^{k}\left[s_i - s_{d_i}\right]. \tag{24}$$

Since the pair $(\boldsymbol{A},\boldsymbol{b})$ is controllable and $\boldsymbol{c}^T\boldsymbol{b} \neq 0$, solving (24) for u_k gives

$$u_k = -(\boldsymbol{c}^T \boldsymbol{b})^{-1}\left[\boldsymbol{c}^T A\boldsymbol{x}_k - s_k + a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) + d_{\mathrm{avg}} + \sum_{i=1}^{k}\left[s_i - s_{d_i}\right]\right], \tag{25}$$

where s_{d_k} is the predicted sliding variable in the absence of disturbance, given by

$$s_{d_k} = s_{k-1} - a\,\mathrm{sgn}(s_{k-1})\min\left(|s_{k-1}|^q, \frac{|s_{k-1}|}{a}\right). \tag{26}$$

The disturbance reconstruction relies solely on past values of s_k and is therefore implementable in real time. Substituting the control law (25) into (23) yields the closed-loop sliding dynamics

$$s_{k+1} = s_k - a\,\mathrm{sgn}(s_k)\min\left(|s_k|^q, \frac{|s_k|}{a}\right) + \Delta d_k\,,$$

which exactly matches the compensated reaching law (15). Under the assumption that $|\Delta d_k| \le \delta$, it follows from Theorem 1 that there exists a finite integer $k_0 \le K(s_0)$ such that

$$|s_k| \le \delta, \qquad \forall k \ge k_0\,.$$

Hence, the proposed controller guarantees finite-time convergence of the switching function to a bounded neighborhood of the sliding manifold, establishing the existence of a quasi-sliding mode with band width of 2δ.

6. SIMULATION RESULTS

In this section, the proposed control strategy is applied to a mechatronic actuator, described with the following state representation

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{\Phi}\boldsymbol{x}(t) + \boldsymbol{\Gamma}[u(t) + f(t)],$$

where the state variables denote: $x_1(t)$ – the angular displacement measured in radians (rad), $x_2(t)$ – the angular velocity in radians per second (rad/s). The control $u(t) \in \mathbb{R}$ corresponds to the input voltage, $f(t) = 10\sin(3t)$ represents matched external disturbances in volts (V). The state matrix $\boldsymbol{\Phi} \in \mathbb{R}^{n\times n}$ and the input vector $\boldsymbol{\Gamma} \in \mathbb{R}^n$ are

$$\boldsymbol{\Phi} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{k_t k_e + RB}{J} \end{bmatrix}, \qquad \boldsymbol{\Gamma} = \begin{bmatrix} 0 \\ \dfrac{k_t}{JR} \end{bmatrix}.$$

The above system is characterized by resistance $R = 1\,\Omega$, moment of inertia $J = 0.2\ \mathrm{kg\cdot m^2}$, torque constant $k_t = 0.15\ \frac{\mathrm{N\cdot m}}{\mathrm{A}}$, electrical constant $k_e = 0.15\ \frac{\mathrm{V\cdot s}}{\mathrm{rad}}$ and viscous friction coefficient $B = 0.01\ \frac{\mathrm{N\cdot m\cdot s}}{\mathrm{rad}}$. The system is discretized with the sampling period $T = 0.1$ s, resulting in

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{b}u_k + \boldsymbol{f}_k\,,$$

where appropriate matrices are obtained as follows

$$\boldsymbol{A} = e^{\boldsymbol{\Phi}T}, \qquad \boldsymbol{b} = \boldsymbol{\Gamma}\int_0^T e^{\boldsymbol{\Phi}\tau}\,\mathrm{d}\tau,$$

$$\boldsymbol{f}_k = \boldsymbol{\Gamma}\int_0^T e^{\boldsymbol{\Phi}\tau} f_{(k+1)T-\tau}\,\mathrm{d}\tau.$$

The initial conditions are chosen as $x_1(0) = 1$ rad and $x_2(0) = 3$ rad/s. The controller design parameters are selected as $c_1 = 5\ \frac{1}{\mathrm{rad}}$, $c_2 = 1\ \frac{\mathrm{s}}{\mathrm{rad}}$, which guarantees stability of the closed-loop system. Furthermore, $D_{\max} = 0.94$ and $\delta = 0.28$. Hence, $q = 0.2$ and $a = 0.95$, which satisfies the conditions $a > D_{\max}$ and $a > \delta^{1-q}$, respectively.

Figure 2 illustrates the comparative performance of the proposed disturbance-compensated minimum-operator-based sliding mode controller and the minimum-operator-based uncompensated scheme. The upper plot shows the evolution of the sliding variable s_k, where the proposed method achieves faster convergence and a substantially reduced quasi-sliding-mode band due to disturbance increment compensation. In the nominal case, the quasi-sliding mode band is described with $|s_k| \le D_{\max}$, while

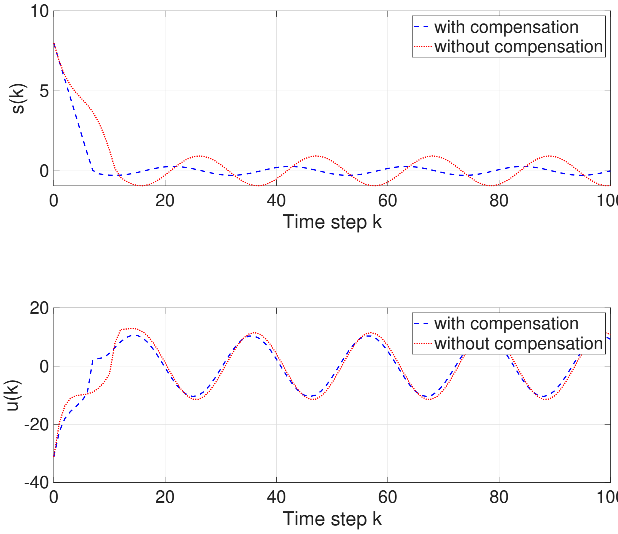

Fig. 2. Comparative performance: (top) sliding variable (bottom) control input

the disturbance compensation scheme reduced the band's width to $|s_k| \leq \delta$. The lower plot depicts the corresponding control input $u(k)$. It can be clearly seen from the figure that improved robustness is achieved without increasing the magnitude of the control signal.

Figure 3 presents the time responses of the actuator states under both considered control strategies. The states converge rapidly to the vicinity of the equilibrium despite the presence of matched external disturbances, confirming the effectiveness of the proposed reaching law in ensuring finite-time convergence and stable closed-loop behavior. It may be seen that the system output error, associated with the first state variable x_1 is significantly reduced by the compensated control algorithm.

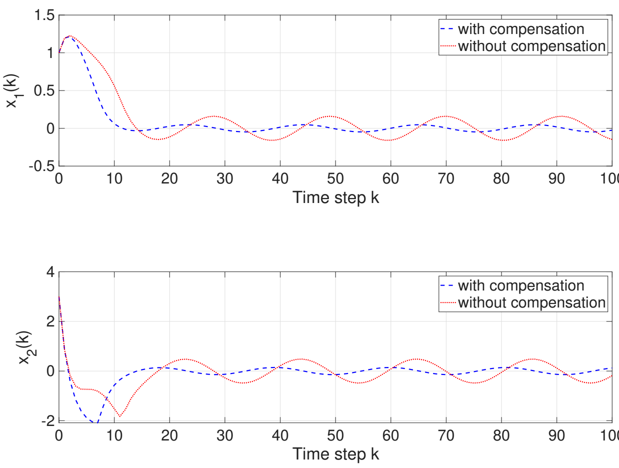

Fig. 3. Comparative performance of the state evolution

Finally, Fig. 4 shows the phase portrait of the state trajectories for both compensated and uncompensated control schemes. The trajectories corresponding to the proposed method converge to a compact invariant set around the origin, whereas the uncompensated controller exhibits larger steady-state oscillations.

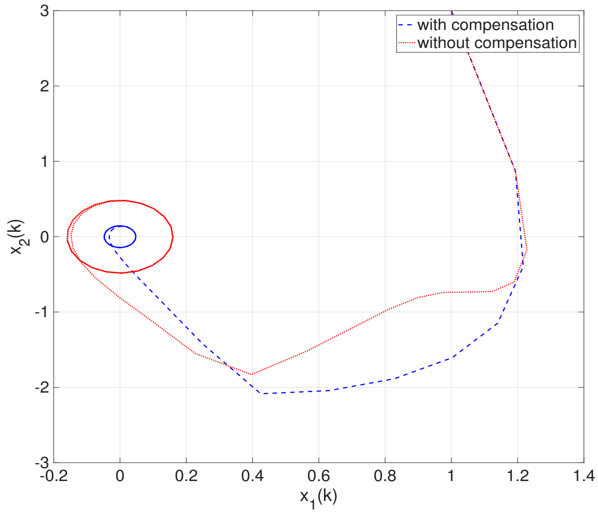

Fig. 4. Phase portrait of the state trajectories

To clearly present the benefits obtained by a disturbance compensated control scheme, well-known performance quality indices, such as the Integral Absolute Error (IAE), Integral Time-Weighted Absolute Error (ITAE), Integral Squared Error (ISE) and Integral Time-Weighted Squared Error (ITSE) were calculated for both, the sliding variable and the system output x_1. The discrete time indices were obtained as follows:

$$\begin{aligned} \mathrm{IAE} &= \sum_{k=0}^{N} |e_k| T, \quad \mathrm{ITAE} = \sum_{k=0}^{N} (kT|e_k|)T, \\ \mathrm{ISE} &= \sum_{k=0}^{N} e_k^2 T, \qquad \mathrm{ITSE} = \sum_{k=0}^{N} (kTe_k^2)T, \end{aligned} \tag{27}$$

where T denotes the discretization period and e_k represents the error signal, in our case s_k or x_{1k}, respectively. The aforementioned criteria have been compared in Table 1. It may be directly verified that the proposed disturbance compensator results in a significant reduction of the control error.

Table 1

Performance indices comparison for both control strategies

Index	Sliding variable s		System output x_1	
	Without compensation	With compensation	Without compensation	With compensation
IAE	10.39	4.85	2.02	1.06
ITAE	31.94	9.48	5.55	1.80
ISE	30.60	18.54	1.22	0.77
ITSE	29.64	4.47	1.08	0.26

7. CONCLUSIONS

This paper presented an advanced disturbance-compensated discrete-time sliding mode control framework based on a minimum operator reaching law. By integrating finite-time minima-based convergence with difference-based disturbance compensation, the proposed method effectively transforms the original matched disturbance into its increment, thereby substantially reducing the quasi-sliding mode band width. Rigorous analysis established an explicit settling-time bound and demonstrated finite-time convergence of the switching function to a compact invariant set whose size is determined by the disturbance variation. A constructive control design was developed to realize the compensated reaching law. Compared with conventional discrete-time sliding mode controllers and uncompensated minima-based schemes, the proposed approach achieves improved steady-state accuracy while maintaining rapid transient response. Future work will focus on extending the proposed framework to multi-input multi-output systems, incorporating actuator constraints, and exploring adaptive tuning of the reaching law parameters.

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