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PRINCIPLES OF MEASURING POWER COMPONENTS IN A THREE-PHASE THREE-WIRE UNBALANCED CIRCUIT IN LIGHT OF THE CPC THEORY

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Abstract

The article presents an analysis of a three-phase electric circuit in terms of describing the power components measured and determined on a three-wire line connecting an unbalanced power source with periodic, non-sinusoidal waveforms with an unbalanced receiver. In this study, the case was considered when the load admittance changes its value when the direction of rotation of the symmetrical voltage and current components is changed.

This is a typical situation where three-phase power must be measured on a three-wire line connecting a real source to a load that contains three-phase rotating machines. In real systems, such machines often change their current/voltage relationship when the direction of rotation changes.

This paper presents a modification of the currents' physical components (CPC) power theory to decompose currents and powers into symmetrical components. This constitutes the theoretical framework of this study. Using the determined dependencies, it is possible to develop a new measurement algorithm. As a result of these measurements, it is possible to determine the electromechanical efficiency of the rotating machine, *i.e.*, the efficiency that does not take into account the influence of higher harmonics and power source asymmetry. This is important information needed to optimise the active power consumption of electromechanical machines. Measurement of active power using classical methods does not allow one to observe the influence of supply voltage asymmetry on the power that affects active conversion to mechanical power. Thanks to such measurements, it is possible to select optimal parameters of the electric motor in order to improve the efficiency of converting electrical energy into mechanical energy. In this approach, the asymmetry of the power supply and the introduction of higher voltage harmonics are perceived as external interferences in the optimisation process. This fragment of the article constitutes the practical side, *i.e.* the application of the theoretical considerations derived. In the presented calculation example, it was shown that the efficiency of the machine strongly depends on the asymmetry of the voltage source, and less on the harmonics.

Keywords: power theory, power measurement, currents' physical components (CPC), engine efficiency.

1. Introduction

Contemporary technological development and global energy policy strive for pro-ecological activities [1, 2]. In light of global trends to optimise electricity consumption, several actions are possible to improve the efficiency of converting electrical energy into mechanical energy. Such actions are necessary because, on a global scale, they affect the world energy economy. The safe use of electrical devices also comes down to ensuring protection against overvoltage [3, 4].

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In practice, the measurement required the most frequently is the three-phase active and reactive power [5], especially in the case of economic settlements [6]. However, more accurate mathematical analyses require more precise measurements. For example, to gain insight into how energy is used efficiently, a more detailed description is needed. Using the *Currents' Physical Components* (CPC) power theory [7, 8], it is possible to decompose the current into physical components which are an interpretation of the physical properties of the circuit. This approach provides insight into all factors that cause increased current flow and increased apparent power. Knowing the elementary physical components, one can propose methods to improve the power factor. In this way, tools will be obtained to select the system structure and the compensating device parameters [9–11]. The problem of selecting the compensator parameters is a complex issue and requires a separate description [12, 13]. As shown in the literature [10], achieving a power factor of 1 in three-phase systems is very difficult or even impossible. It is only possible to approach this value and searching for more accurate solutions is not economically justified.

Referring to the literature, for example, [7–17], it should be noted that the method of describing the CPC power theory for a three-wire circuit in the case of an unbalanced load and a real three-phase source has already been developed. This paper proposes the use of CPC theory for measuring electrical quantities on a line connecting an unknown real source with an unknown unbalanced receiver in a three-phase three-wire network. The current state of knowledge on the CPC power theory did not take into account decomposition into components related to symmetrical components. This article extends the CPC theory to include this new aspect, thanks to which the discussed measurement methodology allows for the observation of the efficiency of the process of converting electrical energy into mechanical energy in the case of power source asymmetry and under the influence of the multiple voltage harmonics.

Voltage asymmetry in the power grid is a consequence of the line being loaded with unbalanced receivers. The operation of such receivers on a common power supply line also results in asymmetry in the currents that supply balanced receivers.

The appearance of higher voltage harmonics in the power supply line is a consequence of impact of non-linear receivers on the common power supply network. Such receivers are able to consume part of the energy of the fundamental component to generate current with higher harmonics. The non-zero internal impedance of the source causes the higher harmonic currents to influence the higher harmonic voltages. As a result, one non-linear receiver may interfere with the operation of other receivers connected to a common power supply network. Higher voltage harmonics can be suppressed or amplified in local resonant connections. In result, even a linear reactance receiver may fall into resonance with the network reactance, and even small current harmonics will be amplified, which will translate into an increase in the voltage amplitude for this harmonic.

Electromechanical devices are constructed in such a way that conversion of electrical energy into mechanical energy takes place only for the fundamental harmonic. In this case, the influence of higher harmonics causes a disturbance in the rotating magnetic flux of this machine. Voltage asymmetry also results in a disturbance of the magnetic field. This can lead to a situation in which some of the electrical energy forces the field to rotate in the opposite direction. This means that both voltage asymmetry and higher current harmonics are undesirable for the operation of electromechanical devices.

Both of these phenomena result in the need to oversize the rated power, increase losses, reduce efficiency, and deteriorate the power factor. When optimising the parameters of electromechanical devices, these are key problems that have been taken into account so far only in an approximate manner, taking into account the constant safety factor.

2. Mathematical description of the circuit

The method of describing a three-phase, three-wire circuit is consistent with existing measurement methods used in the CPC theory. To enable further investigation, it is necessary to consider a circuit containing a real three-phase source, assume that this source is unbalanced, and take into account the possibility of higher voltage harmonics. The task of the measuring system is to record the instantaneous values of voltage and current in the circuit from Fig. 1.

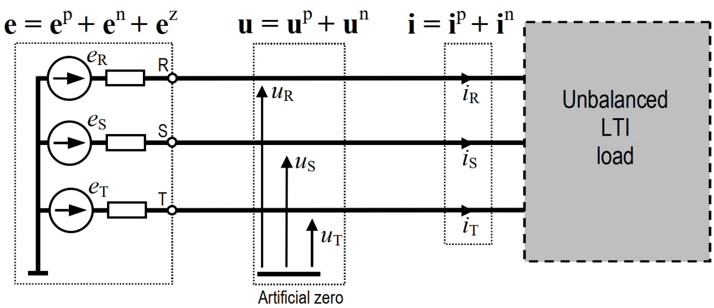

Fig. 1. Measuring system on a three-wire line.

In real operating conditions, the power system is often characterized by the asymmetry of voltage **u** on the power source. This is particularly noticeable in the case of powering unbalanced receivers from a three-phase source with non-zero internal impedance.

The electrical quantities of this circuit, when decomposed into harmonics, can be described vectorially:

$$\boldsymbol{e}_n = \begin{bmatrix} e_{\mathrm{R}n} \\ e_{\mathrm{S}n} \\ e_{\mathrm{T}n} \end{bmatrix}, \quad \boldsymbol{u}_n = \begin{bmatrix} u_{\mathrm{R}n} \\ u_{\mathrm{S}n} \\ u_{\mathrm{T}n} \end{bmatrix}, \quad \boldsymbol{i}_n = \begin{bmatrix} i_{\mathrm{R}n} \\ i_{\mathrm{S}n} \\ i_{\mathrm{T}n} \end{bmatrix}. \tag{1}$$

Assuming that the internal impedances of the measuring channels are the same, the zero-sequence symmetrical component of the voltage $\boldsymbol{u}^z$, which is the interpretation of the voltages determined with respect to the "artificial zero", is equal to zero. In addition, line currents in a three-wire system do not contain a zero-sequence component $\boldsymbol{i}^z$. In the case of harmonic waveforms, the supply voltage vector $\boldsymbol{u}$ with respect to artificial zero can be decomposed into symmetrical components of the positive $(^p)$ and negative $(^n)$ sequence:

$$\begin{aligned} \boldsymbol{u} &= \sum_{n\in N} \boldsymbol{u}_n = \sqrt{2} Re \left\{ \sum_{n\in N} \underline{\boldsymbol{U}}_n e^{jn\omega_1 t} \right\} = \sqrt{2} Re \left\{ \sum_{n\in N} \left(\underline{\boldsymbol{U}}_n^p + \underline{\boldsymbol{U}}_n^n \right) \cdot e^{jn\omega_1 t} \right\} \\ &= \sqrt{2} Re \left\{ \sum_{n\in N} \left(\mathbf{1}^p \underline{U}_n^p + \mathbf{1}^n \underline{U}_n^n \right) \cdot e^{jn\omega_1 t} \right\} = \boldsymbol{u}^p + \boldsymbol{u}^n, \end{aligned} \tag{2}$$

where: $\mathbf{1}^p$, $\mathbf{1}^n$ – three-phase unit vectors defined in the Fortescue transformation [7], $\underline{\boldsymbol{U}}_n$ – the vector of three-phase complex voltages (crms) of the n^{th} harmonic, which is equal:

$$\underline{\boldsymbol{U}}_n = \begin{bmatrix} \underline{U}_{\mathrm{R}n} \\ \underline{U}_{\mathrm{S}n} \\ \underline{U}_{\mathrm{T}n} \end{bmatrix} = \underline{\boldsymbol{U}}_n^p + \underline{\boldsymbol{U}}_n^n, \underline{\boldsymbol{U}}_n^p = \begin{bmatrix} \underline{U}_{\mathrm{R}n}^p \\ \underline{U}_{\mathrm{S}n}^p \\ \underline{U}_{\mathrm{T}n}^p \end{bmatrix}, \quad \underline{\boldsymbol{U}}_n^n = \begin{bmatrix} \underline{U}_{\mathrm{R}n}^n \\ \underline{U}_{\mathrm{S}n}^n \\ \underline{U}_{\mathrm{T}n}^n \end{bmatrix}. \tag{3}$$

The symmetrical components of the n^{th} harmonic voltages are determined from the relationship [18]:

$$\begin{bmatrix} \underline{U}_n^p \\ \underline{U}_n^n \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & \alpha^n & \alpha^{2n} \\ 1 & \alpha^{2n} & \alpha^n \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_{\mathrm{R}n} \\ \underline{U}_{\mathrm{S}n} \\ \underline{U}_{\mathrm{T}n} \end{bmatrix}, \tag{4}$$

where $\alpha = e^{j120^o} = e^{j\frac{2\pi}{3}}$ *i.e.* for harmonics with a positive sequence $n = 3k + 1$, the voltages have a positive sequence, while for the order $n = 3k + 2$ they are of negative sequence. Therefore, for harmonics of the order $n = 3k + 2$, the symmetrical components $\underline{U}_n^p$ and $\underline{U}_n^n$ are cross-interchanged [18]. This is the result of raising the rotation vector α to the power n (Fig. 2). The previous description of such a circuit did not take into account the possibility of changing the direction of the rotation vector α as a function of the order of the n^{th} harmonic [14]. An improved mathematical description was developed and presented only in [18].

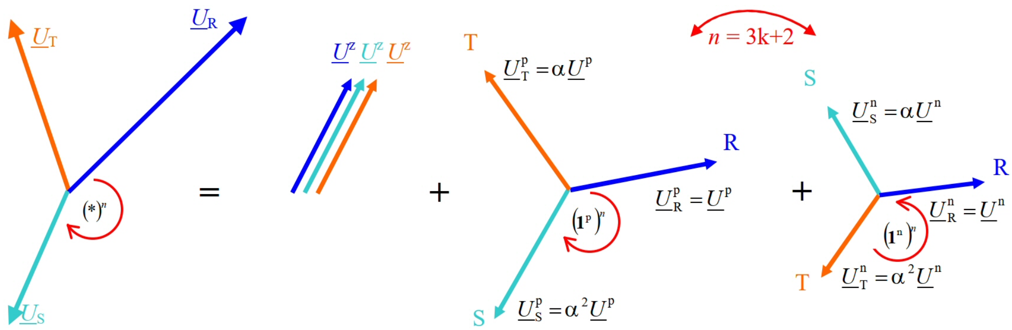

Fig. 2. Graphical interpretation of multiplied three-phase unit vectors, where k is a natural number $k \in N$.

Similarly to voltage $\boldsymbol{u}$, the Fortescue transformation is performed for the three-phase current vector $\boldsymbol{i}$. It is known that in a three-wire system the relation $i_R + i_S + i_T = 0$ is valid. This means that the current vector $\boldsymbol{i}$ also does not have a zero-sequence symmetrical component. The remaining components are determined analogously to (4).

In order to apply Kirchhoff's law for individual load phases, for each n^{th} harmonic the potentials must be equal: the star point in the three-phase voltmeter system V_N and the star point in the load system $V_{N'}$ (Fig. 3). Such an equality will occur, for example, when a balanced receiver $\underline{Y}_b$ is powered from a symmetrical source. In the case the power source is asymmetric and the load admittance is unbalanced, further analysis should be performed using the Fortescue transformation. For the n^{th} harmonic, symmetrical components with positive $\underline{U}_n^p$, $\underline{I}_n^p$ and negative $\underline{U}_n^n$, $\underline{I}_n^n$ sequence are determined from the measured voltage $\underline{U}_n$ and current $\underline{I}_n$ vectors.

Without knowing the load parameters and its connection topology, it is possible to determine the equivalent phase admittances $\underline{Y}_R$, $\underline{Y}_S$ and $\underline{Y}_T$ (5).

$$\frac{\underline{I}_{Ln}}{\underline{U}_{Ln}} = \underline{Y}_{Ln}, \tag{5}$$

where $L = \{R, S, T\}$.

This means selecting such parameters of the unbalanced load for the asymmetric source that the relationship $V_N = V_{N'}$ is satisfied. The load parameters $\underline{Y}_L$ determined in this way will be unbalanced for the n^{th} order harmonic with asymmetric source voltage $\boldsymbol{u}$.

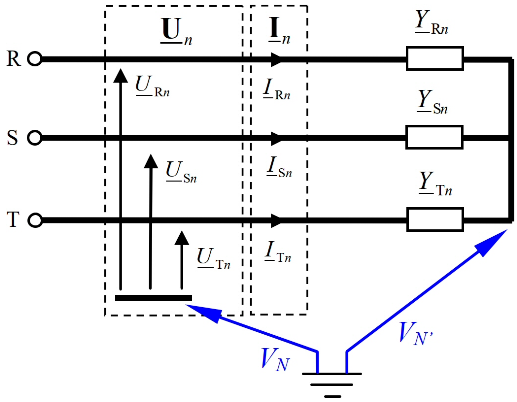

Fig. 3. Equality of star potentials of reference points V_N and $V_{N'}$.

For each n^{th} harmonic there is a balanced load with admittance $\underline{Y}_{bn}$ which will be the response of the system to the resultant active power P_n and reactive power Q_n (Fig. 4). The value of this admittance is:

$$\underline{Y}_{bn} = G_{bn} + jB_{bn} = \frac{P_n - jQ_n}{\|\boldsymbol{u}_n\|^2} = \frac{\underline{C}_n^*}{\|\boldsymbol{u}_n\|^2}, \tag{6}$$

where the effective values of the n^{th} harmonic voltages are equal:

$$\|\boldsymbol{u}_n\| = \sqrt{\left|\underline{U}_{Rn}\right|^2 + \left|\underline{U}_{Sn}\right|^2 + \left|\underline{U}_{Tn}\right|^2}, \tag{7}$$

while the instantaneous current values are:

$$\boldsymbol{i}_n = \sqrt{2}Re\left\{\underline{Y}_{bn} \cdot \underline{\boldsymbol{U}}_n e^{jn\omega_1 t}\right\}. \tag{8}$$

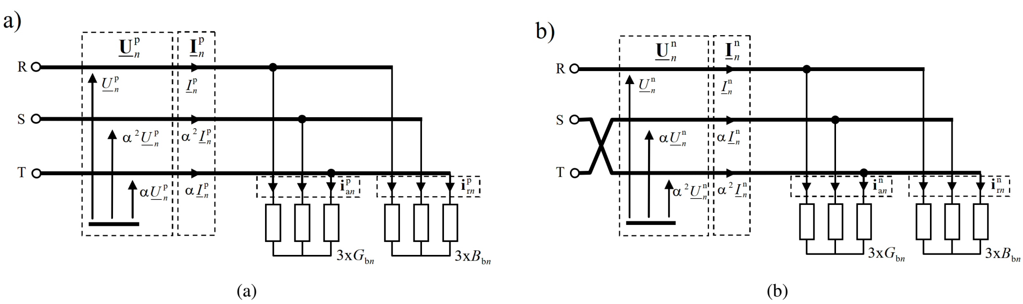

Fig. 4. Balanced load equivalent to the primary in terms of power P_n and Q_n for the n^{th} order harmonic after transposition into symmetrical components: positive (a) and negative (b).

According to the literature [14], the symbol "C" was used in (6) instead of the symbol "S" to avoid confusing the complex power $P_n + jQ_n$ with the apparent power S_n, which may also contain components other than only active and reactive power.

The components P_n and Q_n are determined from the relationship:

$$P_n = Re\left\{\underline{\boldsymbol{U}}_n^T \underline{\boldsymbol{I}}_n^*\right\}, \quad Q_n = Im\left\{\underline{\boldsymbol{U}}_n^T \underline{\boldsymbol{I}}_n^*\right\}, \quad \underline{C}_n = P_n + jQ_n. \tag{9}$$

According to the assumption in this paper, the load is *linear and time-invariant* (LTI), which means a stationary value of the admittance $\underline{Y}_{\mathrm{b}n}$ (6) for stationary values of $\underline{U}_n$ and $\underline{I}_n$. However, it should be remembered that $\underline{Y}_{\mathrm{b}n}$ will change its value when the asymmetry of the power source changes. The load admittance changes its value when the direction of rotation of the symmetrical component of voltage and current changes. When changing the $\underline{U}_p/\underline{U}_n$ ratio, the load admittance behaves quasi-stationary (LTIq).

$$\underline{Y}_{\mathrm{b}n} = f\left(\underline{U}^p, \underline{U}^n, \underline{I}^p, \underline{I}^n\right). \tag{10}$$

The load shown in Fig. 1 is equivalent in terms of active power P to the balanced resistive load shown in Fig. 5 with conductance equal:

$$G_b = \frac{P}{\|\boldsymbol{u}\|^2}, \quad \|\boldsymbol{u}\| = \sqrt{\|u_R\|^2 + \|u_S\|^2 + \|u_T\|^2}. \tag{11}$$

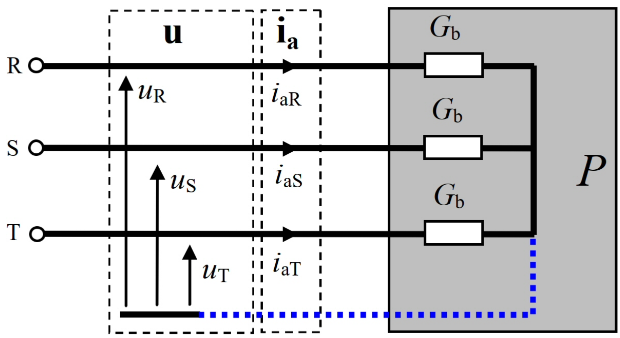

Fig. 5. Balanced resistive load equivalent in terms of active power P.

The active component of the n^{th} harmonic current according to the CPC theory after decomposition into symmetrical components is as follows:

$$\begin{aligned}
\boldsymbol{i}_{an}^p &= G_{\mathrm{b}n}\boldsymbol{u}_n^p = \sqrt{2}Re\left\{G_{\mathrm{b}n}\underline{\boldsymbol{U}}_n^p \cdot e^{jn\omega_1 t}\right\} = \sqrt{2}Re\left\{G_{\mathrm{b}n}\mathbf{1}^p\underline{U}_n^p \cdot e^{jn\omega_1 t}\right\},\\
\boldsymbol{i}_{an}^n &= G_{\mathrm{b}n}\boldsymbol{u}_n^n = \sqrt{2}Re\left\{G_{\mathrm{b}n}\underline{\boldsymbol{U}}_n^n \cdot e^{jn\omega_1 t}\right\} = \sqrt{2}Re\left\{G_{\mathrm{b}n}\mathbf{1}^n\underline{U}_n^n \cdot e^{jn\omega_1 t}\right\},\\
\boldsymbol{i}_{an} &= \boldsymbol{i}_{an}^p + \boldsymbol{i}_{an}^n,
\end{aligned} \tag{12}$$

while the reactive components of the n^{th} harmonic current are equal:

$$\begin{aligned}
\boldsymbol{i}_{rn}^p &= B_{\mathrm{b}n}\boldsymbol{u}_n^p\left(t + \frac{T}{4n}\right) = \sqrt{2}Re\left\{jB_{\mathrm{b}n}\underline{\boldsymbol{U}}_n^p \cdot e^{jn\omega_1 t}\right\} = \sqrt{2}Re\left\{jB_{\mathrm{b}n}\mathbf{1}^p\underline{U}_n^p \cdot e^{jn\omega_1 t}\right\},\\
\boldsymbol{i}_{rn}^n &= B_{\mathrm{b}n}\boldsymbol{u}_n^n\left(t + \frac{T}{4n}\right) = \sqrt{2}Re\left\{jB_{\mathrm{b}n}\underline{\boldsymbol{U}}_n^n \cdot e^{jn\omega_1 t}\right\} = \sqrt{2}Re\left\{jB_{\mathrm{b}n}\mathbf{1}^n\underline{U}_n^n \cdot e^{jn\omega_1 t}\right\},\\
\boldsymbol{i}_{rn} &= \boldsymbol{i}_{rn}^p + \boldsymbol{i}_{rn}^n.
\end{aligned} \tag{13}$$

Metrol. Meas. Syst., Vol. 33 (2026), No. 1, pp. 1–16
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The admittance $\underline{Y}_{bn}$ is the admittance of an equivalent balanced load for the n^{th} order harmonic, while the load in Fig. 1 may be unbalanced. As a consequence, the n^{th} order harmonic of the load current may contain an unbalanced current:

$$\begin{aligned} \boldsymbol{i}_{un}^{p} &= \boldsymbol{i}_{n}^{p} - \left(\boldsymbol{i}_{an}^{p} + \boldsymbol{i}_{rn}^{p}\right) = \sqrt{2} Re\left\{\left(\underline{\boldsymbol{I}}_{n}^{p} - \underline{Y}_{bn} \mathbf{1}^{p} \underline{U}_{n}^{p}\right) \cdot e^{jn\omega_1 t}\right\}, \\ \boldsymbol{i}_{un}^{n} &= \boldsymbol{i}_{n}^{n} - \left(\boldsymbol{i}_{an}^{n} + \boldsymbol{i}_{rn}^{n}\right) = \sqrt{2} Re\left\{\left(\underline{\boldsymbol{I}}_{n}^{n} - \underline{Y}_{bn} \mathbf{1}^{n} \underline{U}_{n}^{n}\right) \cdot e^{jn\omega_1 t}\right\}, \\ \boldsymbol{i}_{un} &= \boldsymbol{i}_{un}^{p} + \boldsymbol{i}_{un}^{n}. \end{aligned} \tag{14}$$

Therefore, for the n^{th} harmonic, the three-phase current was decomposed into components:

$$\boldsymbol{i}_{n} = \boldsymbol{i}_{n}^{p} + \boldsymbol{i}_{n}^{n} = \boldsymbol{i}_{an}^{p} + \boldsymbol{i}_{an}^{n} + \boldsymbol{i}_{rn}^{p} + \boldsymbol{i}_{rn}^{n} + \boldsymbol{i}_{un}^{p} + \boldsymbol{i}_{un}^{n}. \tag{15}$$

Ultimately, the three-phase current is equal:

$$\boldsymbol{i} = \sum_{n \in N} \boldsymbol{i}_{n} = \sum_{n \in N} \left(\boldsymbol{i}_{an}^{p} + \boldsymbol{i}_{an}^{n} + \boldsymbol{i}_{rn}^{p} + \boldsymbol{i}_{rn}^{n} + \boldsymbol{i}_{un}^{p} + \boldsymbol{i}_{un}^{n}\right). \tag{16}$$

The i_r component due to the phase shift of the load current harmonic relative to the supply voltage harmonic is called **the reactive current** and is equal:

$$\boldsymbol{i}_{r} = \sum_{n \in N} \boldsymbol{i}_{rn} = \sum_{n \in N} \left(\boldsymbol{i}_{rn}^{p} + \boldsymbol{i}_{rn}^{n}\right) = \sqrt{2} Re\left\{\sum_{n \in N} jB_{bn} \left(\mathbf{1}^{p} \underline{U}_{n}^{p} + \mathbf{1}^{n} \underline{U}_{n}^{n}\right) \cdot e^{jn\omega_1 t}\right\}. \tag{17}$$

The i_u component due to load unbalance is called **the unbalanced current**:

$$\boldsymbol{i}_{u} = \sum_{n \in N} \boldsymbol{i}_{un} = \sum_{n \in N} \left(\boldsymbol{i}_{un}^{p} + \boldsymbol{i}_{un}^{n}\right) = \sqrt{2} Re\left\{\sum_{n \in N} \left[\underline{\boldsymbol{I}}_{n}^{p} + \underline{\boldsymbol{I}}_{n}^{n} - \underline{Y}_{bn} \left(\mathbf{1}^{p} \underline{U}_{n}^{p} + \mathbf{1}^{n} \underline{U}_{n}^{n}\right)\right] \cdot e^{jn\omega_1 t}\right\}, \tag{18}$$

The i_a component, historically introduced by Prof. Stanisław Fryze, is related to active power – it is called **the active current**:

$$\boldsymbol{i}_{a} = G_{b} \boldsymbol{u} = \sqrt{2} G_{b} Re\left\{\sum_{n \in N} \left(\mathbf{1}^{p} \underline{U}_{n}^{p} + \mathbf{1}^{n} \underline{U}_{n}^{n}\right) e^{jn\omega_1 t}\right\} = \boldsymbol{i}_{a}^{p} + \boldsymbol{i}_{a}^{n}, \tag{19}$$

$$\boldsymbol{i}_{a}^{p} = \sqrt{2} G_{b} Re\left\{\sum_{n \in N} \mathbf{1}^{p} \underline{U}_{n}^{p} e^{jn\omega_1 t}\right\}, \quad \boldsymbol{i}_{a}^{n} = \sqrt{2} G_{b} Re\left\{\sum_{n \in N} \mathbf{1}^{n} \underline{U}_{n}^{n} e^{jn\omega_1 t}\right\}. \tag{20}$$

When conductances $G_{bn} \neq G_b$, the component i_a cannot be determined from the sum of the components from (12) because:

$$\boldsymbol{i}_{a} \neq \sum_{n \in N} \boldsymbol{i}_{an}. \tag{21}$$

In the case of a change in the equivalent conductance G_{bn} together with a change in the n-harmonic, a non-zero current value appears – called **the scattered current**:

$$\sum_{n \in N} \boldsymbol{i}_{an} - \boldsymbol{i}_{a} = \sqrt{2} Re\left\{\sum_{n \in N} \left(G_{bn} - G_{b}\right) \cdot \left(\mathbf{1}^{p} \underline{U}_{n}^{p} + \mathbf{1}^{n} \underline{U}_{n}^{n}\right) \cdot e^{jn\omega_1 t}\right\} = \boldsymbol{i}_{s} = \boldsymbol{i}_{s}^{p} + \boldsymbol{i}_{s}^{n}, \tag{22}$$

$$\begin{aligned} \boldsymbol{i}_s^p &= \sqrt{2}Re\left\{\sum_{n\in N}(G_{\mathrm{b}n}-G_b)\cdot\mathbf{1}^p\underline{U}_n^p\cdot e^{jn\omega_1 t}\right\}, \\ \boldsymbol{i}_s^n &= \sqrt{2}Re\left\{\sum_{n\in N}(G_{\mathrm{b}n}-G_b)\cdot\mathbf{1}^n\underline{U}_n^n\cdot e^{jn\omega_1 t}\right\}. \end{aligned} \tag{23}$$

Combining the individual current components gives the solution:

$$\boldsymbol{i}=\boldsymbol{i}_a+\boldsymbol{i}_s+\boldsymbol{i}_r+\boldsymbol{i}_u. \tag{24}$$

Harmonic components are mutually orthogonal, whereas the Fortescue transformation does not produce orthogonal vectors. The effective values of individual currents are determined from the relationship:

$$\|\boldsymbol{i}_a\|=G_b\|\boldsymbol{u}\|,\quad \left\|\boldsymbol{i}_a^p\right\|=G_b\|\boldsymbol{u}^p\|=G_b 3\cdot\sqrt{\sum_{n\in N}\left|\underline{U}_n^p\right|^2},\quad \left\|\boldsymbol{i}_a^n\right\|=G_b 3\cdot\sqrt{\sum_{n\in N}\left|\underline{U}_n^n\right|^2}, \tag{25}$$

$$\begin{aligned} \|\boldsymbol{i}_r\| &= \sqrt{\sum_{n\in N}\|\boldsymbol{i}_{rn}\|^2}=\sqrt{\sum_{n\in N}B_{\mathrm{b}n}^2\|\boldsymbol{u}_n\|^2}, \\ \left\|\boldsymbol{i}_r^p\right\| &= 3\cdot\sqrt{\sum_{n\in N}B_{\mathrm{b}n}^2\left|\underline{U}_n^p\right|^2},\quad \left\|\boldsymbol{i}_r^n\right\|=3\cdot\sqrt{\sum_{n\in N}B_{\mathrm{b}n}^2\left|\underline{U}_n^n\right|^2}, \end{aligned} \tag{26}$$

$$\begin{aligned} \|\boldsymbol{i}_s\| &= \sqrt{\sum_{n\in N}\|\boldsymbol{i}_{sn}\|^2}=\sqrt{\sum_{n\in N}(G_{\mathrm{b}n}-G_b)^2\|\boldsymbol{u}_n\|^2}, \\ \left\|\boldsymbol{i}_s^p\right\| &= 3\cdot\sqrt{\sum_{n\in N}(G_{\mathrm{b}n}-G_b)^2\left|\underline{U}_n^p\right|^2},\quad \left\|\boldsymbol{i}_s^n\right\|=3\cdot\sqrt{\sum_{n\in N}(G_{\mathrm{b}n}-G_b)^2\left|\underline{U}_n^n\right|^2}, \end{aligned} \tag{27}$$

$$\|\boldsymbol{i}_u\|=\sqrt{\sum_{n\in N}\|\boldsymbol{i}_{un}\|^2}=\sqrt{\|\boldsymbol{i}\|^2-\left(\|\boldsymbol{i}_a\|^2+\|\boldsymbol{i}_s\|^2+\|\boldsymbol{i}_r\|^2\right)}. \tag{28}$$

According to the CPC theory, the powers in a three-phase system are determined by multiplying the individual current components (25), (26), (27), (28) by the effective voltage value (11), which gives:

$$S=\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}\|,\quad D_s=\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}_s\|,\quad Q=\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}_r\|,\quad D_u=\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}_u\|,\quad P=\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}_a\|. \tag{29}$$

Due to the lack of orthogonality between the symmetrical components, there is no mathematical relationship between the effective value of the original variable $\left\|\underline{X}\right\|$ and the effective values of the positive $\left\|\underline{X}^p\right\|$ and the negative $\left\|\underline{X}^n\right\|$ symmetrical components. Only for the complex power $\underline{C}$ (9), due to the reduction of the real and imaginary parts to a common Cartesian coordinate system, the following relations can be written:

$$P^p=3\cdot Re\left\{\sum_{n\in N}\underline{U}_n^p\cdot\underline{I}_n^{p*}\right\},\quad P^n=3\cdot Re\left\{\sum_{n\in N}\underline{U}_n^n\cdot\underline{I}_n^{n*}\right\},\quad P=P^p+P^n. \tag{30}$$

Reactive power can change sign, therefore:

$$Q_n^p=3\cdot Im\left\{\underline{U}_n^p\cdot\underline{I}_n^{p*}\right\},\quad Q_n^n=3\cdot Im\left\{\underline{U}_n^n\cdot\underline{I}_n^{n*}\right\},\quad Q\neq Q^p+Q^n. \tag{31}$$

Metrol. Meas. Syst., Vol. 33 (2026), No. 1, pp. 1–16
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3. Practical consequences

A very important cognitive value is obtained by analysing the values of active powers P^p and P_n because these numbers directly show the influence of current and voltage asymmetry on the effective part of the power transmitted in a rotating machine operating as a receiver.

Knowing the useful part of the power, it becomes possible to determine the **electromechanical efficiency of rotating machine**:

$$\eta_{em} \cong \frac{P_{\text{mech}}}{P_1^p} = \frac{P_{\text{mech}}}{P - P^n - P_H^p}, \tag{32}$$

where: P_{mech} – mechanical power at the motor shaft, P_n – negative sequence active power loss, P_H^p – active power loss of higher harmonics with positive sequence, P_1^p – effective active power transferred to the electromechanical machine.

This is the efficiency of a motor when powered by a symmetrical three-phase sinusoidal voltage. It is consistent with the catalogue efficiency provided by the motor manufacturers. Subtracting P_n and P_H^p in the denominator causes the η_{em} value to ignore the effects of higher harmonics and power source unbalance. Equation (32) shows the efficiency of the machine itself, without taking into account external factors from the power source.

There is a known description of power losses [19, 20] generated in an asynchronous machine (Fig. 6). As noted in this article, the power losses in this machine are also affected by interference from the power grid.

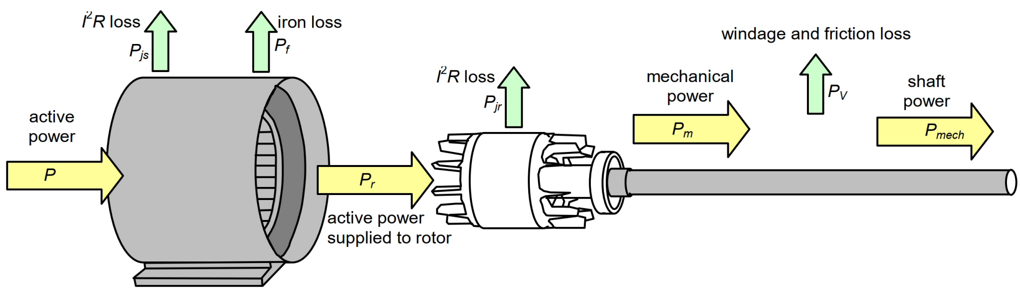

Fig. 6. Power losses in an induction motor.

The principle of operation of the machine from Fig. 6 is based on the conversion of electrical energy with precisely defined parameters. The energy that actively influences the conversion of electrical energy into mechanical energy is the value P_1^p. The remaining power components (Fig. 7) disturb the stator field symmetry.

Ignoring the influence of reactive power and other powers that are complementary to the apparent power balance (written down according to specific principles of the adopted power theory), the relationship (32) can be used. The appearance of components P_n and P_H^p worsens the value of coefficient η_{em}. These powers, after summing up $P^n + P_H^p$, despite being part of the active power P, do not have a positive impact on energy transformations. It can be called non-effective active power. They contribute to the losses shown in Fig. 6, so it is difficult to associate the images shown in Fig. 6 and Fig. 7 together into a common figure.

Increasing the source voltage frequency will increase hysteresis losses in both the stator and the rotor. The opposite direction of the symmetrical component P_n causes an increase in the braking electromagnetic torque.

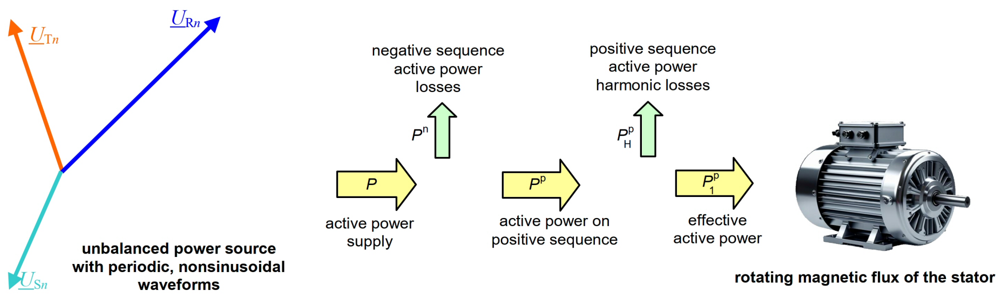

Fig. 7. Powering the electric motor from an asymmetric power source.

3.1. Calculation example

In the three-wire line in Fig. 1, the voltages in phase R and S were measured. Then, using a Fourier series, the interpretation of these voltages was determined in the form of mathematical notation in the time domain:

$$u_R = \sqrt{2} \cdot \{300 \sin(\omega_1 t) + 30 \sin(5\omega_1 t) + 3 \sin(7\omega_1 t)\} \text{ V},$$
$$u_S = \sqrt{2} \cdot \{250 \sin(\omega_1 t - 90°) + 25 \sin(5\omega_1 t + 90°) + 2.5 \sin(7\omega_1 t - 60°)\} \text{ V}.$$

As a result of these operations, the voltage waveforms can be presented in Fig. 8.

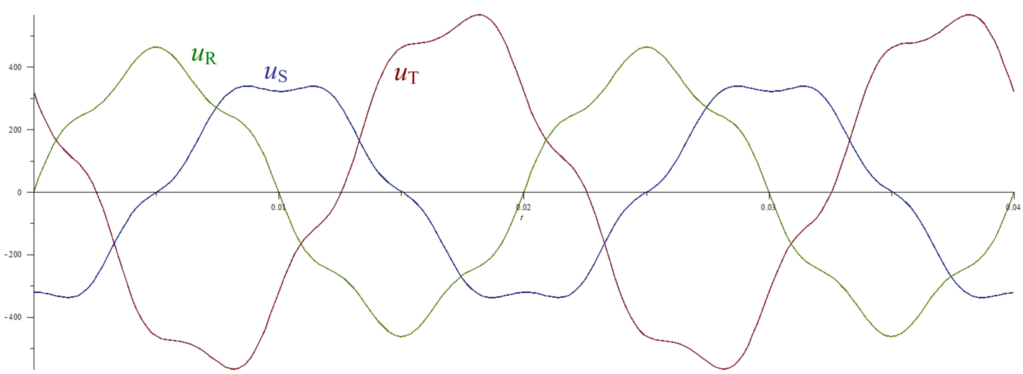

Fig. 8. Voltage waveforms of phases.

In phase T, the voltage is determined according to the relationship $\underline{U}_{Tn} = -\underline{U}_{Rn} - \underline{U}_{Sn}$ for each harmonic individually:

$$\underline{\boldsymbol{U}}_1 = \begin{bmatrix} \underline{U}_{R1} \\ \underline{U}_{S1} \\ \underline{U}_{T1} \end{bmatrix} = \begin{bmatrix} 300 \\ 250e^{-j90°} \\ 50\sqrt{61}e^{j140.2°} \end{bmatrix} \text{ V}, \quad \underline{\boldsymbol{U}}_5 = \begin{bmatrix} \underline{U}_{R5} \\ \underline{U}_{S5} \\ \underline{U}_{T5} \end{bmatrix} = \begin{bmatrix} 30 \\ 25e^{j90°} \\ 5\sqrt{61}e^{-j140.2°} \end{bmatrix} \text{ V},$$
$$\underline{\boldsymbol{U}}_7 = \begin{bmatrix} \underline{U}_{R7} \\ \underline{U}_{S7} \\ \underline{U}_{T7} \end{bmatrix} = \begin{bmatrix} 3 \\ 2.5e^{-j60°} \\ 0.5\sqrt{91}e^{j153°} \end{bmatrix} \text{ V}.$$

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The symmetrical voltage components are determined from (4) and are:

$$\underline{U}_n^p = \frac{1}{3}\left(\underline{U}_{R,n} + \alpha^n \underline{U}_{S,n} + \alpha^{2n}\underline{U}_{T,n}\right),$$

$$\left[\underline{U}_1^p \middle| \underline{U}_5^p \middle| \underline{U}_7^p\right] = \left[306.81 e^{j16.4^\circ} \middle| 30.68 e^{-j16.4^\circ} \middle| 3.18 e^{j30^\circ}\right] \text{ V},$$

$$\underline{U}_n^n = \frac{1}{3}\left(\underline{U}_{R,n} + \alpha^{2n} \underline{U}_{S,n} + \alpha^{n}\underline{U}_{T,n}\right),$$

$$\left[\underline{U}_1^n \middle| \underline{U}_5^n \middle| \underline{U}_7^n\right] = \left[86.79 e^{-j86.26^\circ} \middle| 8.68 e^{j86.26^\circ} \middle| 1.61 e^{-j81.05^\circ}\right] \text{ V}.$$

Analogously to the voltage measurement method, currents were measured in phases R and S. The current in phase T is equal $\underline{I}_{\mathrm{T}n} = -\underline{I}_{\mathrm{R}n} - \underline{I}_{\mathrm{S}n}$. Then, complex forms and values of symmetrical components were determined:

$$i_R = \sqrt{2} \cdot \{47544.75 \sin(\omega_1 t - 85.43^\circ) + 39.87 \sin(5\omega_1 t - 89.96^\circ) + 3.79 \sin(7\omega_1 t - 89.97^\circ)\} \text{ mA},$$

$$i_S = \sqrt{2} \cdot \{4658.53 \sin(\omega_1 t - 178.91^\circ) + 16.7 \sin(5\omega_1 t + 0.04^\circ) + 1.16 \sin(7\omega_1 t - 149.97^\circ)\} \text{ mA},$$

$$\underline{\boldsymbol{I}}_1 = \begin{bmatrix} \underline{I}_{\mathrm{R}1} \\ \underline{I}_{\mathrm{S}1} \\ \underline{I}_{\mathrm{T}1} \end{bmatrix} = \begin{bmatrix} 47544.75 e^{-j85.43^\circ} \\ 4658.53 e^{-j178.91^\circ} \\ 47489.83 e^{j88.95^\circ} \end{bmatrix} \text{mA}, \quad \underline{\boldsymbol{I}}_5 = \begin{bmatrix} \underline{I}_{\mathrm{R}5} \\ \underline{I}_{\mathrm{S}5} \\ \underline{I}_{\mathrm{T}5} \end{bmatrix} = \begin{bmatrix} 39.87 e^{-j89.96^\circ} \\ 16.7 e^{j0.04^\circ} \\ 43.23 e^{j112.76^\circ} \end{bmatrix} \text{mA},$$

$$\underline{\boldsymbol{I}}_7 = \begin{bmatrix} \underline{I}_{\mathrm{R}7} \\ \underline{I}_{\mathrm{S}7} \\ \underline{I}_{\mathrm{T}7} \end{bmatrix} = \begin{bmatrix} 2.79 e^{-j89.97^\circ} \\ 1.16 e^{-j149.97^\circ} \\ 3.51 e^{j73.39^\circ} \end{bmatrix} \text{mA}.$$

The graphical form of the current waveforms is shown in Fig. 9.

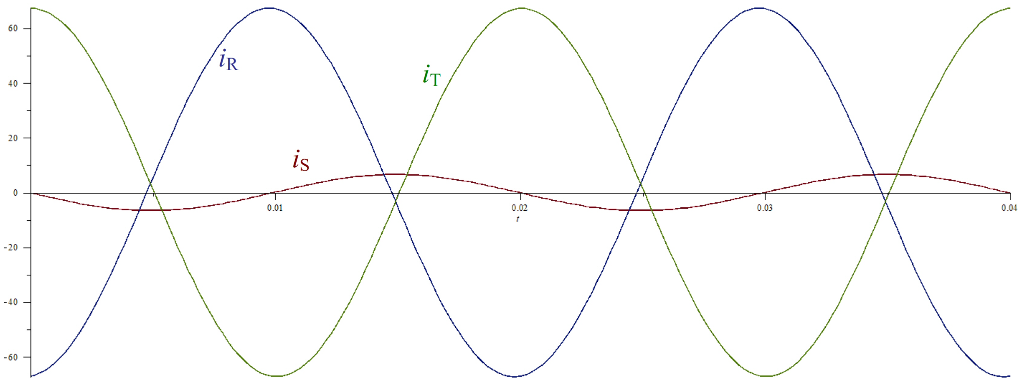

Fig. 9. Current waveforms of phases.

$$\underline{I}_n^p = \frac{1}{3}\left(\underline{I}_{R,n} + \alpha^n \underline{I}_{S,n} + \alpha^{2n}\underline{I}_{T,n}\right),$$

$$\left[\underline{I}_1^p \middle| \underline{I}_5^p \middle| \underline{I}_7^p\right] = \left[29730.25 e^{-j58.29^\circ} \middle| 31.74 e^{-j111.22^\circ} \middle| 2.28 e^{-j59.97^\circ}\right] \text{ mA},$$

$$\underline{I}_n^n = \frac{1}{3}\left(\underline{I}_{R,n} + \alpha^{2n} \underline{I}_{S,n} + \alpha^{n}\underline{I}_{T,n}\right),$$

$$\left[\underline{I}_1^n \middle| \underline{I}_5^n \middle| \underline{I}_7^n\right] = \left[25072.05 e^{-j118.17^\circ} \middle| 15.44 e^{-j41.77^\circ} \middle| 1.4 e^{-j144.5^\circ}\right] \text{ mA}.$$

The load admittances $\underline{Y}_{Ln}$ are determined from relationship (5). The effective value of the three-phase voltage is determined from (7). Active and reactive power are determined from (9). The equivalent admittance of the balanced load is determined from equation (6).

Table 1. Calculation results.

	n	1	5	7
$\underline{Y}_{Rn}$	mS	$158.48\ e^{-j85.43°}$	$1.33\ e^{-j89.96°}$	$0.93\ e^{-j89.97°}$
$\underline{Y}_{Sn}$	mS	$18.63\ e^{-j88.91°}$	$0.67\ e^{-j89.96°}$	$0.46\ e^{-j89.97°}$
$\underline{Y}_{Tn}$	mS	$121.61\ e^{-j68.63°}$	$1.11\ e^{-j86.15°}$	$0.74\ e^{-j79.97°}$
$\boldsymbol{u}_n$	V	552.27	55.23	6.16
P_n	W	12768.2	–0.49	0.003
Q_n	VAr	29844.39	3.23	0.03
G_{bn}	mS	41.86	–0.16	0.08
B_{bn}	mS	–97.85	–1.06	–0.73

A negative value of active power means that for the 5$^{\text{th}}$ harmonic there is a change in the direction of energy transfer. Thus, the total active power is:

$$P = \sum_n P_n = 12767.71 \text{ W}.$$

The resultant effective value of the three-phase voltage is as follows:

$$\|\boldsymbol{u}\| = \sqrt{\|u_1\|^2 + \|u_5\|^2 + \|u_7\|^2} = 555.06 \text{ V}.$$

The equivalent conductance of the load, determined from (11), is $G_b = 41.44$ mS.
From (25): $\|\boldsymbol{i}_a\| = G_b\,\|\boldsymbol{u}\| = 23$ A,
from (26): $\|\boldsymbol{i}_r\| = \sqrt{B_{b1}^2\,\|\boldsymbol{u}_1\|^2 + B_{b5}^2\,\|\boldsymbol{u}_5\|^2 + B_{b7}^2\,\|\boldsymbol{u}_7\|^2} = 54$ A,
from (27): $\|\boldsymbol{i}_s\| = \sqrt{(G_{b1} - G_b)^2\,\|\boldsymbol{u}_1\|^2 + (G_{b5} - G_b)^2\,\|\boldsymbol{u}_5\|^2 + (G_{b7} - G_b)^2\,\|\boldsymbol{u}_7\|^2} = 2.32$ A.
The resultant effective value of the three-phase current is as follows:
$\|\boldsymbol{i}\| = \sqrt{\sum_{L=\text{R,S,T}} \sum_n \left|\underline{I}_{Ln}\right|^2} = 67.36$ A,
from (28): $\|\boldsymbol{i}_u\| = \sqrt{\|\boldsymbol{i}\|^2 - \left(\|\boldsymbol{i}_a\|^2 + \|\boldsymbol{i}_s\|^2 + \|\boldsymbol{i}_r\|^2\right)} = 32.9$ A.

According to relations in (29), the obtained current values are multiplied by the effective voltage value, resulting in the following:
- the apparent power is $S = \|\boldsymbol{u}\| \cdot \|\boldsymbol{i}\| = 37389$ VA,
- the scattered power is $D_s = \|\boldsymbol{u}\| \cdot \|\boldsymbol{i}_s\| = 1289.6$ VA,
- the reactive power is $Q = \|\boldsymbol{u}\| \cdot \|\boldsymbol{i}_r\| = 29995$ VAr,
- the unbalanced power is $D_u = \|\boldsymbol{u}\| \cdot \|\boldsymbol{i}_u\| = 18263.7$ VA.

According to (30), the active power can be divided into two powers:
- $P^p = 3 \cdot Re\left\{\underline{U}_1^p \cdot \underline{I}_1^{p*} + \underline{U}_5^p \cdot \underline{I}_5^{p*} + \underline{U}_7^p \cdot \underline{I}_7^{p*}\right\} = 7226.98$ W,
- $P^n = 3 \cdot Re\left\{\underline{U}_1^n \cdot \underline{I}_1^{n*} + \underline{U}_5^n \cdot \underline{I}_5^{n*} + \underline{U}_7^n \cdot \underline{I}_7^{n*}\right\} = 5540.74$ W,
- $P^p + P^n = P = 12767.71$ W.

Metrol. Meas. Syst., Vol. 33 (2026), No. 1, pp. 1–16
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The power factor is: $\lambda = \frac{P}{S} = 0.341482, \quad \cos\phi = \frac{P_1}{S} = 0.341495.$

The efficiency of the electrical system is: $\eta_{el} = \frac{P_1^p}{S} = \frac{3 \cdot Re\left\{\underline{U}_1^p \cdot \underline{I}_1^{p*}\right\}}{S} = 0.1933.$

The power system delivers electricity to the load with an efficiency of 19%. To determine the efficiency of the process of converting electrical energy into mechanical energy, the relationship (32) should be used. For a mechanical power on the shaft equal to P_{mech}=6 kW, the electromechanical efficiency of the rotating machine is:

$$\eta_{em} = \frac{P_{\text{mech}}}{3 \cdot Re\left\{\underline{U}_1^p \cdot \underline{I}_1^{p*}\right\}} = 0.8302.$$

The overall efficiency of this engine is: $\eta = \frac{P_{\text{mech}}}{P} = 0.4699.$

Power losses resulting from harmonics and asymmetry are:

– $P_H^p = 3 \cdot Re\left\{\underline{U}_5^p \cdot \underline{I}_5^{p*} + \underline{U}_7^p \cdot \underline{I}_7^{p*}\right\} = 0.25$ W,
– $P^n = 5540.74$ W.

The asymmetry of the power source causes a decrease in the efficiency η, which is visible in the disturbance of the magnetic flux symmetry and the creation of an additional braking torque.

4. Conclusions

The use of the CPC power theory, together with the Fortescue transformation, allows for the determination of power components that depend on physical phenomena that increase the current flow in a three-phase circuit. It becomes possible to estimate the influence of the source asymmetry factor, with unbalanced load, scattered conductance and phase reactance, on the supply current value. Knowing the useful power, it becomes possible to determine the electromechanical efficiency of the rotating machine $\frac{P_{\text{mech}}}{P_1^p}$, *i.e.* one that does not take into account the influence of higher harmonics and power source asymmetry. Thanks to this information, it is possible to select the optimal power of the machine, assuming symmetry or lack thereof in the three-phase voltage. According to the analysis presented, it can be seen that in practice, the greatest impact on the deterioration of the efficiency of a rotating machine is the asymmetry of the power source. This is due to the fact that the negative sequence components produce a rotating flux vector correlated opposite to the direction of machine operation. The effect of this vector works analogously to the countercurrent braking process. The fundamental component in the opposite direction P_1^n has the greatest influence on the braking process. Higher harmonics contribute less to efficiency degradation. Higher harmonics increase iron losses, while voltage asymmetry increases copper losses.

SYMBOLS

$\mathbf{1^p}$,$\mathbf{1^n}$,$\mathbf{1^z}$	three-phase unit vectors: positive/negative/zero sequence
α	complex rotation coefficient
B	susceptance, S
B_b	susceptance of an equivalent balanced load, S
D_s	scattered power, VA
D_u	unbalanced power, VA
G	conductance, S
G_b	conductance of a balanced resistive load, S
i, u, e	instantaneous value of the current/voltage/electromotive force, A, V, V

$\boldsymbol{i}, \boldsymbol{u}, \boldsymbol{e}$	three-phase vector of instantaneous *values of the current/voltage/ electromotive force*, A, V, V
I, U	rms value of current/voltage, A, V
$\boldsymbol{I}, \boldsymbol{U}$	three-phase vector of rms values of the current/voltage, A, V
$\underline{\boldsymbol{I}}, \underline{\boldsymbol{U}}$	three-phase vector of crms values of the current, A, V
$\boldsymbol{i_a}$	active component of the current - three-phase vector, A
$\boldsymbol{i_r}$	reactive component of the current - three-phase vector, A
$\boldsymbol{i_s}$	scattered component of the current - three-phase vector, A
$\boldsymbol{i_u}$	unbalanced component of the current - three-phase vector, A
k	natural number, 0,1,2,...
λ	power factor
η_{el}	efficiency of the electrical system
η_{em}	electromechanical efficiency of the rotating machine
N	set of harmonics
P	active power, W
P_{mech}	mechanical power at the motor shaft, W
Q	reactive power, VAr
S	apparent power, VA
t	time, s
ω_1	fundamental pulsation, rad/s
$\underline{Y}$, $\underline{Z}$	complex value of admittance/impedance, S, Ω
Y_b	admittance of an equivalent balanced load, S
V_N	star point potential, V

Subscripts and superscripts

R,S,T,N	phase and neutral wires
L	phase indication{R, S, T}
$(^p)$, $(^n)$, $(^z)$	symmetrical components of the positive/negative/zero sequence
$(_n)$	harmonic order

Acronyms

CPC	Current's Physical Components
crms	Complex Root Mean Square
rms	Root Mean Square
LTI	linear and time-invariant
LTIq	linear and quasi-stationary and time-invariant

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