

THEORETICAL DERIVATION AND ANALYSIS OF THE TRANSFER FUNCTION FOR A HEAT FLUX SENSOR BASED ON ONE-DIMENSIONAL HEAT CONDUCTION

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Abstract

The dynamic response characteristics of sensors in heat flux measurement, such as rising time and frequency band width, determine the capture ability and measurement error of transient thermal load. In order to quantitatively evaluate the dynamic response characteristics of the heat flux sensor, including rise time and frequency band width, based on the differential equation constructed by one-dimensional heat conduction, an analytical expression of the temperature difference between the front and rear surfaces of the sensor sensitive element is derived. Furthermore, the heat flux-temperature transfer function model is established by differential operation and the Laplace transform, and the amplitude-frequency response characteristics of the system are analysed accordingly. The results of theoretical analysis are verified by finite element simulation, and the dynamic analysis characteristics of two types of heat flux sensors are analysed. Finally, the dynamic performance index of a plug calorimeter with an adiabatic rear wall is defined. The modelling method and quantitative conclusion put forward in this study provide a theoretical basis for the dynamic characteristic optimisation and system design of high-precision heat flux sensors.

Keywords: heat flux measurement, dynamic response, frequency band width, one-dimensional heat conduction.

1. Introduction

As important parameters to characterise heat transfer per unit area in unit time, the measurement and application of the heat flux have been widely concerned in the research and development of energetic materials and aerospace fields in recent years [1–3].

In these application scenarios, the heat flux changes rapidly, reaching a peak in a few milliseconds, which puts strict demands on the dynamic performance of measurement. In order to improve the dynamic response of heat flux measurement, transient heat flux sensors usually adopt non-water cooling difference layered structure and measure the temperature gradient of components with known thermal properties (such as thermal conductivity, specific heat capacity, density and geometry) to quantify the heat flux absorbed by the sensitive surface [4–6]. The heat flux sensor based on the thin film thermopile and the heat flux sensor based on the atomic layer thermopile are representative examples of transient sensors with fast response ability.

The thin-film thermal resistance heat flux sensor is a variant of the Schmidt-Bolter heat flux sensor which calculates the heat flux by the temperature difference between the front and back surfaces of the thermal resistance layer [7–10]. Because the thinner thermal resistance layer corresponds to better dynamic characteristics, the micron thickness thermal resistance layer is deposited on the millimetre thickness substrate by the micromachining process, and the surface temperature difference between the two sides of the thermal resistance layer is measured by a thermopile. The thermal resistance layer is usually made of low thermal conductivity materials, such as SiO_2 , while the substrate is made of ceramic materials, such as alumina or aluminium nitride. This configuration gives the sensor response time on the order of microseconds.

The transient heat flux sensor of the atomic layer thermopile also works according to the principle of one-dimensional heat conduction in solid. Specifically, this can be achieved using materials [11–13] whose thermal resistance layer has a lateral Seebeck effect, such as YBCO [4, 14] or PbTe. The transverse voltage generated in the thermal resistance layer is perpendicular to the temperature gradient. The atomic layer thermopile film plays the dual role of thermal resistance layer and temperature difference signal output, without additional temperature-sensitive elements. Therefore, the thickness of the thermal resistance layer can be kept at sub-micron level (hundreds of nanometres).

The reported research on the transient heat flux sensor focuses mainly on structural design and static calibration [15–18]. The basic analysis is usually based on the Fourier law of heat conduction, which defines heat flux as the product of the temperature gradient and thermal conductivity [19, 20]. However, the boundary conditions in the process of heat flux measurement are rarely considered, and there is still a lack of a general theoretical model to quantify the dynamic characteristics. The dynamic response parameters, such as rise time and frequency bandwidth, directly determine the sensor's ability to capture transient thermal load and its measurement accuracy, so they are very important for sensor design and application [21–23]. In this paper, the dynamic response characteristics of the heat flux sensor based on the temperature gradient are studied by analysing one-dimensional heat conduction in a solid. By introducing the boundary conditions with heat flux measurement, the differential equation is established and the transfer function of the system is derived. The dynamic behaviour in the process of heat flux measurement is further analysed by the finite frequency domain response. The proposed dynamic characteristic modelling framework provides a quantifiable design guide for the structure of a high-precision heat flux sensor.

The organization of this paper is as follows: In the Section 2, the principle and mathematical model of the heat flux sensor based on temperature gradient are expounded. In Section 3, the temperature distribution of sensitive elements under step heat flux input is solved, and the system transfer function is obtained. In Section 4, the dynamic characteristics of two typical heat flux sensor configurations are analysed.

2. Principle of heat flux measurement based on temperature gradient

The sensitive part of a heat flux sensor typically comprises a thermal resistance layer flanked by thermoelectric conversion elements. The thermal resistance layer transforms the absorbed heat flux into a temperature gradient, that is, the temperature difference between its two sides. Thermoelectric conversion elements are typically implemented using thin-film thermopiles or thin-film thermistors to convert temperature signals into measurable electrical signals. Given that the thermoelectric elements are much thinner than the thermal resistance layer, their thermal influence is negligible. Accordingly, the term “sensitive element” in this paper specifically refers to the thermal resistance layer. Based on the above simplification, the basic structure of the heat flux

sensor based on one-dimensional temperature gradient is shown in Fig. 1. The sensitive element is usually made of metal or ceramic materials, the front surface of which is subjected to time-varying thermal load through a radiation or convection energy transfer mechanism, and the back side is an adiabatic boundary. By measuring the axial temperature gradient of the sensitive element, we inferred the time-varying heat flux $q(t)$ absorbed by the front surface.

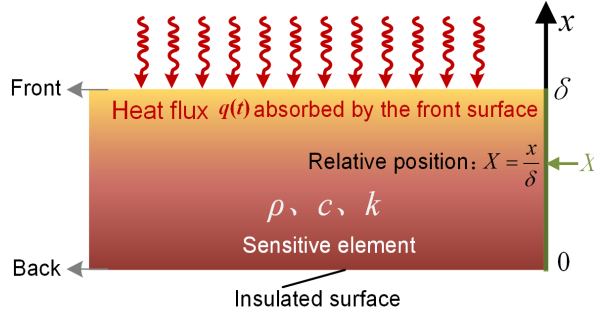


Fig. 1. Schematic diagram of the heat flux measurement model based on the temperature gradient.

In Fig. 1 where k denotes the thermal conductivity of the sensitive element [$\text{W}(\text{m} \cdot \text{K})$]; ρ is the density [kg/m^3]; c denotes the specific heat capacity [$\text{J}/(\text{kg} \cdot \text{K})$]; α is the thermal diffusivity $\alpha = k/\rho c$ [m^2/s]; δ is the thickness of the sensitive element [m]; $q(t)$ represents the heat flux absorbed by the front surface [W/m^2]; X is the is a dimensionless number, which is the ratio of the axial position (x) to the thickness of the sensitive element.

A partial differential equation describing the above heat transfer process is as follows:

$$\frac{\partial T(x, t)}{\partial t} = \alpha \frac{\partial^2 T(x, t)}{\partial x^2}. \quad (1)$$

The boundary condition of the front surface of the sensitive element is modelled as:

$$k \frac{\partial T(x, t)}{\partial x} \Big|_{x=\delta} = q(t). \quad (2)$$

Conversely, the back-surface boundary condition of the sensitive element is modelled as:

$$\frac{\partial T(x, t)}{\partial x} \Big|_{x=0} = 0. \quad (3)$$

The initial conditions of the heat conduction process are as follows:

$$T(x, 0) = T_0. \quad (4)$$

(1) through (4) jointly satisfy the principles of homogeneity and superposition. Assuming that the thermal properties of the sensitive elements do not change with temperature, this dual satisfaction of linearity and time invariance makes the system a *linear time-invariant* (LTI) system. Therefore, the dynamic characteristics of the system can be fully represented by the transfer function.

3. Transfer function

For the transfer function model of uncooled calorimeters based on temperature gradients, the input is the heat flux absorbed by the front surface of the sensitive element and the output is the temperature difference between two distinct axial cross-sections. This input-output relationship enables the derivation of the transfer function of the measurement system, which quantifies the system's dynamic response to thermal excitation. The transfer function is derived through the following three-step procedure:

- 1) The step response is first obtained by calculating the transient temperature distribution under a step heat flux excitation.
- 2) The impulse response is then derived by differentiating the step response.
- 3) Finally, the transfer function is obtained by applying the Laplace transform to the impulse response.

3.1. Solution of equation under constant heat flux boundary condition

Replace the time-varying heat flux of the front surface with a constant heat flux, that is $q(t) = q_0$, solve the partial differential (1) by separating variables. Define dimensionless quantities: relative temperature θ , relative position X and the Fourier number Fo :

$$\theta = \frac{k(T - T_0)}{(q_0\delta)}, \quad X = \frac{x}{\delta}, \quad Fo = \frac{\alpha t}{\delta^2}. \quad (5)$$

Equations (1) to (4) are transformed into (6) to (9), respectively:

$$\frac{\partial \theta(X, Fo)}{\partial Fo} = \frac{\partial^2 \theta(X, Fo)}{\partial X^2}, \quad (6)$$

$$\left. \frac{\partial \theta}{\partial X} \right|_{X=0} = 0, \quad (7)$$

$$\left. \frac{\partial \theta}{\partial X} \right|_{X=1} = 1, \quad (8)$$

$$\theta(X, 0) = 0, \quad (9)$$

The solution of (6) that is, (10), establishes an explicit analytical relationship between dimensionless temperature θ and the Fourier number Fo .

$$\theta(X, Fo) = \frac{k(T - T_0)}{q_0\delta} = Fo + \frac{X^2}{2} - \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2}{k_n^2} (-1)^{n+1} \cos(k_n X) e^{-k_n^2 Fo}, \quad (10)$$

where $k_n = n\pi$ ($n = 1, 2, \dots, n$).

The relationship between the dimensionless temperature θ and the Fourier number Fo is illustrated in Fig. 2.

Furthermore, by substituting the defined dimensionless number (5) into (10), the temperature distribution of the sensitive element is obtained:

$$T(x, t) = T_0 + \frac{q_0\delta}{k} \left[\frac{\alpha t}{\delta^2} + \frac{1}{2} \left(\frac{x}{\delta} \right)^2 - \frac{1}{6} + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi)^2 \frac{\alpha t}{\delta^2}} \cos \left(n\pi \frac{x}{\delta} \right) \right]. \quad (11)$$

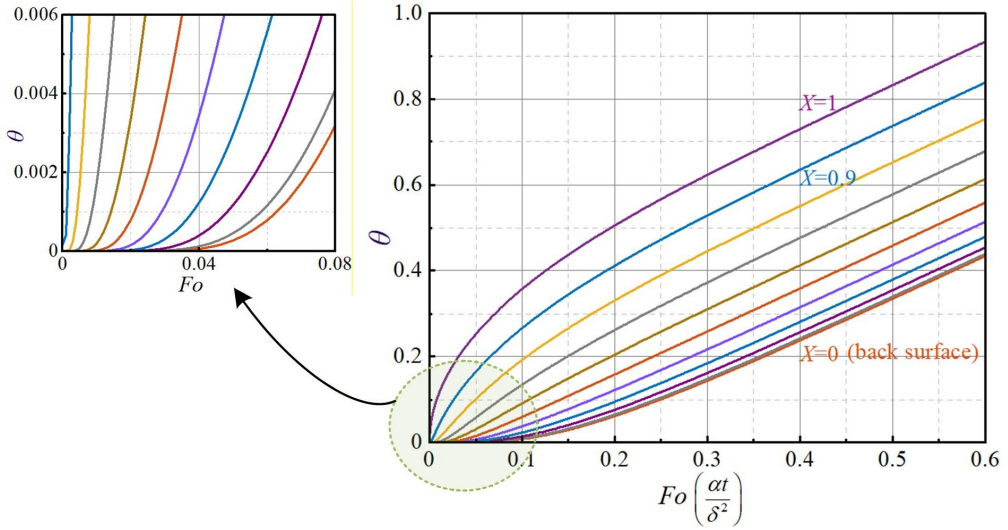


Fig. 2. Relationship between relative temperature and the Fourier number.

Heat flux at different positions is obtained by differentiating (11) according to the Fourier law of heat conduction:

$$\frac{\partial T}{\partial x} = \frac{q_0}{k} \left[\frac{x}{\delta} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{-(n\pi)^2 \frac{at}{\delta^2}} \sin(n\pi \frac{x}{\delta}) \right]. \quad (12)$$

For $Fo < 0.04$, temperature of the back surface of the sensitive element hardly changes ($\theta(0, 0.04) \approx 2.9 \times 10^{-5}$), indicating that the front surface heat flux exerts negligible influence on the back surface. When Fo exceeds 0.6, the system achieves a regular regime in which both the temperature increase rate and heat flux remain consistent. Fig. 3 illustrates the spatial distributions of temperature and heat flux within the sensitive element under this regular regime.

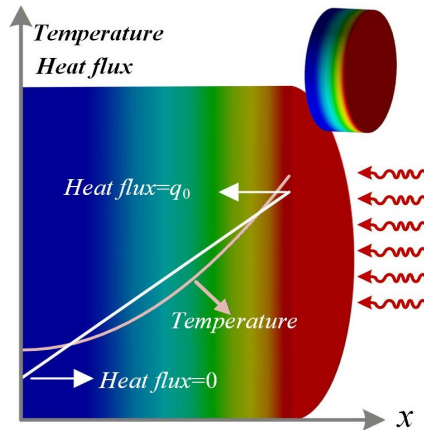


Fig. 3. Distribution of temperature and heat flux after transient heat transfer enters the regular regime.

3.2. Transfer function of the system

By substituting $x = X\delta$ into (11), the temperature distribution at the relative position is derived as follows:

$$T(X, t) = T_0 + \frac{q_0\delta}{k} \left[\frac{\alpha t}{\delta^2} + \frac{1}{2}X^2 - \frac{1}{6} + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi)^2 \frac{\alpha t}{\delta^2}} \cos(n\pi X) \right]. \quad (13)$$

The temperature difference $\Delta T(X_1, X_2, t)$ between X_1 and X_2 is:

$$\Delta T(X_1, X_2, t) = \frac{q_0\delta}{k} \left[\frac{1}{2} (X_1^2 - X_2^2) + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi)^2 \frac{\alpha t}{\delta^2}} (\cos(n\pi X_1) - \cos(n\pi X_2)) \right]. \quad (14)$$

Further, the temperature difference of the system under unit heat flux input is as follows:

$$\Delta T_u(X_1, X_2, t) = \frac{\delta}{k} \left[\frac{1}{2} (X_1^2 - X_2^2) + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} e^{-(n\pi)^2 \frac{\alpha t}{\delta^2}} (\cos(n\pi X_1) - \cos(n\pi X_2)) \right]. \quad (15)$$

In (15), when $t \rightarrow \infty$ or $Fo > 0.6$, the steady-state value is only determined by the non-decaying linear term, and $\Delta T_u(X_1, X_2, \infty)$ represents the steady-state response of the system to the unit heat flux, that is, the sensitivity of the system is shown in (16).

$$\Delta T_u(X_1, X_2, \infty) = \frac{\delta}{2k} (X_1^2 - X_2^2). \quad (16)$$

The unit impulse response of the system is obtained by taking the derivative of (16):

$$\frac{d(\Delta T_u)}{dt} = \frac{2\alpha}{k\delta} \sum_{n=1}^{\infty} (-1)^n e^{-(n\pi)^2 \frac{\alpha t}{\delta^2}} (\cos(n\pi X_1) - \cos(n\pi X_2)). \quad (17)$$

The Laplace transformation is performed on (17), and the transfer function of the system is obtained as follows:

$$H(s) = \mathcal{L} \left\{ \frac{d(\Delta T_u)}{dt} \right\} = \frac{2\alpha}{k\delta} \sum_{n=1}^{\infty} \frac{(-1)^n (\cos(n\pi X_1) - \cos(n\pi X_2))}{s + \frac{\alpha(n\pi)^2}{\delta^2}}. \quad (18)$$

Equation (18) shows that the transfer function of the system is a series composed of countless first-order systems. If the first N terms are truncated, the following equation is obtained:

$$H(s) = \frac{2\alpha}{k\delta} \sum_{n=1}^N \frac{(-1)^n (\cos(n\pi X_1) - \cos(n\pi X_2))}{s + \frac{\alpha(n\pi)^2}{\delta^2}} \quad (19)$$

Essentially, the transfer function can strictly describe the dynamic behaviour of the system only when N tends to infinity. However, considering resource consumption in the calculation and the allowable error in the engineering application, it is very important to understand the approximation ability of the model when N is expressed in finite terms. When N increases, more items are included and the weight of each item is determined by the numerator and denominator.

Although the denominator increases with n^2 , it cannot be simply assumed that only the first few terms of the series can describe the low-frequency characteristics of the system more accurately, especially when X_1 and X_2 are close, because the molecules of the subitems may amplify some frequency components when N increases.

Substituting $s = 0$ into the transfer function (19) yields (20), which represents the DC gain of the system. It is equal to (16), represents the static proportional relationship between the input signal and the output signal, which is the sensitivity of the system. When $N \rightarrow \infty$

$$H(0) = \frac{\delta}{2k} (X_1^2 - X_2^2). \quad (20)$$

4. Frequency characteristics of the system

Letting $s = j\omega$ in (19), the frequency response is derived as (21):

$$H(j\omega) = \frac{2\alpha}{k\delta} \sum_{n=1}^N \frac{(-1)^n (\cos(n\pi p_1) - \cos(n\pi p_2))}{j\omega + \frac{\alpha(n\pi)^2}{\delta^2}} \quad (21)$$

Further, the amplitude-frequency and phase-frequency characteristic equations of the system are obtained as shown:

$$|H(j\omega)| = \left| \frac{2\alpha}{k\delta} \sum_{n=1}^N \frac{(-1)^n (\cos(n\pi p_1) - \cos(n\pi p_2))}{j\omega + \frac{\alpha(n\pi)^2}{\delta^2}} \right| \quad (22)$$

$$\angle H(j\omega) = \arg \left(\frac{2\alpha}{k\delta} \sum_{n=1}^N \frac{(-1)^n (\cos(n\pi p_1) - \cos(n\pi p_2))}{j\omega + \frac{\alpha(n\pi)^2}{\delta^2}} \right) \quad (23)$$

For any dynamic measurement system, it should be used under the condition of meeting the requirements of undistorted measurement, that is, its amplitude-frequency response should be constant and its phase-frequency response should be linear in a certain frequency band. Generally, the bandwidth of the system is quantized with a -3 dB bandwidth, which indicates the frequency range when the amplitude of the frequency response drops to $1/\sqrt{2}$ of the maximum.

In the following section, the dynamic responses of two representative heat flux sensor configurations are analysed. The first configuration ($X_1 = 1$ & $X_2 = 0$) determines the heat flux by measuring the surface temperature gradient across the sensitive element, this is essentially an extension of the slug calorimeter, whose heat flux is inferred from the change rate of the sensitive element's back surface temperature. The second configuration (X_1 & X_2 is close to 1) represents the thermal gradient quantization in shallow depth (usually < 100 microns), and this is the typical feature of modern thin film heat flux sensor. It should be noted that these two types of heat flux sensors are designed for measuring spatial heat flux, rather than the actual heat transfer rate to the target object. For such applications, the thermal resistance of the heat flux sensor should be close to or (ideally) match the thermal resistance of the material it is mounted on or embedded in.

4.1. Temperature difference between the front and back surfaces of the sensitive element

A closed-loop Nyquist diagram can show the gain and phase information of the system at different frequencies, while a Bode diagram shows the amplitude-frequency and phase-frequency characteristics, respectively. Combining the two can comprehensively evaluate the system's performance. Assuming that the sensitive element material is copper, the material parameters are listed in Table 1.

Table 1. Thermophysical parameters of the copper.

k [W/(m · K)]	ρ [kg/m ³]	c [J/(kg · K)]	α [m ² /s]	δ [m]
400	8933	385	1.163×10^{-4}	0.005

The closed-loop Nyquist diagram and Bode diagram of the system are illustrated in Figs. 4(a) and (b), respectively, when $X_1 = 1$, $X_2 = 0$ and N take different values.

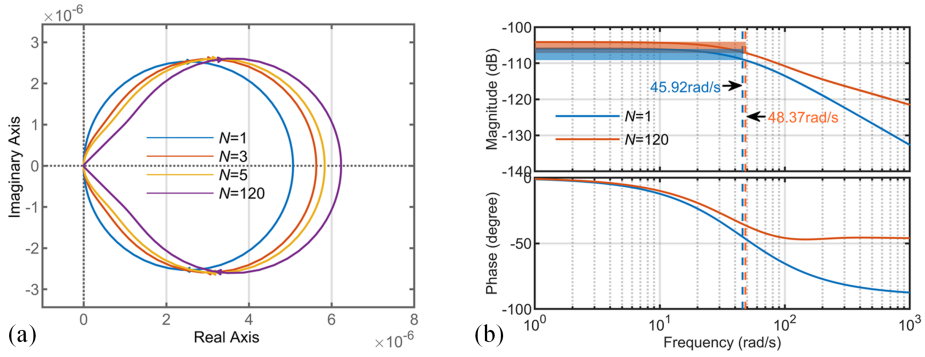


Fig. 4. Frequency characteristics of the system with different values of N : (a) closed-loop Nyquist diagram, (b) Bode diagram of the system.

In the Nyquist diagram, when the imaginary axis is 0, the value of the real axis represents the DC gain of the system. Taking the first term of the series $N = 1$, the transfer function is shown in (24):

$$H(s) = \frac{4\delta}{k\pi^2} \frac{1}{1 + \frac{\delta^2}{\alpha\pi^2}s} \quad (24)$$

This system is a typical first-order system, thus its dynamic performance can be characterised by a time constant and a bandwidth of -3 dB. The time constant of this system is as follows:

$$\tau = \frac{\delta^2}{\alpha\pi^2} \quad (25)$$

The Fourier number corresponding to the above time constant is:

$$Fo = \frac{1}{\pi^2} = 0.101 \quad (26)$$

The -3 dB bandwidth $Bandwidth_{-3dB}$ is:

$$Bandwidth_{-3dB} = \frac{1}{\tau} = \frac{\alpha\pi^2}{\delta^2} \quad (27)$$

For the material parameters in Table 1, when N is 1, the time constants of the system are as follows:

$$\tau = 129.97 \text{ ms}, \quad (28)$$

$$\text{Bandwidth}_{-3 \text{ dB}} = 45.92 \text{ rad/s}. \quad (29)$$

4.1.1. Time domain response of the system under step signal input

The zero-state response of the system can be obtained by (14), which needs to be approximated by infinite series. In order to evaluate the response of the system under step heat flux comprehensively, an Ansys Workbench Transient Thermal Analysis module (Release 2020 R2) is used for simulation. The sensitive element is defined as a three-dimensional plate, 15 [mm] in diameter and 5 [mm] in thickness, and is composed of copper; its thermophysical parameters are listed in Table 1. A hex-dominant meshing scheme is adopted, with local refinement applied to the front surface where the heat flux is loaded, in order to accurately resolve the thermal gradient. The mesh layout and the simulated temperature distribution are presented in Fig. 5. The global element size is controlled by the solver with adaptive sizing enabled. The resulting mesh consists of 46355 nodes and 6807 cells. A step heat flux of $q = 1 \text{ [MW/m}^2\text{]}$ is applied to the front surface, while all other surfaces are treated as adiabatic. The simulation solves the three-dimensional, unsteady Fourier heat conduction equation using the direct time-integration method available in the transient thermal analysis module.

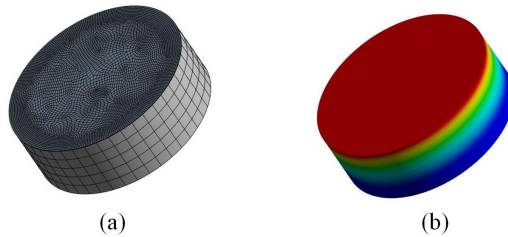


Fig. 5. Grid layout and temperature distribution of the simulation model; (a) grid layout of the sensitive element, (b) temperature distribution in the sensitive element.

The temperature distribution in the sensitive element is shown in Fig. 6.

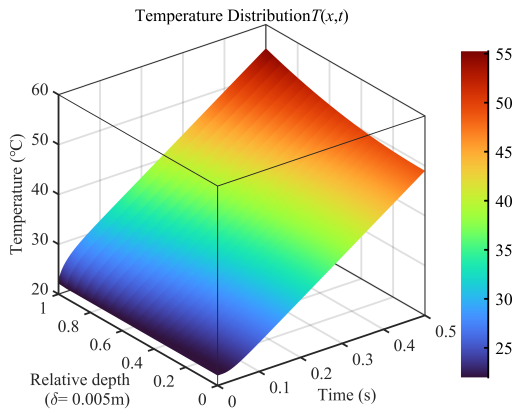


Fig. 6. Temperature distribution under step heat flux input.

When $T_f = T(\delta, t)$, $T_b = T(0, t)$, the simulation results for the temperature difference between the front and back surface are shown in Fig. 7.

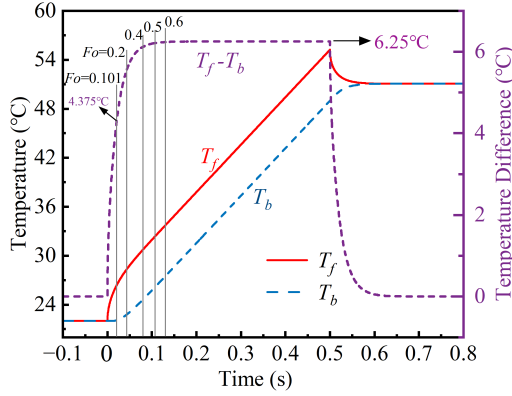


Fig. 7. Simulation results of the temperature difference between the front and back surfaces.

As illustrated in Fig. 8, with time increasing, the temperature distribution in the sensitive element gradually stabilises into a regular regime state. When the Fourier number exceeds 0.6, the temperature difference between the front and back surface remains constant, with a measured value of 6.25 [°C]:

$$\Delta T(X_1, X_2, \infty) = \frac{q_0 \delta}{2k}. \quad (30)$$

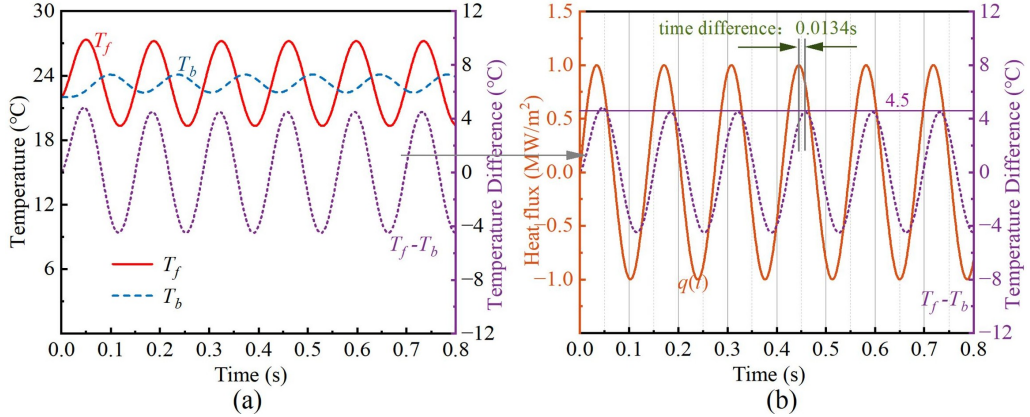


Fig. 8. Temperature difference resulting from a dynamic heat flux applied to the front surface; (a) temperature and temperature difference between the front and back surfaces, (b) phase correlation between the heat flux of the front surface and the temperature difference between the front and back surfaces.

For the standard first-order system, the step response of the system at the time constant is 0.707 of the steady-state value, while according to the simulation results, the step response of the system at the time constant is 0.7, which reflects the slight error caused by the truncation of the transfer function.

4.1.2. Response of the system under periodic signal input

When sinusoidal heat flux excitation as shown in (31) is applied to the front surface of the sensitive element:

$$q_{\sin}(t) = q_0 \sin [(\omega_{-3dB})t] . \quad (31)$$

Let $\omega_{-3dB} = 45.92$ [rad/s], which is the result of calculation of (29). Temperature and temperature difference between the front and rear surfaces of the system, and curves between temperature difference and input heat flux are shown in Fig. 7a and Fig. 7b, respectively.

As illustrated in Fig. 8b, in the steady-state stage, the expression of the temperature difference between the front and rear surfaces is as follows:

$$T_f - T_b = 4.5 \times \sin [45.92(t - 0.0134)] . \quad (32)$$

Based on the analysis of the derived model, the output of the system shows 0.0134 [s] output delay and 0.614 [rad] phase shift (equivalent to 35°) relative to the input heat flux. The phase obviously deviates from the value of the first-order system 45°, indicating that the phase distortion caused by the system approximation exceeds the amplitude distortion.

The frequency characteristics of the system under the varying parameter N at $\omega = 45.92$ [rad/s] are quantitatively summarised in Table 2.

Table 2. Amplitude and phase of the system with different values of N when the angular frequency is 45.92 rad/s.

Truncation number	Frequency [rad/s]	Magnitude $\times 10^{-6}$	Phase [degree]
$N = 1$	45.92	3.58	-45
$N = 60$	45.92	4.52	-35.25
Ansys Simulation	45.92	4.5	-35.15

The analytical findings demonstrate that with an increase in N , the -3 dB bandwidth of the system gradually widens, the DC gain gradually increases, and the phase corresponding to the -3 dB bandwidth gradually decreases. Numerical simulation shows that the fidelity of numerical calculation data is higher than that of first-order model approximation. Nevertheless, when the first-order modal approximation is applied to the transfer function, the system bandwidth becomes narrower than the actual -3 dB bandwidth. This indicates that the approximated system demonstrates superior amplitude-frequency characteristics compared to the physical system. Therefore, the -3 dB bandwidth derived from the first-order modal approximation can effectively characterise the system's dynamic performance.

4.2. Shallow resistance heat flux sensor

In this case, a thermal resistance layer with low thermal conductivity is usually deposited on a high thermal conductivity substrate, as shown in Fig. 9. In order to simplify the analysis of the dynamic response of the system, it is assumed that the thermal resistance layer and the substrate have the same material characteristics.

For this kind of structure, $X_1 = 1$, and X_2 is a value close to 1, since the thermal resistance layer is generally thin. When $X_2 = 0.99$, the step response is shown in Fig. 10. The frequency response characteristics of different values of N are shown in Fig. 11.

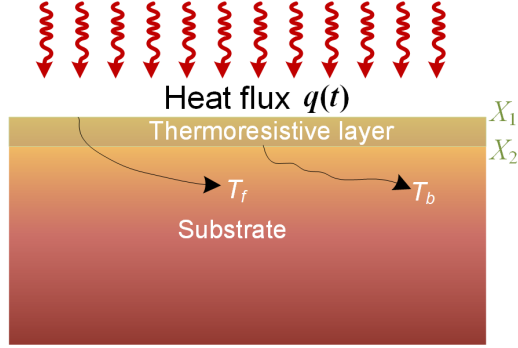


Fig. 9. Schematic diagram of the sensor with the substrate.

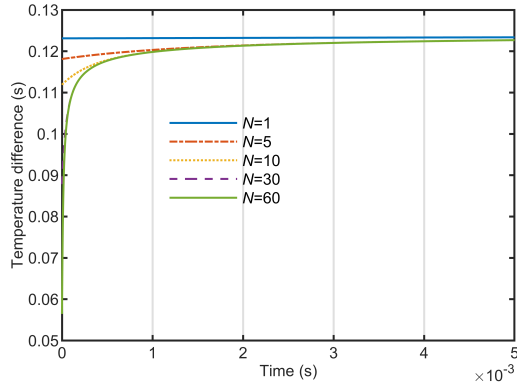


Fig. 10. Step response with different values of N

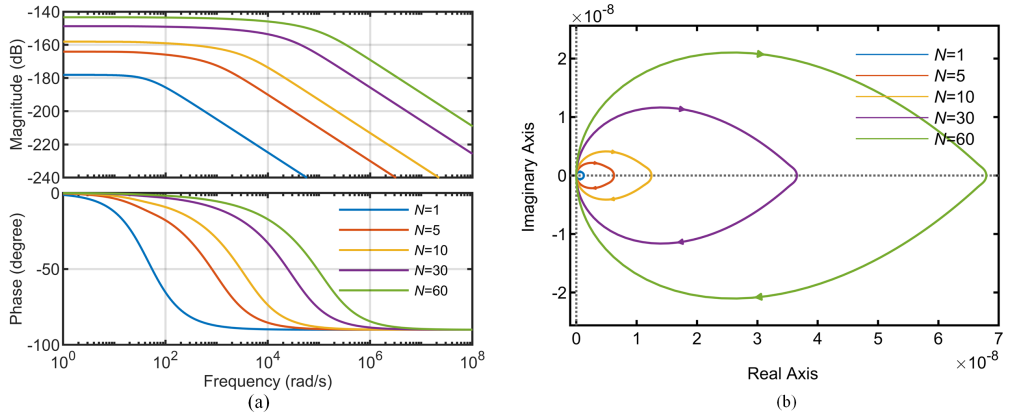


Fig. 11. Frequency domain response of the system when using different N to approximate the model; (a) Bode diagram, (b) closed-loop Nyquist diagram.

Analysis results demonstrate that as the number of participating modes increases, the system's –3 dB bandwidth exhibits significant enhancement, at the same time, the DC gain is monotonically increased until reaching the system's theoretical limit:

$$\Delta T(X_1, X_2, \infty) = \frac{q\delta}{2k}(X_1^2 - X_2^2). \quad (33)$$

For $N = 120$, the transient term becomes so small that it underflows the effective range of floating-point numbers, introducing numerical errors, therefore, use $N = 60$ to ensure computational stability. Under $N = 60$, the frequency response of the system at different X_2 is shown in Fig. 12. As the thickness of the thermal resistance layer decreases, the system bandwidth gradually increases while the sensitivity continuously diminishes, which means that the temperature difference becomes smaller. To compensate for the diminished signal, multiple thermopiles may be arranged on both sides of the thermal resistance layer. However, this enhancement strategy requires balancing improvements in sensitivity with potential increases in response time.

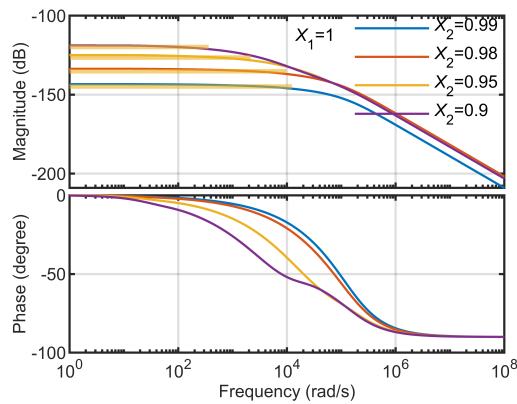


Fig. 12. Bode diagram of the thermal resistance layer system with varying thickness.

Assuming the material parameters specified in Table 1 with $N = 60$, Table 3 presents the system bandwidth values corresponding to the varying thickness of the thermal resistance layer.

Table 3. Bandwidth with various thickness of the thermal resistance layer.

X_2	Sensitivity $\times 10^{-7}$ [K/(W/m ²)]	–3 dB Bandwidth [rad/s]	Phase [degree]
0.99	1.24	13759.46	-20.25
0.98	2.48	8395.36	-19.33
0.95	6.09	1798.84	-19.42
0.9	11.88	553.44	-20.23

The finite-term approximation methodology imposes inherent constraints on the estimation of bandwidth based on the transfer function. Specifically, the actual system bandwidth invariably exceeds the calculated value derived from this truncated model. Nevertheless, sensors employing this structural configuration represent a viable strategy for improving dynamic performance.

5. Conclusions

In this study, the dynamic response characteristics of the non-cooling heat flux sensor based on a one-dimensional temperature gradient are quantified, and the following conclusions are drawn by constructing a heat transfer model and a system transfer function:

- 1) The sensitivity of the system can be accurately quantified by solving the DC gain of the transfer function, and the effective working frequency band of the system can be clearly defined by combining the modal decomposition theory to approximate the frequency response, which establishes a mathematical criterion for the design of the frequency response characteristics of the sensor.
- 2) In the temperature difference measurement mode between the absorption surface and the insulation surface, the dynamic characteristics of the system can be equivalent to the first-order inertia link, and the distortion of the phase-frequency characteristics of the system is greater than that of the amplitude-frequency characteristics. This simplified model provides a theoretically solvable engineering realisation path for transient heat flux measurement.
- 3) For the heat flux sensor with surface-deep gradient temperature measurement configuration, its high-order modal components have a significant impact on the dynamic response of the system, so the multi-order modal superposition method must be adopted to accurately characterise the essential characteristics of the system. Theoretical analysis shows that this type of structure can increase the bandwidth of the sensor by 1 to 2 orders of magnitude, which is an effective means to improve the measurement of transient heat flux.
- 4) Under some complex shapes and boundary conditions, it is difficult to establish and solve the mathematical model of the system, that is, partial differential equations. Then, finite element simulation tools can be used for auxiliary analysis. Nevertheless, the design methodology of the heat flux measurement system presented in this paper emphasises the matching relationship among material parameters, boundary conditions, and spectral characteristics of the measured heat flux in many aspects, which is still of reference significance for the design and simulation of this kind of heat flux measurement system.

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