

Truss Systems: From Structural Idealization to Monitoring-Based Assessment

Marcin KAMIŃSKI^{1*}, Rafał BŁOŃSKI¹

¹ Lodz University of Technology, Faculty of Civil Engineering, Architecture and Environmental Engineering,
Department of Structural Mechanics, Aleje Politechniki 6, 93-590 Lodz, Poland

Abstract. Truss systems are widely used in construction as lightweight and efficient load-bearing elements for roofs, bridges, industrial facilities, towers, and large-span structures. Their structural performance is influenced not only by the axial forces acting on individual members, but also by system-level effects related to geometric imperfections, joint behavior, nonlinear response, environmental actions, and variations in other design parameters. This paper reviews engineering approaches to the modeling and assessment of truss systems, with particular emphasis on their significance for the reliability assessment of civil engineering structures. The review covers classical idealizations, normative modeling requirements, experimental and full-scale evidence, finite element modeling strategies, stochastic analysis, system reliability, surrogate-based methods, and monitoring-based updating. The reviewed studies show that simplified deterministic models are often insufficient when structural response is influenced by instability, load redistribution, or uncertainty-sensitive behavior. Stochastic and reliability-based methods provide a consistent framework for quantifying these effects, although their direct integration with detailed numerical models may become computationally demanding. The review identifies three principal research gaps: efficient representation of nonlinear, imperfection-sensitive, and system-level truss behavior; integration of reliability analysis with surrogate, data-driven, and physics-informed approaches; and systematic incorporation of structural health monitoring data into numerical and probabilistic models for service-life reliability assessment.

Key words: truss systems; structural reliability; geometric imperfections; finite element modeling; monitoring-based updating.

1. INTRODUCTION

Truss structures can be understood not so much as a specific type of engineering object, but rather as a universal structural concept based on the deliberate arrangement of linear members and their mutual connections. Their mechanical behavior results primarily from the organization of load-transfer paths so that axial forces in members dominate and bending is minimized. From this perspective, the geometry and topology of the system, rather than solely the material properties of individual elements, become the key carriers of information about the behavior of the entire structure. This principle was already identified in classical studies on the statics of bar structures and the equilibrium of reciprocal figures [1,2].

In civil engineering, trusses are among the most widely used load-bearing systems. This is due to their favorable strength-to-weight ratio, high stiffness with limited material consumption, and ability to span very large distances. Their modular character facilitates prefabrication and assembly, while the dominance of axial member behavior results in relatively predictable structural performance, which is important for long-term durability and safety. Consequently, trusses have found extensive application in road and railway bridges, roofs of

sports and industrial facilities, towers, masts, and special engineering structures. This widespread use is confirmed by both design standards [3,4] and numerous experimental studies and analyses of real structures [5-9]. Representative applications of truss systems are shown in Fig. 1. Despite their long history of application, contemporary modeling of truss structures goes far beyond classical analyses based on linear elasticity theory and idealized static schemes. The development of numerical methods, in particular the finite element method, has enabled the inclusion of geometric nonlinearity associated with member instability, material nonlinearity resulting from plasticity, fatigue, or corrosion degradation, as well as realistic modeling of joints and connections [10-13]. At the same time, there is a growing integration of classical computational methods with artificial intelligence (AI)-based tools, enabling design automation, geometric optimization of trusses, and the exploration of new structural solutions [14].

Alongside the development of deterministic methods, probabilistic approaches are gaining increasing importance. In this framework, truss structures are treated as systems subject to uncertainty arising from variability in material properties, geometric imperfections, manufacturing tolerances, and the random nature of environmental actions and operational

*e-mail: marcin.kaminski@p.lodz.pl

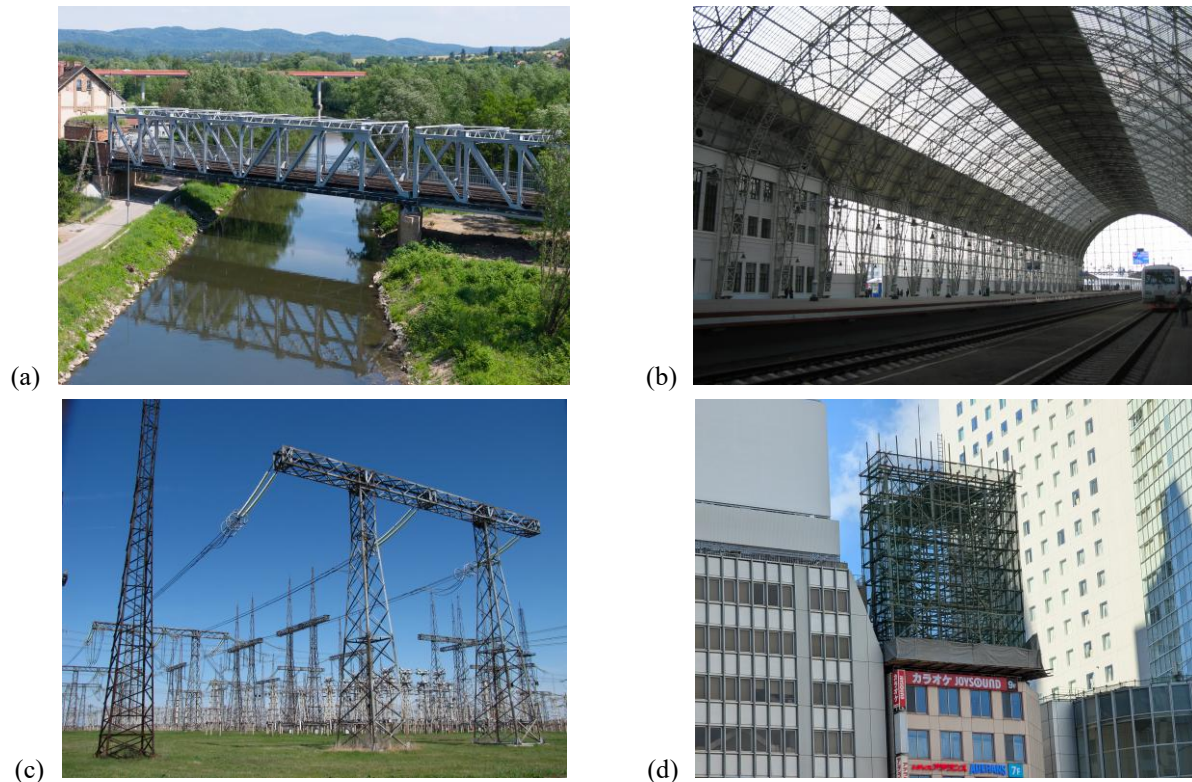


Fig. 1. Truss systems in civil engineering: (a) railway truss bridge in Kłodzko, Poland (Jacek Halicki, CC0 1.0, Wikimedia Commons); (b) steel roof structure of the Kievsky Railway Station in Moscow (Kucharek, public domain, Wikimedia Commons); (c) high-voltage switchgear structure (Novoklimov, CC0 1.0, Wikimedia Commons); (d) scaffolding structure in Shibuya (Syced, CC0 1.0, Wikimedia Commons). Images cropped by the authors.

processes. Reliability analysis enables a quantitative assessment of this variability, leading to the estimation of failure probabilities, identification of critical elements, and design of structures in accordance with risk-based design concepts [15-18]. For existing structures, structural health monitoring provides an additional source of information on in-service structural response and changing environmental or operational conditions, supporting more informed assessment during the service life.

1.1. Scope and motivation of the review

Despite the wide range of available research, classical structural idealization, experimental observation, numerical modeling, reliability analysis, and monitoring-based assessment are often treated as separate methodological areas. A more integrated approach is needed to clarify how assumptions made in structural modeling influence the representation of uncertainty and reliability of truss systems. This review aims to fill this gap by integrating these areas within a common framework for the assessment of truss structures.

This paper reviews contemporary methods for modeling and assessment of truss structures, from classical idealizations and experimental observations to advanced numerical and probabilistic approaches. The work also identifies key directions for future research, including integrating physical models with artificial intelligence methods [19] and developing adaptive computational models based on data from structural health monitoring systems [20].

Consequently, the review is organized as a sequence of stages, starting with structural idealization and standards-based modeling, through experimental and numerical analysis, to uncertainty quantification, reliability analysis, and monitoring-based updating.

1.2. Literature review methodology

The literature review was conducted as a structured narrative review. Relevant publications were identified mainly using Google Scholar, supplemented by studies already known to the authors and by references cited in the selected publications. The search covered terms related to truss structures, structural modeling, geometric imperfections, nonlinear analysis, reliability, stochastic methods, structural health monitoring, and data-driven approaches.

The aim of the selection was to include both the latest research reflecting the current state of knowledge and selected earlier works presenting alternative or historical perspectives on the discussed issues. Classic works and fundamental publications used in structural mechanics and structural analysis were also included. They provide the theoretical foundation for the concepts discussed in this review.

The adopted approach was intended to provide a broad and representative overview, rather than conducting an exhaustive, systematic review of the available literature. Therefore, publications were selected based on their relevance to the specific topics discussed and their contribution to illustrating the development, potential, and limitations of the considered approaches.

2. STRUCTURAL CHARACTERISTICS OF TRUSS SYSTEMS

2.1. Fundamental structural behavior

In civil engineering, a truss is a load-bearing structure composed of straight, slender members connected at nodes in a geometrically stable form, in which internal loads are transferred mainly in the form of axial forces - tension or compression. In the classical engineering idealization, it is assumed that nodes are hinged and that external loads are transferred exclusively to the nodes, allowing each member to be treated as a uniaxial element without bending moments or transverse forces. The structural efficiency of a truss arises from its reliance on triangular geometry, the simplest form that maintains shape without deformation due to internal hinging. By appropriately arranging triangular modules, truss girders achieve high stiffness-to-weight ratios.

2.2. Idealization assumptions and practical limitations

Classical truss theory relies on three basic assumptions [21]:

- Joints are perfectly pinned.
- Loads act only at nodes.
- Members are straight, prismatic, and pin-connected.

In addition, deformations are assumed to be small enough to neglect geometric nonlinearity.

In reality, practical truss structures deviate from these idealizations. Joint rigidity introduces bending moments; connections may be eccentric relative to member centerlines; loads from roof sheeting and purlins often induce bending; residual stresses and initial crookedness reduce compressive capacity. Consequently, accurate modeling of modern truss girders frequently requires semi-rigid joint models, stability analyses that account for P- Δ effects, and, in many cases, full finite element analyses using beam-column or shell elements.

2.3. Classical and contemporary truss types

In a review of truss systems, typological classification is significant not only from the historical perspective, but also because differences in geometry have a direct impact on load-transfer mechanisms, force distributions, stability, and assumptions made during structural modeling. Representative configurations are a useful framework for relating classical structures with modeling aspects discussed in the following parts of this study. The selected configurations include: (i) basic triangulated systems used to explain the fundamental force path in chord and web members, (ii) canonical truss families distinguished by the predominant axial state of diagonals (tension vs compression-dominated), (iii) refined variants introduced to improve stiffness and performance under concentrated or moving loads, and (iv) special geometries that combine truss action with arch behavior. Based on these principles, a set of representative truss types is discussed below, using the Historic American Engineering Record (HAER) truss identification poster as the typological reference [22]. The poster is presented in Fig. 2.

The King Post and Queen Post trusses represent the simplest triangulated bridge systems and serve as baseline configurations for introducing member roles and axial force transfer in truss idealization. In the King Post system, a single central post links the apex to the tie chord, forming two primary triangular panels and establishing a clear and direct axial load path. The Queen Post scheme extends this concept by using two vertical posts connected with a straining beam, enabling multi-panel behavior and a more distributed stiffness pattern while retaining a structurally transparent force path. The HAER scheme describes the King Post as a traditional wooden truss with medieval origins and the Queen Post as an extended version of the King Post. They provide a simple visual and mechanical explanation of how a truss transforms a distributed load into axial forces in the chords, columns and diagonal members.

The classical Howe and Pratt trusses provide a key engineering comparison within truss typology families, as they adopt opposite diagonal orientations, thereby altering the dominant axial states in the web. In the Howe configuration, the diagonals are typically compression-dominated under gravity loading, making buckling resistance and member slenderness the primary design drivers. In contrast, the Pratt configuration typically shifts the gravity-load demand toward tension in diagonal members, enabling the use of slender diagonal elements without compression-buckling constraints. Together, these two families provide a fundamental reference framework for discussing structural efficiency, sensitivity to stability, and the impact of these differences on modeling assumptions.

The Warren family exemplifies efficient, repetitive triangulation, in which alternating diagonals form a series of triangular panels. This geometry results in a highly repetitive web layout with a clear axial-force transfer mechanism, while the diagonals often alternate between tension and compression depending on load distribution. However, a purely triangular web system may be sensitive to concentrated loading, resulting in less favorable force distribution in certain panels. The Warren with vertical configuration refines the web by introducing vertical members, improving force transfer at panel points, and typically increasing global stiffness and service performance. Beyond the basic truss families, several refined types illustrate how truss geometry and member subdivision were historically and practically adapted to span length, stiffness demand, and live-load effects. In the HAER typology the Parker, Camelback, Baltimore, and Pennsylvania/Petit trusses can be viewed as a group of improvements based on the Pratt truss. The Parker truss modifies the Pratt system by introducing a polygonal top chord, allowing for variable truss heights along the span. The Camelback truss can then be regarded as a more regular variant of the Parker truss, typically defined by a polygonal top chord with a limited number of slopes. The Baltimore and Pennsylvania/Petit trusses develop the same logic based on the Pratt system by introducing secondary cross-braces, secondary columns or other secondary elements that divide the main fields and improve performance under concentrated and live loads. From the perspective of this

work, they illustrate how geometric improvements can affect the level of detail required in the models.

From a stability and slenderness perspective, the K-Truss provides an engineering-relevant web arrangement in which members form characteristic K-shaped patterns within each panel. The key concept is the subdivision of the web, which reduces the effective unsupported lengths of compression members and improves buckling resistance. As a result, K-type web layouts are often discussed in the context of stability-driven member arrangement and web refinement in truss systems.

In parallel with these basic and refined systems, the Town Lattice truss represents a structurally distinct concept based on a dense network of overlapping diagonals. Unlike panelized Pratt, Howe, and/or Warren systems, the lattice layout offers multiple alternative load paths and smooth shear transfer across numerous intersecting members, introducing a high level of redundancy.

The Lenticular truss and the Bowstring Arch-Truss cover hybrid geometries in which truss action is combined with arch-like behavior. The Lenticular (fish-belly) system employs opposing curved top and bottom chords forming a lens-shaped profile; the upper chord typically carries compression with arch-like action while the lower chord acts as a tension tie, resulting in reduced bending effects compared to straight-chord arrangements. The Bowstring configuration similarly combines a curved or polygonal top chord with a straight tie chord, producing an arch–tie mechanism where compression is carried mainly in the upper chord and thrust is resisted by tension in the bottom chord. These systems broaden the typological coverage toward geometry-controlled force flow and stiffness, providing an engineering comparison with purely straight-chord truss families.

For brevity, the principal classical truss configurations discussed above are summarized in Table 1, which highlights their dominant structural characteristics, key advantages, and primary engineering motivations. This synthesis provides a theoretical framework for further discussion about structural idealization, stability, numerical modeling and reliability assessment. Representative bridge structures corresponding to selected truss systems are shown in Fig. 3.

3. EVOLUTION OF MODELING APPROACHES

The development of truss analysis spans over 150 years and illustrates the evolution of structural engineering from intuitive geometric approaches to advanced numerical and computational techniques. This evolution progressively extended the range of effects that could be modeled – from axial-force equilibrium in idealized trusses to stability, geometric and material nonlinearities and complex numerical models. The main stages of this development are presented below to provide a methodological background for the contemporary modeling approaches.

One of the earliest important contributions to structural analysis was made by James Clerk Maxwell, who introduced the concept of reciprocal figures and demonstrated a geometric relationship between a truss and its force diagram [1].

Later, Luigi Cremona expanded these ideas in his book *Graphical Statics* [2]. He organized the process of finding forces in structural members using diagrams, which became a key reference for engineers in the late nineteenth century. Cremona’s force diagrams were widely used to analyse large trusses before analytical methods became common. Further improvements in graphical methods were made by Maurice Lévy, who applied Maxwell’s and Cremona’s principles to more complex structural systems [23]. His work expanded the

TABLE 1. Synthesis of representative truss types and their structural characteristics

Truss type	Dominant structural characteristic	Key advantage	Primary engineering motivation
King Post	Simple axial load transfer	Structural clarity and minimal material use	Basic chord–web interaction
Queen Post	Multi-panel axial behavior	Increased span and stiffness	Extension of simple triangulated systems
Howe	Compression-dominated diagonals	Efficient use of compression members	Adaptation to compression-resistant materials
Pratt	Tension-dominated diagonals	Reduced buckling sensitivity	Efficient exploitation of tensile members
Warren	Alternating axial forces in diagonals	Repetitive and material-efficient geometry	Uniform triangulation
Warren with verticals	Enhanced panel-point force transfer	Improved stiffness under concentrated loads	Improved service performance
Parker	Variable chord depth	Reduced chord force demand	Geometry adapted to the bending-moment distribution
Baltimore	Subdivided web system	Improved shear distribution and stiffness	Control of peak internal forces
K-Truss	Reduced effective compression length	Improved buckling resistance	Stability-driven web modification
Town Lattice	Multiple overlapping load paths	High redundancy and robustness	Alternative force redistribution
Lenticular	Combined truss–arch action	Reduced bending effects	Efficient force flow through geometry
Bowstring	Arch–tie mechanism	Efficient thrust–tie action	Combination of truss and arch behavior

range of applications of graphical statics and helped shift from descriptive geometry toward early analytical approaches. At the same time, developments in elasticity theory and the understanding of internal forces were advanced by Otto Mohr. His contributions, particularly Mohr's circle, helped unify stress analysis and influenced the later development of analytical structural models [24]. This analytical development was subsequently extended toward problems of structural efficiency and optimal material distribution. Michell showed that the minimum volume of a material structure subjected to specific external forces is achieved when the structure corresponds to a statically permissible continuous stress field of uniform strength, in which tensile and compressive elements are distributed along two orthogonal systems of curves aligned with the principal stress directions [25], Fig. 4. Michell's formulation provides an early theoretical basis for structural optimization aimed at material efficiency. Contemporary truss optimization extends this concept through size, shape, and topology optimization, in which member cross-section areas, nodal positions, and connectivity are treated as design variables. These approaches are discussed further in Section 7.5 in the context of reliability-based design and optimization.

Fritz Engesser introduced early concepts of second-order theory and structural stability, which later became essential for analysing slender truss members prone to geometric nonlinearity [26]. Stephen P. Timoshenko described classical methods and extended them with ideas from elasticity, stability, and materials science [27,28]. His detailed studies showed the transition from empirical design rules to a more precise mathematical theory and placed truss analysis in the broader context of structural mechanics.

Further mathematical rigor underpinning modern numerical methods was provided by Richard Courant, whose variational principles established the theoretical foundations for discretization and approximation techniques that were later adopted in finite element formulations [29]. Building on these mathematical concepts, Argyris and Kelsey developed matrix-based methods for structural analysis that enabled the systematic numerical treatment of trusses, frames, and continuum structures, including nonlinear, dynamic, and three-dimensional problems [30]. A major milestone was the formulation of the Finite Element Method (FEM) by Turner, Clough, Martin, and Topp. Their stiffness-based approach provided a computational framework for analysing complex trusses and frames, enabling high-precision evaluation of deformations, internal forces, and stability effects [31]. Olgierd Zienkiewicz's works were essential in transforming FEM from a promising mathematical concept into a widely used engineering tool. His contributions allowed FEM to become the main method for analysing truss systems, shells, plates, and three-dimensional structures [32].

The development outlined above is summarized in Table 2 for clarity and conciseness. These developments established the analytical and numerical foundations of modern structural modeling. The following section discusses how these concepts are reflected in current modeling approaches and design

standards. It describes structural idealizations, second-order effects, imperfections, and material nonlinearity.

4. THEORETICAL MODELING APPROACHES

4.1. Idealization of truss behavior and modeling scope

In the theoretical modeling of truss structures, the adopted level of idealization defines the structural mechanisms represented in the global analysis model and conditions the admissibility of the applied analysis procedure. Both EN 1993-1-1 and AISC 360-22 allow simplified representations of truss behavior at the system level; however, they define the conditions for their use differently [3,4].

In classical truss theory, members are idealized as straight, axially loaded elements connected by pinned joints, with bending moments and joint rigidity neglected. This representation clearly describes force transfer and is suitable for conceptual assessment and preliminary global analysis. Its applicability within a normative framework depends on whether the adopted model adequately represents the mechanisms governing strength, stiffness, and stability.

Design standards for steel structures permit the use of simplified global models in which truss members are represented as line elements, provided that the adopted idealization reflects the structural behavior relevant to the analysis purpose. At the same time, practical truss structures deviate from ideal assumptions due to joint flexibility, connection eccentricities, fabrication tolerances, and initial geometric imperfections. These deviations must be considered when defining the modeling scope and selecting the appropriate level of idealization.

Simplified truss idealizations may be acceptable for systems with limited sensitivity to global displacements and instability, whereas structures influenced by sway effects or force-displacement interaction require a more extensive modeling approach consistent with stability verification requirements.

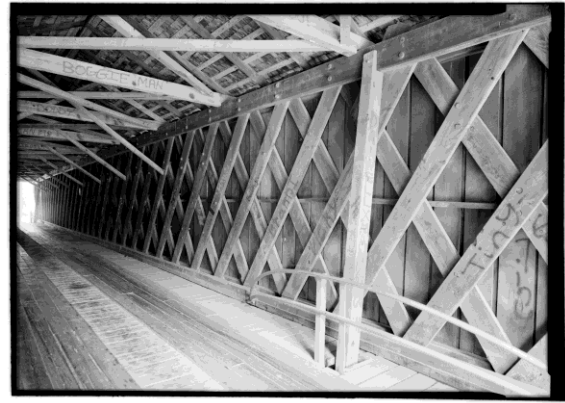
4.2. Choice of discretization and element formulation

The choice of discretization and element formulation determines which structural mechanisms are explicitly included in the analysis model. In both EN 1993-1-1 and AISC 360-22, this choice affects the applicability of first-order analysis, the treatment of imperfections, and the need to consider second-order effects.

The most common formulation in truss analysis is the axial force model based on bar (truss) elements, which have translational degrees of freedom only and transmit axial forces without resisting bending or torsion. This formulation is consistent with classical truss idealization and is appropriate for structures in which joint stiffness, member bending, and spatial effects do not significantly affect the global response. When bending stiffness of members, joint semi-rigidity, or spatial interaction contribute to the structural behavior, discretization based solely on axial-force elements may not adequately represent displacements or internal force redistribution. In such cases, beam elements with rotational degrees of freedom or



(a)



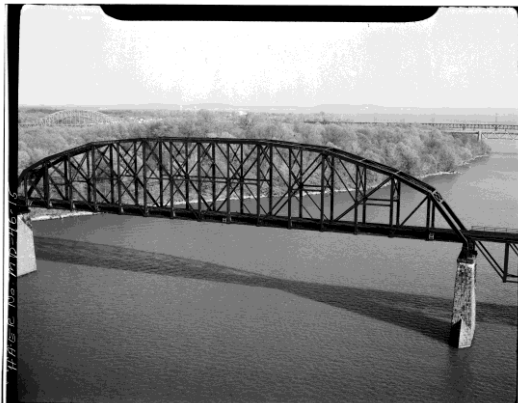
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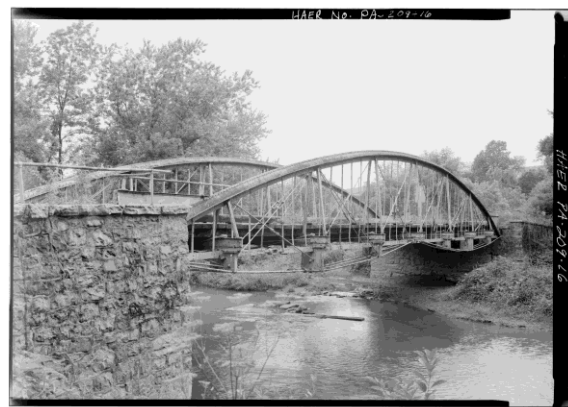
(e)



(f)



(g)



(h)

Fig. 3. Selected bridge structures illustrating different truss systems: (a) King Post, (b) Town Lattice, (c) Warren, (d) Pratt, (e) Parker, (f) Pennsylvania, (g) Lenticular, (h) Bowstring Arch-Truss. Sources: Library of Congress, Historic American Engineering Record (HAER); no known restrictions on publication: hhh.wi0206.photos.171251p; ga0911.photos.195281p; hhh.ca1352.photos.016399p; wi0197.photos.171297p; mo1803.photos.197233p; hhh.md0961.photos.082067p; hhh.nj0902.photos.109859p; hhh.pa2262.photos.356538p.

hybrid discretization combining different element types are required.

Design standards for steel structures address the admissibility of element idealization using different normative strategies. While European standards define this admissibility through conditional criteria based on the structural behavior of the system, North American standards adopt prescribed analysis procedures to ensure an adequate representation of the governing response mechanisms.

4.3. Second-order effects: EN 1993-1-1 and AISC

In EN 1993-1-1, the distinction between first-order and second-order analysis is based on whether equilibrium is formulated with respect to the undeformed or the deformed geometry of the structure. First-order analysis is permitted only if the effects of deformed geometry are sufficiently small, which is assessed using the elastic critical factor

$$\alpha_{cr} = F_{cr}/F_{Ed} \quad (1)$$

where F_{cr} is the elastic critical load associated with a global instability mode and F_{Ed} denotes the design load applied in the analysis model. Physically, α_{cr} expresses the proximity of the structure to a global instability state: large values indicate low sensitivity to second-order effects, whereas low values indicate high sensitivity to geometric nonlinearity.

In truss structures dominated by axial forces, global second-order effects associated with sway (P-Δ effects) and local member curvature (P-δ effects) may significantly influence

internal force distribution and stability. When such effects are relevant, second-order analysis based on the deformed configuration is required. AISC 360-22 does not introduce a stability parameter analogous to α_{cr} . Instead, Chapter C defines general stability requirements and presents the Direct Analysis Method. For the elastic implementation of the Direct Analysis Method, Section C2 requires second-order analysis considering P-Δ and P-δ effects, together with initial system imperfections and stiffness adjustments. Alternative stability design methods are provided in Appendix 7 and may be applied subject to their specified limitations.

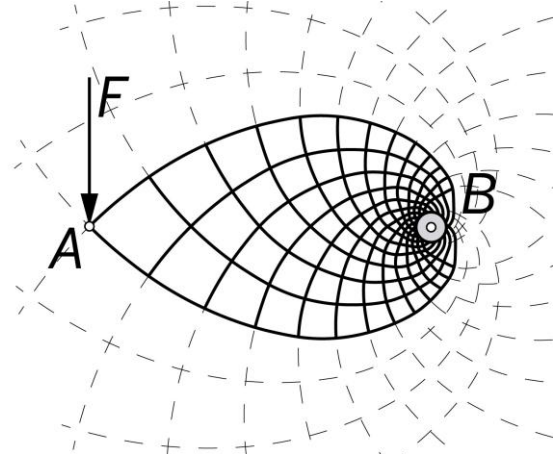


Fig. 4. Michell's cantilever structure (Arek Mazurek, public domain, Wikimedia Commons).

TABLE 2. Key contributors to the historical development of truss theory and structural analysis

Author	Period	Contribution	Significance	References
J.C. Maxwell	1864	Reciprocal figures and force diagrams	Introduced a geometric framework for determining internal forces in pin-jointed trusses.	[1]
L. Cremona	1890	Formalization of graphical statics	Provided systematic graphical procedures for truss force analysis.	[2]
M. Lévy	1898	Extension of graphical methods	Extended graphical statics to more complex structural systems.	[23]
O. Mohr	1870–1900	Stress theory and energy-based concepts	Contributed analytical tools for stress representation and structural analysis.	[24]
A.G.M. Michell	1904	Minimum-weight structural design	Established theoretical principles for minimum-material truss structures.	[25]
F. Engesser	1889–1910	Stability and second-order effects	Introduced early formulations addressing buckling and geometric nonlinearity.	[26]
S.P. Timoshenko	1920–1950	Synthesis of structural mechanics theories	Systematized elasticity, stability, and structural analysis methods.	[27, 28]
R. Courant	1943	Variational principles	Provided mathematical foundations for variational and discretization methods.	[29]
J. Argyris	1950–1970	Matrix-based structural analysis	Developed matrix formulations for numerical analysis of structures.	[30]
M.J. Turner	1956	Stiffness-based finite element formulation	Proposed an element-based stiffness approach for structural analysis.	[31]
O.C. Zienkiewicz	1960–2000	Development of finite element methodology	Systematized and disseminated finite element formulations for engineering applications.	[32]

Thus, EN 1993-1-1 uses the elastic critical factor to assess the significance of second-order effects, whereas AISC 360-22 addresses these effects through prescribed stability analysis procedures and their applicability requirements.

4.4. Modeling of imperfections

In EN 1993-1-1, the modeling of geometric imperfections is a fundamental requirement for global stability verification and directly conditions the admissibility of the global analysis method. Appropriate allowances for the effects of residual stresses and geometric imperfections should be incorporated in the structural analysis, unless the relevant effects are already included in the resistance formulae for member design.

EN 1993-1-1 distinguishes between global imperfections and local bow imperfections of members. Global imperfections may be represented either through an imperfect initial geometry or through equivalent horizontal forces. Local member imperfections are generally incorporated in the member buckling-resistance formula. Where member stability is fully accounted for by second-order global analysis, the relevant member imperfections are included directly in the analysis.

AISC 360-22 adopts a more prescriptive approach to geometric imperfections in stability analysis. The effects of initial system imperfections in the positions of points of intersection of members are taken into account either by direct modeling or by the application of notional loads. The treatment of member imperfections depends on the adopted analysis procedure and the extent to which their effects are represented directly in the structural analysis.

4.5. Material nonlinearity and advanced analysis in truss modeling

In EN 1993-1-1, material nonlinearity may be considered within global analysis. Internal forces and moments may be determined using either elastic or plastic global analysis. Elastic global analysis assumes linear stress-strain behavior, whereas plastic global analysis allows material nonlinearity and the redistribution of internal forces to be represented explicitly. The application of plastic analysis is subject to requirements concerning deformation capacity and member stability.

AISC 360-22 permits the Direct Analysis Method to be based on either elastic or inelastic analysis. The stability assessment accounts for stiffness reductions associated with inelasticity, including partial yielding and the effects of residual stresses. Appendix 1 provides advanced analysis procedures in which geometric imperfections, second-order effects, and inelastic behavior may be represented directly in the structural analysis.

4.6. Combined imperfections and interaction effects in truss analysis

EN 1993-1-1 permits second-order effects and imperfections to be accounted for either entirely in the global analysis or partly through global analysis and partly through individual member stability checks. Where the relevant member imperfections and second-order effects are fully represented in the global analysis, separate member stability checks are not required for those effects. Otherwise, the effects not represented in the global

analysis shall be verified through appropriate member stability checks.

Under the AISC Direct Analysis Method, second-order effects, initial system imperfections, and stiffness reductions are considered jointly within the stability analysis. Appendix 1 extends this approach by permitting direct modeling of system and member imperfections and, where applicable, explicit representation of inelastic effects. Alternative stability design methods are provided in Appendix 7.

4.7. Normative synthesis and implications for truss analysis

From the perspective of truss modeling, both EN 1993-1-1 and AISC 360-22 lead to the same fundamental conclusion. A purely first-order analysis of a geometrically perfect system is not admissible when global instability, sway, or displacement-force interaction governs the structural response. In both normative frameworks, the use of simplified modeling assumptions must be justified rather than assumed. For truss structures whose response is sensitive to second-order effects, instability, or interactions between local and global behavior, these effects and the relevant imperfections shall be accounted for using an analysis procedure permitted by the applicable standard. Material nonlinearity shall also be considered where required by the selected analysis method.

The principal modeling approaches discussed in Sections 4.1-4.6 are summarized in Table 3 in terms of the represented effects, conditions of applicability, and their role in truss analysis.

5. EXPERIMENTAL INVESTIGATIONS

Experimental research on truss girders has been conducted for more than a century, primarily to investigate member instability, global buckling behavior, load redistribution, and sensitivity to geometric imperfections. Early systematic experimental studies were closely linked to the development of classical stability theory and to the formulation of design rules for axially loaded structural members [33].

As experimental research developed, attention shifted from isolated members to structural systems. This shift was especially relevant to truss girders, whose global response is governed by the interactions among individual members, joints, and boundary conditions. Due to their mainly axial-force-driven behavior, truss systems exhibit pronounced sensitivity to geometric imperfections, support displacements, and second-order effects, which may influence stiffness, internal force distribution, and load-carrying capacity. Experimental investigations play a crucial role in capturing these interactions, as they provide direct evidence of the combined influence of member behavior, connection response, and global system effects. Unlike idealized analytical or numerical models, experiments allow observation of the progressive development of deformation and failure processes, offering insight into the governing mechanisms at different structural levels.

For these reasons, the studies reviewed in this section are discussed in relation to the main mechanisms observed experimentally, including instability, joint and connection

TABLE 3. Comparison of modeling approaches for truss analysis

Modeling level / approach	Effects represented	Conditions / limitations	Role in truss analysis
Axial force model	Axial forces, bending and torsion neglected	Applicable when joint stiffness, member bending, and spatial effects do not significantly affect the global response	Conceptual assessment and preliminary global analysis
Beam / hybrid discretization	Member bending, joint semi-rigidity, and spatial interaction	Required when axial-force elements do not adequately represent displacements or internal-force redistribution	Representation of displacements and internal-force redistribution
First-order analysis	Equilibrium based on the undeformed geometry	Applicable when the effects of deformed geometry are sufficiently small	Global analysis when second-order effects may be neglected
Second-order analysis with imperfections	P- Δ and P- δ effects, geometric imperfections	Requires appropriate representation of relevant global and/or member imperfections	Stability assessment of systems sensitive to second-order effects
Elastic / inelastic analysis	Linear elastic response or material nonlinearity and internal-force redistribution	Inelastic analysis requires adequate deformation capacity and stability	Representation of elastic or inelastic material behavior in global analysis

behavior, load redistribution, and sensitivity to geometric imperfections.

5.1. Observed failure mechanisms in truss girders

Experimental investigations and detailed analyses of structural performance reported in the literature indicate that failure of truss girders rarely develops as a single, isolated mechanism. Instead, observed collapse or limit-state behavior typically emerges from the interaction between member instability, joint and connection behavior, and global system response. Across reported experimental programs, the governing failure mode has been shown to depend primarily on: (i) the slenderness and bracing conditions of compression members, (ii) the deformation capacity and detailing of joints and connections, and (iii) the presence of geometric imperfections and support displacements.

A dominant class of observed failure mechanisms is the buckling of compression members, frequently initiating in web members or compression chords. In a detailed study reported in Engineering Failure Analysis [11], compression-member buckling in a space-truss roof system was shown to be associated with differential support settlements resulting from consolidation of the foundation soil. The observed distribution of buckled truss bars showed reasonably good agreement with the nonlinear analysis accounting for the measured support settlements. Experimental investigations of roof bracing systems [34,35] demonstrate that initial imperfections of compression chords generate loads in bracing members and affect their distribution. This observation highlights the need to account for imperfection-induced secondary forces in both experimental interpretation and numerical modeling of truss systems. These findings indicate that both deviations in support conditions and geometric imperfections can influence the stability response of truss systems.

A second major class of failure modes involves out-of-plane instability and lateral-torsional buckling of chord systems. Experimental results reported in [36] showed that the critical load depends on the stiffness and spacing of lateral bracing and indicated that an out-of-plane buckling length smaller than the conventional value of $0.9L$ may be justified for both rigid and flexible bracing. Full-scale tests of cold-formed steel roof trusses provide complementary system-level evidence. In the tested systems, steel sheathing attached to the top chords provided lateral restraint, while observed failure modes included top-chord buckling, web-member buckling and connection failures [17]. These observations indicate that the stability response of roof trusses is influenced by the restraint conditions provided by the surrounding structural system.

A third class of failure mechanisms is governed by the behavior of joints and connections. Full-scale tests of a cold-formed steel roof truss indicated that failure began with local buckling and joint failure before buckling of the compression member occurred [5]. The study also showed that connection eccentricity generated additional torsional effects that were not represented by a purely axial-force idealization. Experimental tests of gusseted bracing connections showed that the connection configuration influenced both force transfer and the governing failure mode. Gusset-plate buckling occurred in specimens with strong-axis column connections, whereas tearing occurred in the weak-axis configuration [37]. Experimental tests of angle specimens subjected to tension and connected by one leg to gusset plates showed that the eccentricity of the bolt group introduces an additional bending moment. The study also demonstrated that specimen length affects the behavior of the angles, with a greater effect observed for larger angle sections, which were associated with greater load eccentricity. The observed failure modes included net cross-section rupture, bolt shear with ovalisation of bolt holes due to bearing, and combined bolt shear and edge tearing [9]. For tubular steel truss structures, large-scale fatigue tests of

welded circular hollow-section joints showed the importance of local stress concentrations and bending effects in the connected members, with fatigue cracks developing in the weld region [38].

5.2. Deformation patterns and load redistribution

Experimental studies on truss girders show that the structural response preceding failure may involve progressive deformation and redistribution of internal forces, although the character of failure depends on the governing mechanism and the configuration of the structure. The experimental evidence discussed below shows that deformation may develop locally in individual members, while load redistribution may allow the structural system to continue carrying load after the first component failure [17,39].

In the full-scale roof truss test reported in [5], the net deflection of the mid-joint was approximately proportional to the applied joint load. At higher load levels, deformation developed in selected top-chord members. As the load increased further, the deformation of one of these members increased rapidly and failure subsequently occurred in this member. The test also showed torsional deformation of adjacent members caused by fabrication eccentricity at one of the joints.

The full-scale test reported in [17] included both single cold-formed steel roof trusses and truss systems connected by corrugated steel decking. In the single-truss tests, the first failures occurred in the top chords due to local buckling, followed by failure of web members before the trusses reached their ultimate capacity. The conclusions of the study indicate that the single truss tests exhibited approximately three failure sequences. In the system tests with corrugated b-decking, at least two failure sequences were observed, with some tests reaching three.

Full-scale ultimate-load tests of an old steel truss bridge reported in [6] presented two distinctly different failure responses. In the first test, the bridge failed by sudden rupture of a forged diagonal tension member, the failure was brittle and occurred without warning. After the failed member and other forged tension members were retrofitted, the second test resulted in a more ductile response, with failure developing gradually and with ample warning. In the second test, failure was governed by out-of-plane buckling of a top-chord compression member.

Field testing of a high-truss bridge reported in [39] provided further evidence of load redistribution after local failure of a member. The experiment showed that the initially near-linear load-deflection response changed as member yielding developed. When one hanger subsequently failed, part of the applied load was transferred to the other loading point. The load was then reapplied and the truss continued to carry additional load despite the substantial deformation.

Taken together, these studies show that the response preceding failure may involve local deformation, successive component failures and redistribution of load, but the observed sequence depends on the governing failure mechanism and structure configuration. The experimental evidence also shows that local

member failure does not necessarily result in immediate loss of load-carrying capacity of the entire truss system.

5.3. Experimental evidence of imperfection sensitivity

Experimental investigations reported in the literature indicate that the structural response of steel truss girders is sensitive to initial geometric imperfections and to deviations in restraint and boundary conditions. Given the predominance of compression-dominated, axially loaded members, relatively small departures from ideal geometry or restraint assumptions may affect the development of out-of-plane response and the correspondence between experimental behavior and simplified analytical representations.

Chan and Cho investigated single-angle web members in laboratory truss tests and compared the results with a second-order $P - \Delta - \delta$ analysis incorporating initial member curvature and semi-rigid connection behavior [10]. In both tests, failure occurred by flexural buckling about the principal minor axis, with out-of-plane deformations exceeding the in-plane deformations. The tests also showed that the connection condition affected the buckling resistance, with the double-bolted specimen reaching a higher failure load than the single-bolted specimen.

Sensitivity associated with lateral and torsional restraint conditions is documented in laboratory testing of braced steel truss systems. Wongjeeraphat and Helwig report an experimental and numerical investigation of the bracing behavior of steel trusses, including laboratory tests on a twin-truss system with lateral and torsional bracing [40]. The results show that truss buckling behavior was affected by brace stiffness and cross-sectional distortion. The authors also showed that idealizing the web-member connections as pinned provides a conservative estimate of buckling capacity compared with models accounting for the actual connection stiffness.

Imperfection sensitivity with direct relevance to roof-truss practice is further documented through experimental evaluation of bracing actions induced by imperfect truss chords. Piątkowski reports experimental research on steel truss models that addresses the magnitude and distribution of loads in transverse roof bracing generated by a planar steel truss girder with geometric imperfections in the compressed chord. The experimental results showed that the bracing load was non-uniform and sign-changing, differing from the uniformly distributed notional load adopted in the standard model [34].

At the connection-component level, Dowsell reviewed available experimental and finite element data on gusset-plate buckling [41]. In discussing sources of inaccuracy in the design model, the study refers to measured out-of-flatness of fabricated gusset plates and cites previous finite element results showing that the magnitude of initial out-of-flatness significantly affects gusset-plate compressive capacity. These findings show that the representation of initial out-of-flatness can affect the accuracy of gusset-plate buckling assessment.

The reviewed studies show that imperfection sensitivity in steel truss girders occurs at several structural levels. At the member level, initial geometric imperfections and connection restraint

affect buckling behavior [10]. At the truss level, lateral and torsional bracing conditions influence the buckling response [40], while geometric imperfections of the compressed chord generate non-uniform and sign-changing forces in transverse roof bracing [34]. At the connection level, initial out-of-flatness is identified as a relevant source of inaccuracy in the assessment of gusset plates in compression [41]. Taken together, these findings indicate that geometric imperfections and restraint conditions should be represented explicitly and consistently with the structural configuration when stability-sensitive truss behavior is modeled.

5.4. Full-scale tests and field evidence from existing truss structures

Full-scale testing and field investigations of existing truss structures provide experimental evidence at the system level under realistic boundary, restraint, and operational conditions. In contrast to laboratory studies on isolated members or idealized subassemblies, these programs document the global response of complete truss systems and how local distress accumulates and propagates when load paths, restraints, and support conditions reflect those of real structures.

Classical field programs on highway truss bridges combined service-level testing with ultimate-load assessment of complete spans and selected subcomponents. Klaiber et al. showed close agreement between analytical and experimental member forces, supporting the pin-ended member assumption for the tested truss [39]. The predicted capacity of the failed hanger also agreed closely with the load causing fracture. However, the bridge rating represented only about one-fifth of the capacity reached in the full-scale test, indicating substantial conservatism at the system-assessment level.

Further full-scale evidence was provided by Blanksvärd et al., who tested an existing steel truss railway bridge to failure [42]. The bridge remained elastic up to about three times the original design load and subsequently developed substantial yielding before buckling of the top girders occurred. The test therefore demonstrated considerable reserve capacity and a progressive, ductile system response prior to instability.

Field testing has also been used directly to validate analytical models for the assessment of existing truss bridges. Spyrakos et al. combined static and dynamic field measurements with laboratory tests to develop a validated three-dimensional model of a historic steel truss railway bridge [43]. The validated model was subsequently used to assess the bridge under heavier train loads as well as wind and seismic actions, and to identify the members requiring strengthening.

Complementary evidence under service loading was provided by Bačinskas et al., who performed full-scale static and dynamic tests on an old riveted railway truss bridge using operating locomotives [44]. The static tests showed that the bridge remained essentially elastic under service loading, while the overall assessment indicated sufficient reserve capacity for continued operation. The experimentally determined dynamic factors agreed closely with theoretical values, while the asymmetric response observed in one span indicated torsional behavior of the deck.

A further field study by Kilikevičius et al. focused specifically on the dynamic behavior of an existing continuous steel truss bridge after a long period in service [45]. The experimental program included resonance, forced-vibration, and free-vibration tests, from which mode shapes, vibration frequencies, damping ratios, accelerations, and dynamic displacements were obtained. These results show that dynamic field testing can complement static load tests by providing measured modal parameters for the condition assessment of existing truss bridges.

Field measurements have also been used to evaluate alternative structural idealizations of steel truss bridges. Hickey et al. compared live-load measurements with three analytical representations: a simple truss model, a frame model containing the truss members, and a frame model additionally including the stringers [46]. The model including the stringers provided the closest agreement with the measured response, leading the authors to recommend that deck truss bridges be modeled as frame systems with the stringers included.

Material characterization provides another important input for the assessment of existing truss bridges. Ambroziak and Malinowski combined laboratory testing of steel extracted from an old riveted railway bridge with three-dimensional finite-element analysis [47]. The study showed that the current mechanical properties of aged steel may differ substantially from their original values. The load-bearing capacity and serviceability of the investigated bridge were governed by the bridge deck structure. The authors therefore emphasized that the assessment of old steel bridges should be based on the current measured material properties.

Long-term structural health monitoring provides complementary evidence under operational conditions. Anastasopoulos and Reynders monitored a historical Vierendeel truss bridge for more than a year using 64 fiber-Bragg grating sensors. The study compared the influence of changing support conditions, environmental and operational variability, and temperature-related material effects on the identified modal characteristics [20].

These full-scale and field-based programs show the value of experimentally grounded system-level evidence for assessing existing truss structures: controlled load tests (including ultimate-load programs), test-to-failure observations where available, and monitoring-based field evidence capturing environmental and boundary-condition variability. Such sources provide the most direct basis for relating modeling assumptions and assessment procedures to observed behavior of real truss systems.

5.5. Implications of experimental findings for modeling strategies

The experimental investigations reviewed in this chapter provide a consistent body of evidence demonstrating that the behavior of steel truss girders is governed by interactions occurring at multiple structural levels. Member instability, restraint effectiveness, connection behavior, and realistic boundary conditions have all been shown experimentally to influence structural response and failure behavior. As a result,

modeling strategies intended to represent truss behavior should be formulated with explicit reference to the experimental configuration and the level of structural idealization adopted.

At the member and truss-modeling level, the reviewed studies emphasize the importance of representing initial geometric imperfections and second-order effects in a manner consistent with the tested geometry. Simplified representations based on idealized straight members or perfect restraints may reproduce global stiffness at low load levels but are insufficient to capture the experimentally observed sensitivity of instability-dominated response. Where experimental data indicate pronounced sensitivity to imperfections, modeling approaches should incorporate imperfection forms and amplitudes consistent with fabrication tolerances, measured geometry, or test assumptions.

Experimental evidence also highlights the influence of restraint and boundary conditions on stability behavior. Laboratory and full-scale investigations show that the effectiveness of lateral and torsional bracing, support flexibility, and interactions with adjacent structural components may significantly affect the buckling response. Consequently, modeling strategies should avoid implicit assumptions of idealized restraints and instead reflect the actual restraint conditions present in the tested or assessed structure, particularly when interpreting experimental results or calibrating numerical models.

At the connection and component levels, experimental observations show that connection configuration, eccentricity, and local instability can influence force transfer and governing failure modes [5, 9, 37]. Accordingly, connection representations used in structural models should be compatible with the experimentally observed behavior and failure mechanisms. In this context, modeling simplifications should be traceable to experimental observations and justified with respect to their influence on structural response.

Finally, full-scale tests and field investigations of existing truss structures underline the value of system-level experimental evidence for validating modeling assumptions. Measured global response, deformation patterns, and monitoring data obtained under realistic conditions provide a direct reference for assessing the applicability and limitations of analytical and numerical models. The experimental findings reviewed in this chapter motivate modeling strategies that are explicitly linked to observed behavior, acknowledge the role of imperfections and restraints, and are calibrated against system-level evidence wherever available.

6. NUMERICAL ANALYSIS OF TRUSS SYSTEMS

Analytical and numerical modeling are fundamental tools for analysing and assessing steel truss girders. Such models are routinely employed to represent load distribution, stability behavior, and structural response under various loading conditions. The formulation of these models involves assumptions concerning structural geometry, boundary conditions, restraint representation, and the idealization of members and connections. The choice and treatment of these assumptions influence the correspondence between the calculated response and the observed structural behavior. In

particular, modeling approaches differ in the extent to which instability phenomena, geometric imperfections, restraint conditions, and interaction effects are represented within the analytical framework.

6.1. Levels of structural idealization

Experimental and applied numerical studies demonstrate that steel truss girders can be modeled with varying degrees of structural idealization, depending on the purpose of the analysis and the behavioral mechanisms of interest. These approaches range from highly simplified bar or beam models to more detailed representations employing shell elements or mixed formulations. The choice of model type has been shown in the literature to influence predicted stiffness, internal force distribution, stability behavior, and sensitivity to imperfections. A direct comparison between different finite element idealizations applied to the same truss configuration is reported by Johansson and Löfberg [48]. The pitched steel truss was modeled using beam and shell elements, with joint eccentricities included in both models. The two models gave similar ultimate loads and first buckling modes, but differed for higher modes. The shell model provided a clearer representation of local twisting, flexural-torsional buckling, connections, and load application. The study also showed that connection stiffness and the assumed buckling length affect calculated capacity.

Krajewski investigates a steel roof truss with elastic lateral restraint and considers both translational and rotational stiffness of the restraint system. The study reports a linear buckling analysis performed for two finite-element idealizations of the same truss - a beam model and a shell model and presents nonlinear static analyses including initial geometric imperfections [49]. The two idealizations differed in representation of connections and structural eccentricities and represented welded connections using rigid elements, whereas the members in the beam model were joined axially at the nodes. The beam and shell models presented different buckling-load predictions, which are attributed to differences in the modeling method. The reported results relate the buckling response and the limit load to the adopted lateral restraint stiffness and show that increasing translational or rotational brace stiffness increases the buckling load, although above a threshold stiffness further increases have only a limited effect. The influence of modeling idealization is illustrated by studies that combine numerical modeling with field measurements of existing steel truss bridges. Shibeshi and Roth report a dynamic investigation of a steel truss railway bridge using field measurements and numerical models of varying complexity, including a generalized single-degree-of-freedom representation and three-dimensional finite element models developed with beam and shell elements [50]. The finite element models were based on physical measurements of the bridge, with riveted connections and abutments assumed to be rigid. The predicted natural frequencies obtained from the simplified and finite element models were compared with values identified from field measurements, showing reasonable agreement across the different modeling approaches. In

addition, the simplified moving-load model was shown to reproduce measured mid-span displacement response with acceptable accuracy. The authors noted that differences in natural frequency could be related to the assumed support stiffness. The study provides a documented example of how different levels of model idealization may be evaluated against field data for the same steel truss structure. Studies on reticulated and truss-type structures employing shell-based finite element formulations illustrate the influence of element formulation on displacement and stress predictions. Selhorst et al. investigate the applicability of shell elements in structures commonly modeled using bar or beam elements by comparing shell- and solid-based finite element models of reticulated systems, including a roof truss configuration [51]. The results show that shell models provide stress and displacement responses in close agreement with solid-element solutions and analytical results. The finite element analyses took into account local stress concentrations at the joints, which were not considered in the analytical solution. The authors noted a greater computational cost for the solid-element model than for the shell-element model.

The practical implications of modeling idealization are also addressed in the AASHTO/NSBA G13.2–2024 Guidelines for Steel Truss Bridge Analysis [7]. The document provides recommendations on the selection and implementation of analysis methods for steel truss bridges, including the development of two-dimensional and three-dimensional analytical models [7]. The document addresses modeling of members, connections, lateral and portal bracing systems, deck–truss interaction, and boundary conditions within different analytical frameworks. In addition, the guidelines include a series of case-based examples illustrating the formulation of 2D and 3D models, covering analysis method selection, model geometry, element idealization, boundary conditions, and load representation.

Taken together, these studies indicate that no single level of idealization is universally applicable. Simplified models can provide results comparable to those of more detailed models for selected global response variables, whereas shell, solid, or three-dimensional models provide additional capabilities for representing local instability, connection effects, out-of-plane behavior, or local stress concentrations. Increasing the level of model detail may also increase modeling complexity and computational effort. Therefore, an appropriate level of idealization should be selected depending on the structural response under investigation and the required level of detail.

6.2. Representation of geometric imperfections

Numerical studies on steel truss girders commonly treat geometric imperfections as an explicit modeling assumption in analyses addressing stability and second-order effects. In truss systems composed predominantly of axially loaded members, deviations from ideal geometry are introduced in numerical models to account for imperfection-related changes in structural response and to enable the assessment of stability behavior within second-order analytical formulations [10,52].

One modeling approach represents imperfections by prescribing initial geometric deviations in beam-based finite element models. Chan and Cho incorporate initial imperfections together with semi-rigid joint behavior in the second-order analysis of steel angle trusses [10]. In their formulation, $P-\Delta$ and $P-\delta$ effects are considered by geometry and curvature updates, while imperfections are represented by imperfect frame geometry and initial member curvature. Connection rotational stiffness is introduced using rotational spring elements at member ends. The method was verified against laboratory tests and avoids the need to assume an effective member length.

A different approach is presented by Gremza and Zamorowski, who compare direct geometric representation of global and local imperfections with several equivalent load formulations for a spatial roof truss system [52]. The reference nonlinear model included global chord imperfections and local bow imperfections of compressed members directly in the structural geometry, while alternative models replaced global imperfections by equivalent horizontal loads. The study showed that the adopted equivalent load representation had little influence on the upper chord, braces, and posts, but it was important for the bottom chord. The authors showed that the spatial response of the structure generates horizontal actions in the bottom chord due to initial truss twisting and that the standard equivalent-load formulation may underestimate the corresponding stresses. They identified the load system including forces applied to both the upper and lower chords as a convenient and correct approach for assessing bottom-chord forces and stresses.

Other numerical investigations focus on the interaction between geometric imperfections and lateral or torsional restraint conditions in steel truss systems. Wongjeeraphat and Helwig report laboratory tests and parametric three-dimensional finite element analyses of steel trusses with different bracing configurations [40]. The numerical models used line elements and considered both moment-transferring and simplified pinned web-to-chord connections. The results showed that cross-sectional distortion and connection stiffness influence the buckling response, while the simplified pinned connection model provides a conservative estimate of buckling capacity. In the associated doctoral research, the author further documents the sensitivity of buckling and bracing behavior to the magnitude and shape of initial geometric imperfections [53]. Measured initial imperfections were introduced into the finite element model and iteratively adjusted until good agreement with the measurements was obtained.

The influence of truss geometry on numerical results is examined explicitly by Piątkowski, who analyses planar steel roof truss girders with a sinusoidal geometric imperfection of the compressed chord [54]. The study employs geometrically and materially nonlinear analyses using beam finite elements, with the imperfection introduced directly by modifying the coordinates of the upper-chord nodes. The results indicate that the influence of truss geometry on the nodal bracing-load distribution is generally limited, although some configurations may modify the distribution and produce sign changes. None of

the numerical load distributions reproduced the uniform distribution assumed in EN 1993-1-1.

Building on numerical evidence showing that geometric imperfections influence force redistribution in transverse roof bracing, further studies address how imperfection modeling interacts with the level of structural idealization adopted in numerical analysis. In this context, Krajewski investigates steel roof trusses with elastic lateral restraints using both beam-based and shell-based finite element models [49]. Linear buckling analyses were performed for both model idealizations, while nonlinear static analyses of the shell model included two forms of initial geometric imperfections: arch curvature of the top chord and an imperfection corresponding to the buckling mode. The reported results demonstrate that the predicted buckling loads depend on the adopted element idealization and brace stiffness. The shape of the initial geometric imperfection also significantly affects the out-of-plane displacement and limit load.

Related work by Krajewski and Iwicki further examines the influence of elastic lateral supports on the stability of roof truss systems [55]. Their numerical and experimental study employs both beam and shell finite element models and reports linear buckling and nonlinear analyses performed for imperfect truss configurations. Two forms of initial geometric imperfections were considered: the first buckling mode and a parabolic out-of-plane imperfection of the top chord. The results explicitly relate the buckling response and limit load to the stiffness of the lateral bracing and show that the assumed shape of the initial geometric imperfection affects the limit load and brace forces. Selected numerical analyses were verified experimentally.

Modeling simplifications related to imperfection representation are also discussed in studies on the practical design of roof-truss bracing systems. Korcz-Konkol and Iwicki analyse bar and diaphragm roof-bracing systems, incorporating imperfection-origin forces and comparing analytical and numerical ways of accounting for truss-girder imperfections [56]. Their geometrically nonlinear three-dimensional roof model employed shell and beam elements and considered four exemplary shapes of initial truss-girder imperfections. The results show that the assumed imperfection shape and the method used to account for imperfections affect the stabilizing forces in purlins and the forces in bar bracing. In subsequent work, the authors examine successive simplifications of the numerical model, including replacement of the three-dimensional shell model of the sheeting by an equivalent orthotropic shell, omission of purlin-to-sheeting connection flexibility, and omission of purlin-sheeting eccentricity [57]. Omitting the purlin-sheeting eccentricity had a greater influence than omitting connection flexibility.

At the level of connection components, geometric imperfections are considered in numerical studies of gusset-plate stability. Dowswell discusses gusset-plate buckling within the effective-length framework and proposes effective-length factors for typical gusset-plate configurations, addressing sources of inaccuracy in simplified stability assessments and the relevance of out-of-plane geometric deviations in gusset-plate behavior [41]. The study identifies initial out-of-flatness,

together with mesh refinement, material model, and boundary conditions, as factors affecting the accuracy of finite element predictions.

The reviewed numerical studies incorporate geometric imperfections into steel truss analysis as explicit modeling inputs, using strategies such as prescribed or measured initial geometry, buckling-mode and code-based imperfection shapes, equivalent stabilizing forces, and component-level stability idealizations for connection plates. Across these publications, the representation of imperfections is reported to influence the predicted stability response, including buckling and limit loads, bracing-force distributions, and connection-component capacity. The studies also show that these effects interact with the adopted level of structural idealization: simplified beam or equivalent-load models can reduce model complexity, while more detailed shell and three-dimensional models provide additional capabilities for representing local, out-of-plane, and connection-related effects. Therefore, imperfection representation and structural idealization should be considered together when selecting a numerical model for stability-sensitive truss analysis.

The principal numerical modeling approaches discussed in Sections 6.1 and 6.2 are summarized in Table 4 in terms of model idealization, imperfection representation, main structural effects, and computational implications or limitations.

7. STOCHASTIC AND RELIABILITY-BASED MODELS

Experimental observations reviewed in Section 5 identify structural mechanisms affecting the response of truss systems, while the numerical approaches presented in Section 6 provide deterministic models for representing these effects. In practice, loads, material properties, geometric imperfections, and boundary conditions are subject to uncertainty. The stochastic and reliability-based approaches discussed in this chapter extend deterministic modeling by accounting for these uncertainties and provide a basis for updating the structural assessment using data obtained from structural monitoring.

7.1. Methodological scope of stochastic and reliability analysis

Stochastic and reliability-based analyses of truss structures include a broad spectrum of modeling strategies that differ in their treatment of uncertainty, level of structural idealization, and computational complexity. Beyond classical stochastic finite element formulations and reliability methods discussed in subsequent sections, numerous alternative and specialized approaches have been proposed to address specific modeling challenges associated with truss systems.

Within this framework, the discussed approaches address different stages of probabilistic structural assessment. Stochastic modeling and stochastic finite element methods (SFEM) are primarily used for uncertainty propagation and quantitative determination of structural response variability. Reliability methods extend this analysis by evaluating failure probabilities and reliability indices with respect to predefined limit states. Reliability-based optimization incorporates these probabilistic safety measures into the design process, whereas surrogate models are usually introduced to reduce the

TABLE 4. Comparative summary of numerical modeling approaches for steel truss systems

Source	Model idealization / element formulation	Imperfections / nonlinear features	Main structural effect	Computational implications / limitations
Johansson and Löfberg [48]	Beam / shell FE	Eccentricities; initial imperfection; second-order effects	Similar ultimate load / first mode; shell better captures local twisting and connections	Convergence problems with combined imperfections, eccentricities, and plasticity
Krajewski [49]	Beam / shell FE	Arch-curvature / buckling-mode imperfection; nonlinear shell analysis	Buckling and limit load depend on brace stiffness and imperfection shape	Shell model uses more elements; connection details simplified
Shibeshi and Roth [50]	Generalized single degree of freedom (SDOF); 3D beam / shell FE	No geometric imperfection; moving-load model without damping	Reasonable frequencies; SDOF reproduces displacement; support stiffness matters	Dynamic FE time history is more complex and time-consuming
Selhorst et al. [51]	Shell / solid FE	No geometric imperfection or nonlinear analysis reported	Similar displacements; FE captures joint stress concentrations	Solid FE has higher computational cost
Chan and Cho [10]	Nonlinear element model	Initial geometry / curvature; $P-\Delta/P-\delta$; semi-rigid joints	Imperfections and joint stiffness affect second-order response	No effective-length assumption
Gremza and Zamorowski [52]	3D spatial model	Direct imperfections or equivalent loads; second order analysis	Equivalent-load choice important for bottom-chord response	Direct imperfection modeling is tedious; equivalent loads are practical
Wongjeeraphat and Helwig [40]	3D line-element FE; BEAM44 / LINK8	Not reported	Cross-sectional distortion and connection stiffness affect buckling; pinned joints are conservative	Line elements omit local gusset / chord distortion
Wongjeeraphat [53]	3D line-element FE; BEAM44	Measured initial imperfection	Buckling / bracing response depends on imperfection magnitude and shape	Measured geometry requires iterative adjustment
Piątkowski [54]	Beam FE	Sinusoidal upper-chord imperfection; geometric / material nonlinearity	Generally limited geometry effect; some configurations change load distribution / sign; none matches EN 1993-1-1 uniform load	Rigid bracing supports and web-to-chord connections assumed
Krajewski and Iwicki [55]	Beam / shell FE	Buckling-mode / parabolic imperfection; geometric / material nonlinearity	Limit load and brace forces depend on brace stiffness and imperfection shape	Not reported
Korcz-Konkol and Iwicki [56]	3D beam / shell FE	Four imperfection shapes; geometric nonlinearity	Imperfection shape / method affect purlin and bar-bracing forces	Beam idealization reduces complexity but omits local effects
Korcz-Konkol and Iwicki [57]	3D beam / shell FE; orthotropic-shell variants	Imperfection-origin stabilizing forces	Model simplifications affect bar-bracing forces	Eccentricity more influential than connection flexibility; local effects omitted
Dowswell [41]	Experimental / FE database; effective-column model	Initial gusset-plate out-of-flatness	Out-of-flatness affects compressive capacity; effective-length factors calibrated	FE accuracy depends on mesh, out-of-flatness, material model, and boundary conditions

computational effort associated with repeated reliability assessments. Methods based on imprecise probability provide an alternative when complete probability distributions for uncertain parameters are not available. Updates based on structural health monitoring (SHM) use measured structural data to refine model parameters and the reliability estimates during service.

Alternative numerical formulations, including meshless methods and response-function-based approaches, have been investigated as substitutes or extensions to standard finite element techniques in stochastic structural analysis. These methods aim to reduce mesh dependency or improve numerical efficiency in problems involving uncertainty [58]. The improved response function stochastic meshless method avoids repeated construction and simulation of system matrices during Monte Carlo sampling and was shown to provide accurate results with reduced computational effort.

Advanced geometric representations have also been introduced into stochastic analysis frameworks. Ding et al. propose an isogeometric stochastic method that enables exact geometric modeling while incorporating uncertainty in material parameters, providing a consistent link between geometric description and probabilistic analysis [59]. Its application in the cited study is limited to material uncertainty and is not truss-specific.

A number of studies have focused on reliability analysis of specific truss typologies, highlighting the influence of structural form on probabilistic safety measures. Mochocki et al. show that the system reliability of steel trusses is affected by topology and the degree of static indeterminacy, with the system approach accounting explicitly for interactions between structural members [60]. Kamiński and Błoński analyse a Pratt steel truss using Monte Carlo simulation and stochastic perturbation methods, with the truss span treated as the uncertain input parameter [61]. Both probabilistic approaches delivered very similar results, while the perturbation-based method required significantly less computational effort in the analysed case. Brites et al. apply Monte Carlo simulation to a timber roof truss subjected to biological decay, allowing deterioration effects to be included directly in the reliability assessment [62]. Potrzyszcz-Sut et al. investigate spatial truss domes susceptible to snap-through instability using First-Order Reliability Method (FORM), classical Monte Carlo simulation, and a hybrid Monte Carlo approach supported by neural-network surrogate models [63].

Beyond these truss-specific applications, stochastic perturbation-based finite element formulations have also been extended to non-Gaussian uncertainty. Kamiński considers uniform and triangular input distributions and demonstrates the applicability of the generalized perturbation approach to uncertainty propagation in solid-mechanics problems [64].

Reliability assessment has also been extended to spatial trusses susceptible to node snap-through and to idealized benchmark systems such as the Mises truss [65,66]. Potrzyszcz-Sut applies a hybrid Monte Carlo approach, in which finite element method simulations are used to generate training and test data for a neural network, which is then used in the Monte Carlo

simulation [65]. This approach addresses the problem of high computational costs associated with classical Monte Carlo analysis by reducing the need to perform multiple nonlinear finite element calculations. The method requires the neural network to be prepared, trained, and tested separately. Radoń applies FORM to analyse a Mises truss under a random load, axial stiffness and node coordinate parameters [66]. The study shows that the reliability index is sensitive to the chosen set of random variables, with particularly strong influence from the uncertainty of the node coordinates.

Benyahi et al. combine a nonlinear mechanical formulation for spatial metallic trusses with a reliability model based on the Hasofer–Lind method to estimate the reliability index and the probability of failure [67]. Hashemolhosseini investigates component- and system-level reliability of truss structures using FORM, Monte Carlo simulation, and the Response Surface Method (RSM), with SFEM linking finite element analysis and reliability assessment [68]. The study also notes the high computational cost of Monte Carlo simulation, the approximation-dependent accuracy of RSM, and the implementation effort required by classical perturbation-based SFEM.

Nakib compares deterministic and reliability-based optimization of truss bridges [69]. In the reliability-based formulation, member strengths and applied loads are treated as random and correlated variables. Sensitivity studies show that their coefficients of variation and correlation significantly influence the optimum bridge weight.

Discrete optimization techniques constitute another extension of reliability-based design methodologies for truss structures. Stocki et al. formulate a mixed discrete–continuous Reliability-Based Optimization (RBO) problem with reliability constraints evaluated using FORM [15]. The main limitation is computational cost, since reliability constraints must be evaluated for a large number of designs. The proposed methods aim to reduce the discrete search space and the number of reliability calculations. Further probabilistic developments include the virtual distortion method proposed by Di Paola for linear elastic trusses with uncertain member parameters [70]. The method provides explicit solutions for statically determinate trusses and an asymptotic expansion for redundant trusses, whose accuracy depends on the range of parameter uncertainty. Other structural assessment approaches include the tangent-stiffness-based method of Feng et al., which identifies key elements for progressive-collapse and safety assessment of spatial truss structures [71].

Together, these approaches provide the methodological basis for the more detailed discussion of uncertainty modeling, reliability assessment, reliability-based optimization, and monitoring-based updating in the following sections.

7.2. Randomness of geometric imperfections in truss structures

Geometric imperfections in truss structures are often modeled as random variables to capture unavoidable deviations due to fabrication and construction processes. In studies on space truss structures, initial curvature imperfections of members are

modeled as random variables, and Monte Carlo simulations are employed to assess the structural response and system reliability of double-layer grid structures [72]. The results show that collapse behavior is sensitive to both random distribution of initial imperfections and the type of supports.

The von Mises planar truss is analysed to investigate the critical and post-critical behavior of a truss structure with initial random imperfections under a vertical static load. In this study, initial imperfections are introduced into the truss geometry as sinusoidal curvature of the bars and a horizontal imperfection at the top joint. Nonlinear load–deflection curves are obtained via geometrically nonlinear finite element analysis, and multiple realizations of random initial imperfections are generated using the Latin Hypercube Sampling method. The results presented show that variations in the amplitude of initial imperfections are associated with different load–deflection curve shapes and different load values obtained in the numerical analysis [73].

In studies addressing optimal truss design under geometric uncertainty, geometric imperfections are treated as stochastic inputs within the optimization framework. Jalalpour et al. incorporate geometric imperfections by modeling nodal coordinate uncertainty and accounting for their effects through an equivalent random-load formulation based on perturbation methods [18]. The authors note that computational efficiency is an advantage of the perturbation-based approach, but also point out that the method is heuristic and provides a linear approximation of nonlinear behavior.

In studies of concrete-filled steel tubular (CFST) trusses, initial imperfections are explicitly accounted for in nonlinear finite element models. Chen et al. state that initial imperfections are inherently present in CFST trusses and that neglecting these imperfections affects the prediction of ultimate strength. In their study, random initial imperfections are generated using Monte Carlo simulation with Latin Hypercube Sampling, and the structural response and reliability are evaluated using nonlinear finite element analysis and the First-Order Reliability Method (FORM) [74]. The probabilistic models of the imperfections are based on experimental and on-site measurement data.

Overall, these studies illustrate different uses of probabilistic geometric imperfection models in nonlinear stability analysis, reliability assessment, and topology optimization.

7.3. Classical stochastic modeling and stochastic finite element methods

Classical stochastic modeling constitutes the foundation of uncertainty quantification in structural mechanics. Stochastic finite element methods extend the classical deterministic finite element approach to problems involving uncertainties in mechanical, geometric, and loading properties, which may be represented using random variables and random fields [75].

Monte Carlo simulation remains the most general and conceptually straightforward stochastic technique. Its application to truss structures allows direct estimation of response statistics by repeated deterministic analyses under sampled realizations of uncertain parameters. Despite its generality, Monte Carlo simulation can be computationally

demanding because a large number of deterministic analyses may be required. This limitation has motivated the development of variance-reduction techniques [76] and hybrid Monte Carlo formulations, in which neural networks can be used to reduce the computational effort associated with repeated nonlinear finite element analyses [65].

SFEM approaches provide an alternative framework by incorporating uncertainty directly into the finite element formulation. Perturbation-based SFEM approaches approximate the stochastic response using Taylor-series expansions of random quantities, offering computational efficiency for problems with relatively small input variability [64,77]. Generalized higher-order perturbation formulations have also been applied to elastic stability problems, including the classical Euler problem, two- and three-dimensional steel frames, and a cylindrical shell [78].

Spectral SFEM formulations may substantially increase the size of the numerical problem. The computational complexity increases rapidly with the number of stochastic dimensions, leading to the curse of dimensionality. To address this limitation, reduction techniques based on global basis functions [79] and proper orthogonal decomposition combined with element-based empirical approximation have been proposed [80].

A sample-based SFEM proposed by Zheng and Dai decouples the stochastic response into a series of deterministic responses with random variable coefficients [81]. The deterministic responses can be computed using existing FEM solvers, facilitating implementation in existing structural analysis software and application to high-dimensional stochastic problems. Numerical examples demonstrate good convergence and substantially lower computational cost than large-sample Monte Carlo simulation.

Advanced stochastic formulations have also incorporated signal-processing techniques. Wavelet-based signal decomposition (DWT, Mallat pyramid) has been used to transform structural response signals (e.g., discrete nodal displacement sets) for damage localization in truss and steel girders. These approaches have been combined with probabilistic estimation methods, including the semi-analytical method (SAM), stochastic perturbation technique (SPT), and Monte Carlo simulation (MCS), together with reliability measures based on Bhattacharyya relative entropy. The authors report very good convergence between the three probabilistic methods [82].

Overall, stochastic modeling and SFEM provide methods for propagating uncertainty into structural response. Their main numerical limitations include the computational cost of repeated simulations and the increasing size of stochastic formulations as the number of uncertain parameters grows. The following section extends the discussion from structural response variability to reliability assessment based on predefined limit states.

7.4. Reliability analysis of truss structures

Reliability analysis provides a probabilistic framework for assessing the safety of truss structures by quantifying failure

probabilities and reliability indices with respect to defined limit states [83,84]. In contrast to the uncertainty-propagation methods discussed in Section 7.3, reliability analysis focuses on whether predefined structural performance criteria are violated. In truss structures, limit states are commonly defined at member or system levels. At the member level, they are commonly defined in terms of member strength, whereas system-level reliability accounts for multiple failure modes and failure sequences in redundant truss systems [85,86]. Their probabilistic formulation enables the computation of reliability indices and failure probabilities while accounting for uncertainties in loads, material strengths, member dimensions, and model uncertainties.

A distinguishing feature of reliability analysis for truss structures is the explicit consideration of system behavior. Due to load redistribution and multiple load paths, the failure of individual members does not necessarily imply system failure. As a result, system reliability models, including series, parallel, and mixed system formulations, have been widely employed to capture the interaction between component failures and global structural performance [83,85,87].

Classical reliability methods, particularly FORM, have been widely applied to truss structures for both element and system levels. FORM is based on the linear approximation of limit-state functions in the standard normal space and enables efficient estimation of failure probabilities. In the study of Kubicka and Radoń, FORM was used as the primary reliability method, while Subset Simulation was used to verify the results [84]. The two methods produced very similar reliability indices; however, the authors note that Subset Simulation is more appropriate for high reliability indices corresponding to small failure probabilities. Monte Carlo simulation has also been applied to reliability assessment of truss systems subjected to degradation, including biological decay [62].

For highly redundant or large-scale truss systems, applying system reliability methods directly may become computationally demanding due to the large number of potential failure sequences. To address this challenge, techniques such as branch-and-bound strategies and dominant failure mode identification have been proposed to reduce the computational effort [85,86], while automatic model updating has been introduced to efficiently capture load redistribution and progressive failure [85,87].

In addition to conventional mechanical limit states, reliability analysis of truss structures has also been applied to deterioration- and environment-related problems. Brites et al. assess the reliability of a timber truss subjected to biological decay [62]. Catbas et al. incorporate long-term environmental monitoring data, including temperature-induced responses, into the reliability assessment of a long-span truss bridge [88]. Cho et al. investigate fatigue failure of a collapsed steel truss bridge using reliability-based analysis [89].

7.5. Reliability-based design and optimization of truss structures

Reliability-based design and optimization (RBDO) extends classical deterministic design frameworks by explicitly

incorporating probabilistic measures of structural safety into the optimization process. In the context of truss structures, RBDO formulations aim to achieve optimal designs while satisfying prescribed target reliability levels expressed in terms of failure probabilities or reliability indices [86,90].

Applications of RBDO to truss structures include size optimization problems, in which the cross-sectional areas of members were treated as design variables subject to probabilistic constraints on stress and displacement limit states. These studies showed that reliability-based formulations can yield optimal designs that differ from those obtained using deterministic optimization when uncertainties in material properties and applied loads are explicitly considered [90,91]. RBDO formulations addressing truss structures have been applied to shape and topology optimization problems.

Torii et al. simultaneously optimize the geometry and topology of statically indeterminate trusses, with nodal coordinates and member cross-sectional areas used as design variables and applied forces and yielding stresses treated as random variables [92]. System-reliability-based sizing and shape optimization has also been developed for trusses considering progressive collapse and large numbers of potential failure sequences [86]. The influence of structural topology on reliability has also been demonstrated by Mochocki et al. [60].

Evolutionary optimization algorithms, such as genetic algorithms, have been applied in multi-objective reliability-based optimization of truss structures. Ho-Huu et al. propose a two-step framework in which the optimization and reliability analyses are performed separately, allowing multiple optimal solutions with different reliability levels to be obtained in a single run [91]. Metaheuristic algorithms have also been applied to reliability assessment of truss structures with natural-frequency constraints [93].

The computational cost of reliability assessment remains an important limitation in RBDO of truss structures [91]. Surrogate-based reliability methods provide one approach to reducing this computational burden. Chen et al. propose an adaptive Polynomial-Chaos Kriging (PC-Kriging) method for system reliability analysis of truss structures, using global and local metamodels to balance accuracy and computational efficiency [94]. Reliability-based design and optimization therefore depend strongly on the efficiency of the reliability assessment methods used within the design process.

7.6. Surrogate-based and data-driven approaches in reliability analysis of truss structures

Surrogate-based and data-driven approaches address different computational and modeling limitations in structural reliability assessment. Surrogate models approximate computationally expensive structural responses or limit-state functions and can therefore substantially reduce the cost of repeated reliability evaluations [94]. Data-driven finite element approaches follow a different concept, in which experimental material data may be used directly in the analysis, bypassing an explicit constitutive material model [95].

Kriging-based surrogate models are employed in reliability analysis due to their ability to provide predictions of structural

response together with associated measures of uncertainty. In the context of truss structures, Kriging techniques have been applied to approximate limit-state functions and to support the estimation of failure probabilities, including formulations that incorporate adaptive sampling or model refinement strategies [94,96].

Polynomial chaos expansions and hybrid PC-Kriging formulations represent an alternative class of surrogate techniques that combine spectral representations with data-driven learning. Such approaches exploit the smooth response behavior and moderate dimensionality of the stochastic space to construct efficient approximations of limit-state functions and have been applied to the reliability analysis of truss structures within PC-Kriging-based frameworks. The balance between accuracy and computational efficiency was demonstrated using two truss systems. For a 25-member planar truss bridge, the system reliability index obtained using the proposed approach had a relative error of 2.93% with respect to Monte Carlo simulation, while the method required on average 141 calls to the performance function compared with 460,330 for Monte Carlo simulation. For a 25-bar space truss, the corresponding relative error was 1.76%, with 172 calls to the performance function compared with 2×10^7 required by Monte Carlo simulation [94].

A comparison between stochastic finite element methods and data-driven finite element approaches reveals a more complex balance between computational efficiency and modeling assumptions. The mathematical properties of SFEM with respect to accuracy and convergence are well established, and the method can generally be implemented within existing finite element frameworks. Its main limitation is computational cost, particularly when Monte Carlo simulation is used for complex problems. In contrast, data-driven FEM bypasses explicit material modeling by operating directly on available experimental data. Although this avoids uncertainties associated with selecting and calibrating a constitutive material model, the search for appropriate data points becomes a computational bottleneck and, particularly in three-dimensional problems, may make the method more expensive than conventional deterministic finite element analysis (FEA) [95]. Machine learning techniques, including artificial neural networks and other regression-based models, have been investigated as surrogate representations for complex structural analyses. In the context of truss structures, such models have been applied to approximate structural response quantities or reliability-related measures, enabling efficient evaluation within reliability analysis or optimization-oriented procedures. For non-linear truss reliability problems, computational time was reduced by up to 25 times compared with conventional Monte Carlo simulation [97].

A key advantage of surrogate-based approaches is their ability to reduce the computational effort associated with repeated numerical simulations. This computational advantage is demonstrated by the reduced number of calls to the performance function [94] and reduced computational time [97]. However, these benefits are accompanied by approximation error, while the accuracy and robustness of

surrogate models depend strongly on the quality and representativeness of the training data, as well as on the adopted sampling and validation strategies [94,97].

Surrogate-based and data-driven approaches provide practical means to reduce the computational burden of reliability analysis for truss structures. Their effective application relies on appropriate surrogate selection, adaptive sampling, and careful validation, which remain central topics of ongoing research.

7.7. Imprecise probability and interval-based models

In practical engineering problems, the probabilistic information required for classical stochastic and reliability analyses is often incomplete or unavailable. In such cases, the assumption of fully defined probability distributions may not be justified, motivating the use of alternative uncertainty modeling frameworks based on imprecise probability concepts.

Interval-based models represent a simple approach to handling incomplete information, in which uncertain parameters are described by bounded intervals rather than by random variables with specified probability distributions. In the analysis of truss structures, hybrid probability-interval formulations have been applied in which variables with sufficient statistical information are represented probabilistically, whereas variables lacking sufficient information are described by variation intervals [90]. More advanced representations of imprecise uncertainty include probability boxes (p-boxes), which combine interval bounds with partial probabilistic information by defining upper and lower cumulative distribution functions. This allows uncertainty to be represented without assuming a single precise probability distribution. In reliability analysis of truss structures, p-box-based approaches have been used to model uncertainty in resistance parameters and joint behavior, particularly when experimental data are limited or modeling uncertainty is significant [98].

Within p-box frameworks, reliability is therefore expressed as bounds on failure or non-failure probability rather than as a single point estimate. Hybrid probability-interval formulations similarly provide a means of extending conventional reliability analysis when the probabilistic description of selected input variables is incomplete [90,98]. The main advantage of these approaches is that they avoid imposing unsupported precise probability distributions when statistical information is insufficient. Their main limitation is that the resulting reliability bounds may become too wide for practical decision-making when the available data are highly limited. Narrowing these bounds requires additional or higher-quality statistical information [98]. Moreover, hybrid random-interval reliability formulations may introduce additional computational effort due to nested probability and interval analyses, although equivalent formulations can reduce this burden [90].

Imprecise probability and interval-based models, therefore, provide a complementary means of representing uncertainty in truss analysis when probabilistic descriptions of input parameters are incomplete or unreliable.

7.8. Structural health monitoring and reliability updating of truss structures

Whereas the preceding reliability-based approaches generally rely on probabilistic and numerical models of structural parameters, loads, and failure mechanisms, SHM provides in-service measurement data that can be incorporated into these models to refine reliability assessment as new information becomes available. SHM-based updating should therefore be regarded not as a separate alternative to reliability analysis, but as a complementary source of operational information within the same assessment framework.

SHM has been applied to the reliability assessment of truss structures, particularly for existing bridges and large-span systems that are subjected to aging, deterioration, and environmental effects. SHM systems provide measured response data, including displacements, strains, accelerations, and environmental parameters, which can be used to support the evaluation of structural condition during service life [88].

In truss structures, monitoring data have been used for damage detection and localization, as well as for identifying changes in stiffness and load-transfer mechanisms. Signal-processing techniques, including wavelet-based methods, have been applied to structural response signals to detect local damage in truss members and girder systems [82,99].

Reliability updating frameworks integrate SHM data with probabilistic and numerical models in order to revise estimates of failure probability or reliability indices as new information becomes available. In the long-span truss bridge investigated in [88], monitoring data and finite element results were used jointly to determine component- and system-level reliability. Incorporating measured temperature-induced responses reduced the reliability index of a critical hanger from 4.60 to 4.26-4.43, while the system reliability index decreased from 3.04 to 2.38-2.72 [88]. This example illustrates a principal advantage of SHM-informed reliability assessment when structural effects that are difficult to represent accurately using conventional analytical models can be incorporated into reliability evaluation using in-service measurements.

Time-dependent reliability analysis is relevant to the assessment of existing truss structures when deterioration mechanisms such as fatigue, corrosion, material aging, as well as environmental effects are considered. Probabilistic degradation and fatigue models have been used to address biological decay in timber trusses [62] and fatigue-related failure mechanisms in steel truss bridges [89].

A related time-dependent perspective has been applied to steel trusses subjected to fire. In [100], the global Hasofer-Lind reliability index was evaluated in successive time steps for both statically determinate and indeterminate truss systems, enabling the probability of structural failure and compliance with the required safety level to be assessed during fire exposure. This concept was further developed in [101], where reliability was monitored during consecutive stages of fire exposure while accounting for changes in the static scheme of the structure caused by the evolving failure process [101]. Although these approaches are not based on measurement-driven SHM, they

illustrate the broader concept of reliability updating and monitoring as the structural state changes with time.

At the system level, SHM-informed reliability updating is particularly relevant to truss structures, given their inherent redundancy and capacity for load redistribution. Monitoring-informed system reliability models enable the identification of critical components and dominant failure paths, supporting assessments for maintenance planning and structural safety management of existing truss bridges [87,88]. Despite these applications, SHM-based reliability updating approaches face limitations related to measurement uncertainty, data incompleteness, model bias, and the need for robust damage indicators. Consequently, SHM data are typically combined with established structural and reliability models rather than used in isolation to ensure consistency between monitoring evidence and analytical predictions.

Thus, SHM complements conventional reliability analysis by providing information on the actual structural condition and its evolution during service.

7.9. Discussion

The reviewed literature demonstrates that stochastic and reliability-based analyses of truss structures encompass a broad range of modeling strategies, representations of uncertainty, and levels of structural idealization. Classical stochastic approaches, including Monte Carlo simulation and stochastic finite element methods (SFEM), provide a rigorous basis for quantifying response variability, but their direct application to large or complex truss systems is often limited by computational cost.

Reliability analysis offers a complementary perspective by focusing on failure probabilities and reliability indices derived from limit-state formulations. In the context of truss structures, the distinction between component-level and system-level reliability is particularly important due to redundancy and multiple load paths. Studies on system reliability show that the failure of individual members does not necessarily imply system failure, and that load redistribution mechanisms play a significant role in determining overall safety.

Reliability-based design and optimization frameworks extend deterministic design concepts by incorporating probabilistic safety measures directly into the design process. Applications to truss structures have shown that reliability constraints may influence optimal design solutions, particularly when uncertainties in material properties and loading conditions are explicitly considered. These formulations are most commonly applied to size optimization problems, with extensions to shape and topology optimization reported in the literature.

From a computational standpoint, integrating reliability analysis into optimization or system-level assessment frameworks presents significant challenges. To address these challenges, various strategies have been employed, including approximate reliability methods, surrogate modeling techniques, and reduced-order representations. Such approaches aim to improve computational efficiency while retaining an acceptable level of accuracy for reliability estimation.

Alternative uncertainty representations have been introduced for situations in which probabilistic descriptions of input parameters are incomplete or unreliable. Interval-based and imprecise probability models provide a means of representing bounded uncertainty without assuming fully specified probability distributions. These approaches have been applied to truss analysis and optimization problems, offering complementary tools to classical probabilistic methods in settings with limited data availability.

Structural health monitoring provides a further extension of reliability assessment for existing truss structures by introducing information obtained during service. Monitoring data can support damage detection, identification of changes in structural behavior, and refinement of component- and system-level reliability estimates. Time-dependent reliability approaches additionally allow changes in structural safety to be followed as deterioration or other structural conditions evolve. Overall, the reviewed studies illustrate that stochastic analysis, reliability methods, surrogate and data-driven approaches, alternative uncertainty representations, and monitoring-based updating are complementary components of a broader methodological landscape for assessing and managing truss structures. Each class of methods addresses specific modeling challenges and practical constraints, and their combined use reflects the diverse requirements encountered in the analysis, design, and assessment of truss systems.

8. DATA-DRIVEN AND MONITORING-BASED DEVELOPMENT DIRECTIONS

Future research on truss structures should increasingly adopt a holistic perspective, treating trusses as complex structural systems rather than merely as vehicles for specific analysis techniques. The previous chapters have shown that further progress requires closer integration of structural mechanics, computational modeling, uncertainty quantification, reliability analysis, and monitoring-based assessment. Particular challenges concern the modeling of nonlinear and system-level behaviors, computationally efficient reliability assessment, and the integration of measurement data with structural models.

One major research direction concerns the continued development of advanced modeling strategies for truss structures, particularly those that bridge discrete bar models and equivalent continuum representations. Recent studies underline the potential of homogenization techniques, reduced-order models, and equivalent continuum formulations to enable efficient simulation of large truss systems while preserving essential mechanical characteristics [12]. An important direction for future research is to extend these approaches to strongly nonlinear regimes, including large displacements, post-buckling behavior, and dynamic response, where current equivalent models are limited in accuracy or applicability.

Nonlinear mechanics remains a central theme in future truss research. While classical linear analysis provides valuable insight for preliminary design, modern applications increasingly require robust treatment of geometric and material nonlinearities. Recent advances in elasto-plastic analysis and optimization of truss structures demonstrate the feasibility of

capturing yielding, large deformations, and imperfection sensitivity within unified computational frameworks [19]. Future investigations should further integrate these nonlinear effects with stability, serviceability, and optimization requirements in computationally efficient design frameworks.

From a design standpoint, optimization of truss structures remains a fertile area for future research. Beyond classical weight minimization, current challenges include multi-objective optimization incorporating stiffness, stability, deformation limits, and manufacturability. Recent work combining nonlinear finite element analysis with evolutionary algorithms illustrates the potential of such approaches [19]. Future research should aim to develop scalable optimization strategies applicable to large and complex truss systems involving nonlinear response and multiple design constraints.

In parallel, the rapid development of machine learning techniques offers powerful tools to support truss modeling and analysis. State-of-the-art reviews in structural engineering indicate that data-driven methods can significantly enhance surrogate modeling, pattern recognition, and model updating [14]. In the context of trusses, future research should emphasize cautious and transparent use of machine learning in structural analysis and design, including approaches that integrate physics-based knowledge to improve reliability, interpretability, and acceptance in practice. Closely related to this is the growing interest in physics-informed and hybrid data-driven approaches. Recent studies demonstrate that combining neural networks with physics-based constraints and Monte Carlo simulation can improve efficiency, accuracy near the failure surface, and robustness [102]. Future work should explore how such approaches can be systematically integrated into truss analysis workflows, particularly for cases with implicit limit-state functions and evaluations based on finite element analysis. In the broader context of truss modeling, reliability analysis remains an important area of research aimed at addressing the impact of uncertainty on structural behavior. Recent advances in surrogate-based reliability methods, including adaptive PC-Kriging approaches, provide effective tools for reducing the computational cost of system reliability analysis while maintaining an appropriate level of accuracy [94]. Future studies may further investigate integrating such reliability methods with finite element-based structural analyses, particularly when the limit-state function is implicitly defined and computationally expensive.

In this context, future reliability-oriented studies are expected to increasingly account for stochastic effects associated with geometric imperfections [74], as these factors play a critical role in the global response and safety assessment of truss systems. Incorporating such sources of uncertainty within advanced computational frameworks represents a natural extension of current research directions.

Further research priority concerns closer integration of structural health monitoring with reliability assessment of existing truss structures. As discussed in Section 7.8, monitoring data can provide structure-specific information for refining component- and system-level reliability estimates and for representing operational and environmental effects that may

not be fully captured by analytical models [88]. Future work should also focus on systematic integration of measured structural responses with numerical and probabilistic models throughout the service life of truss systems.

Recent approaches based on graph theory provide an additional perspective by representing the paths of tensile and compressive load-transfer in lattice metamaterials as independent networks and relating their connectivity to the localization of deformation [103]. This concept can be viewed as complementary to Michell-type approaches. Both approaches emphasize the organization of tensile and compressive load-transfer paths, although they address different objectives. Recent parametric formulations further extend Michell-type principles toward stress-aligned three-dimensional truss layouts [104].

Future research on truss structures is also expected to progress towards closer integration of advanced mechanical modeling, computational strategies, reliability-based methods, monitoring information, and data-assisted approaches, while maintaining a clear physical interpretation of structural behavior.

9. CONCLUSIONS

This article reviews engineering approaches to the modeling and evaluation of truss systems, with a particular focus on the transition from classical structural idealization to reliability-based analysis. The literature review confirms that the behavior of trusses cannot be interpreted only in terms of the axial behavior of individual members. Although classical assumptions of trusses are still useful for conceptual analysis and preliminary design, real structures are affected by geometric imperfections, joint behavior, second-order effects, and uncertainties in material, geometric, and boundary parameters.

Experimental and full-scale studies show that the response of truss systems depends on the interactions between member instability, joint behavior, bracing effectiveness, load redistribution, and global system effects. These results support modeling approaches that are based on observed structural behavior and take into account imperfections, with an appropriate level of idealization.

Numerical modeling is currently the basic tool for truss design. However, its accuracy depends on model assumptions. Depending on their type, finite element models differ in their ability to represent global force distribution, local effects, structural behavior, and sensitivity to imperfections. Therefore, the choice of modeling approach should be suited to the goal of analysis, predicted mechanisms, and the required level of accuracy.

Stochastic and reliability-based methods offer a framework for measuring the impact of uncertainty on truss behavior. They can be used to estimate the probability of failure, identify critical parameters, and determine system reliability under variations in loads, geometry, material properties, and imperfections. Approaches based on surrogate models and data-driven methods are particularly important when direct reliability analysis based on detailed numerical models becomes computationally expensive.

The most important research gaps identified in this review concern three closely related areas. First, further development is needed to efficiently represent nonlinear, imperfection-sensitive, and system-level truss behavior in computational models. Second, reliability assessment requires more efficient integration of detailed numerical models with surrogate, data-driven, and physics-informed approaches, while maintaining adequate accuracy and validation. Third, structural health monitoring data should be more systematically integrated with numerical and probabilistic models to support reliability assessment throughout the service life of truss structures. Addressing these gaps is expected to drive future research towards integrated mechanical-probabilistic and monitoring-informed assessment frameworks.

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