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IDENTIFICATION OF ROTARY AXIS POSITION ERRORS OF CNC MACHINE TOOLS AND THEIR ANALYSIS USING MACHINE LEARNING TECHNIQUES

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Abstract

The paper presents the results of measurements of kinematic centre position errors of rotary axes of five-axis machine tools with a swivel rotary table, using the modern R-Test diagnostic and calibration system. The use of the system was the basis for obtaining experimental data, enabling the assessment of changes in the position of the kinematic centre of the rotary axes of the tested machine tools. The estimation was made on the basis of a statistical analysis of the measurements obtained. The measurements made it possible to determine the position errors of the rotary axes A, B, C in planes Y–Z, X–Z, X–Y and the average values of the calibration ball deviation in specific axes of machine tools. The deviation parameters of the reference ball in the machine tool space were simulated using 3 models - Multi Layer Perceptron, Kolmogorov-Arnold Networks and Multi Output Gaussian Process (MOGP). The simulation results showed that the MOGP model performed best in estimating the deviation parameters. In addition, the MOGP model achieved the best values in all metrics compared to the other models tested. Statistical analysis showed that the largest errors in the position of the kinematic centre of the Lasertec 65 machine tool's rotary axis were recorded for rotary axis A and for the measuring radius R = 150 mm (0.099 mm). The smallest errors were recorded for rotary axis C for the same radius R = 150 mm (0.0179 mm). The monoBLOCK 65 machine tool demonstrated the smallest error in the position of the kinematic centre of rotary axis A and the minimum measuring radius of R = 75 mm (0.0285 mm). The largest error of the monoBLOCK 65 machine tool was recorded for rotary axis B and the measuring radius of R = 300 mm (0.0433 mm). The scientific novelty of this work is the development of a complex expert system that enables in-depth analysis of errors in the position of the kinematic centre of the rotating axes of CNC machine tools, as well as forecasting the wear of machine components and detecting anomalies in machine operation. The expert system was developed not only for laboratory users, but also for industrial users for whom maintenance and production quality management are of paramount importance.

Keywords: CNC machine tool, rotary axes error, CNC machine tools monitoring system, KAN, Gaussian process.

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1. Introduction

The problem of identifying errors that occur in *computer numerical control* (CNC) machine tools is of critical importance to their users as well as manufacturers. Errors of numerically controlled machine tools can be evaluated on the basis of a series of measurements. For this purpose, diagnostic tests and monitoring find their application, which provide important information about the technical condition of the machine. Considering the range of applications of CNC machine tools, their complexity and high requirements, the best solution is to identify errors by monitoring and ensuring the continuous operation of machine tools. Diagnostic testing of CNC machine tools is standardized and described in Polish and foreign standards. These standards describe provisions for testing CNC machine tools and methods for determining selected deviations and tolerances. Proper source identification of CNC machine tool errors is important for optimizing preventive actions at the time of machine damage and their removal from the production process [1].

Determining CNC machine tool errors usually requires long-term measurements and multiple diagnostic systems, while standard calibration methods are mainly based on static measurements which do not reflect machine dynamics [1]. One important group of errors are kinematic centre point position errors of rotary axes which arise from changes in machine kinematics, *e.g.*, after collisions, overloads or improper setups. Such errors significantly affect dimensional and shape accuracy, and their identification and inclusion in the kinematic model allow restoring correct machine motion. However, their analysis is difficult due to complex kinematic dependencies and the lack of simple identification and prediction tools [1].

In [2], the influence of rotary axes on nonlinear errors in five-axis CNC machines was analysed for different machine configurations: *single turret* (ST), *twin turret* (TT), *twin/triple structure* (TST). The nonlinear error, caused by synchronous motion of rotary and linear axes, was shown to depend mainly on tool length, clamping position and workpiece position, with the strongest effects observed for machines with tilting tables or combined table/spindle systems. These results confirm the significant impact of rotary axis configuration and setup conditions on machining accuracy.

The literature widely addresses the determination and compensation of errors in CNC machine tools, with particular emphasis on rotary axes as major sources of machining inaccuracies. A comprehensive review in [3] discusses the growing importance of five-axis machine tools whose additional rotary axes provide significant advantages in machining complex shapes, but also introduce substantial geometric errors. These errors are commonly divided into *position-dependent geometric errors* (PDGE) and *position-independent geometric errors* (PIGE) components. PDGE vary with the rotational position, while PIGE result mainly from assembly imperfections and remain constant. For each rotary axis, ten PDGE and six PIGE components are typically considered. In practice, indirect measurement methods using systems such as R-Test, Ballbar or electrostatic probes are preferred due to their higher efficiency and accuracy compared to direct measurements.

In [4], a method for predicting spatial errors of a five-axis CNC milling machine was proposed, taking into account repeatability errors of measurement positions. Three main approaches to spatial error determination were discussed: prediction based on uniaxial error measurement and modelling, identification of uniaxial errors from spatial error measurements, and direct interpolation or fitting of spatial errors. The introduction of confidence-index weighting improved identification accuracy, reducing the maximum and average spatial error deviations. The authors also pointed out that such approaches can be further enhanced by combining them with machine learning techniques and improved scale design concepts. Related issues of error compensation and data integration in diagnostic and life-cycle support systems are discussed in [5] and [6], highlighting the increasing importance of measurement quality and effective data management in modern engineering systems.

The authors of paper [7] presents the results of geometric error compensation of a five-axis CNC machine tool using the R-Test system. The authors of the paper focused on measuring the position of rotary axes A and C and linear axes X, Y, Z. Error models of A rotary axes (ZOA, BOA, COA) and C (XOC, YOC, YOZ, AOC, BOC) were also developed (Fig. 1).

Several studies demonstrate the practical effectiveness of R-Test-based methods. The use of the R-Test system enabled a reduction of linear and rotary axis position errors by about 10 µm and simplified the verification of angular positioning and eccentricity errors, significantly reducing setup time and improving industrial applicability. In [8], geometric errors of rotary axes A and B in a five-axis machine tool were identified using the R-Test system, achieving maximum deviations between predicted and measured values below 3 µm and average deviations below 0.5 µm, which confirmed the high accuracy of the method.

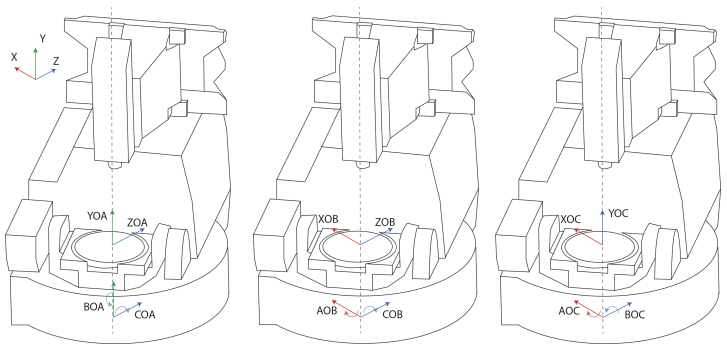


Fig. 1. Errors in the position of the kinematic centres of the rotary axes of a CNC machine tool [9].

More advanced calibration approaches were presented in [10–12]. In [10], a six-dimensional motion measuring device was used to identify PIGE of rotary axes B and C, achieving extremely small deviations, though the method was limited by the measurement range and sensitivity to kinematic modelling errors. In [11], PIGE, PDGE and thermal errors of axis C were identified based on machining tests, with good agreement (about 85–91%) compared to classical tests, confirming the reliability of the proposed approach. Paper [12] introduced a two-stage calibration method using a touch-trigger probe to compensate rotary and linear axis errors, demonstrating robustness and applicability even in special machine tools with difficult-to-measure parameters, and indicating that the method can be extended to general multi-axis CNC machines.

Further investigations using the R-Test system were reported in [13] and [14] where angular positioning errors of rotary tables and the positions of linear and rotary axes were measured over wide angular ranges. These studies showed good agreement between predicted uncertainties and experimental results, reduction of linear axis deviations, and improved perpendicularity between axes, leading to higher accuracy and process efficiency. However, certain limitations were also noted, such as dependence on other calibration systems and reduced suitability when only angular calibration is required [15].

Overall, the reviewed literature confirms that although many effective measurement and calibration methods exist, they often require expert operation and still suffer from practical limitations such as concentricity issues or limited applicability. Therefore, static measurements of the kinematic centres of rotary axes remain important for improving the geometric accuracy of five-axis CNC machine tools, enabling better condition assessment and more reliable error prediction. In parallel, artificial intelligence methods are increasingly applied in engineering, ranging from machine design optimization and tribology to infrastructure monitoring, energy

systems, and materials research [16–27]. These studies demonstrate that machine learning has become a key tool in modern engineering analysis. For this reason, AI-based methods were also adopted in this work for the identification of rotary axis position errors of CNC machine tools.

1.1. Motivation

The main goal of the scientific research is building an intelligent system for monitoring the accuracy and repeatability of CNC machine tool positioning supported by machine learning techniques. The basis for the construction of the system is the data obtained from the results of measurements of position of rotary axes A, B, C in CNC five-axis numerically controlled milling machines. Each value of average deviation of the position of the rotary axes was determined by taking measurements in two rotation directions of the machine tool table and using a fixed value of the measurement stroke α. At the same time, the system for monitoring the accuracy and repeatability of the CNC machine tools positioning will serve, among other things, as a knowledge base about the technical condition of the machine and will enable the prediction of errors in CNC machine tools.

A literature review shows that most of the research to data has focused on the impact of a CNC machine tool errors on shaping accuracy. The subject of the impact of errors in CNC machine tools is relatively known, *i.e.* the impact of linear and angular positioning errors, reversal errors, follow-up errors, cyclic errors or axis perpendicularity errors and many others. While this state of knowledge is insufficient, this study is a novelty due to the originality of the research objective because the subject of changing of the kinematic centre position of the rotary axis of CNC machine tools is not widely discussed and very well known. This is due in part to the lack of measurement tools and the complexity of the process.

2. Materials and methods

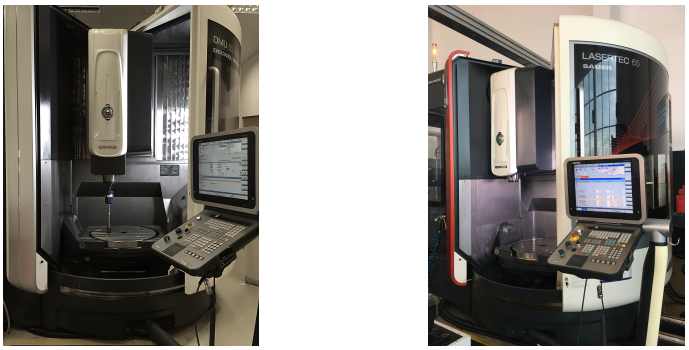
2.1. Methodology

The subjects of the study were the two CNC five-axis milling machines with a tilt-rotary table: monoBLOCK 65 and Lasertec 65, on which measurements of the kinematic centre positions of the rotary axes were realized. The selection of the CNC machine tools was dictated by different intensity of their use. The monoBLOCK 65 machine tool is used for testing cyclically, while the Lasertec 65 machine tool is used periodically. The following measurement conditions were used during the measurements:

- measurement radius R: 75, 150, 225, 300 mm,
- feed rate v_f: 500, 2000, 3500, 5000 mm/min,
- measurement range: 360°,
- measurement stroke α = 30°,
- movement direction: right, left.

A comparison of measurement results made it possible to assess how the accuracy and repeatability of the machine tools positioning affected their wear rate. The research plan was designed so that the experiment could be carried out in an orderly and structured manner. 2 Measurements of errors in the kinematic centres position of the rotary axes were made on five-axis CNC milling machines. The monoBLOCK 65 machine tool is equipped with a Sinumerik 840D control system, with a maximum spindle speed of 30 000 RPM. The Lasertec 65 machine tool is equipped with a Sinumerik 840D control system, with a maximum spindle speed of 24 000 RPM. Figure 2 shows photos of CNC milling machine tools. The technical data of the CNC machine tools were presented in Table 1.

The R-Test system used in the study is equipped with a measuring head, a pin and a reference ball. The measuring head has three 2.4 GHz measuring transducers. During the testing, the head was mounted in a toolholder in the machine tool spindle. A gauge ball was mounted on the machine table at a certain distance from the centre of rotation (distance R). A measurement program was then run to rotate the machine tool table, using four feed rates v_f. As a result, the relative difference between the position of the head gauge ball across the interpolated circle was measured. Table 2 shows the technical data of the R-Test system.



(a) (b)

Fig. 2. Five-axis CNC milling machine tools: monoBLOCK 65 (a) and Lasertec 65 (b).

Table 1. Technical data of the CNC machine tools.

CNC machine tool	**MonoBLOCK 65**	**Lasertec 65**
X-axis displacement [mm]	735	650
Y-axis displacement [mm]	650	650
Z-axis displacement [mm]	560	560
Positioning accuracy [μm]	5	8

Table 2. The technical data of the R-Test system [28].

Device	**IBS Trinity 001 0029**	
Series	T-Nr: 12111	
System resolution, µm	0.2	
Reference sphere's radius, mm	11	
Length of the measuring pin, mm	75	
System accuracy, µm	0.4	
Measurement uncertainty		
Form deviation of sphere, µm	0.3	
Deformation of adjustable sphere mounting, µm	0.14	
Thermal drift, µm	0.15	
Dependent on the range:	Measurement range, µm	
Perpendicularity of the probing planes	2	0.5
Angular error of the probe's direction, µm	1	0.5
Orientation of the R-Test mounted on the machine	0.4	0.1

3. Analysis of results of experimental research

The acquired experimental data were saved in special software of the R-Test system. They were used to read the position error values of the rotary axes of the CNC machine tools. The results are presented in the form of graphs showing the amplitude of the displacement of the linear axes X, Y, Z; the measurements results shown against the background of an ideal circle with a straight line fit to the data; and the position errors of rotary axes A, B, C in three planes. Example of measurements results of the monoBLOCK 65 machine tool are shown in Fig. 3. Analogous graphs were obtained for other parameters and for measurements of the Lasertec 65 machine tool. The system software dashboard shows, among other things, the displacement of the centre point of the calibration sphere during rotary motion relative to rotary axis C relative to the measuring head sensors over the entire measurement cycle (full rotation of the table in two directions) and the results against the background of an ideal circle and the fit of a straight line to the data. Example measurement results for the monoBLOCK 65 machine tool using a R = 75 mm measuring radius are shown in Figure 3.

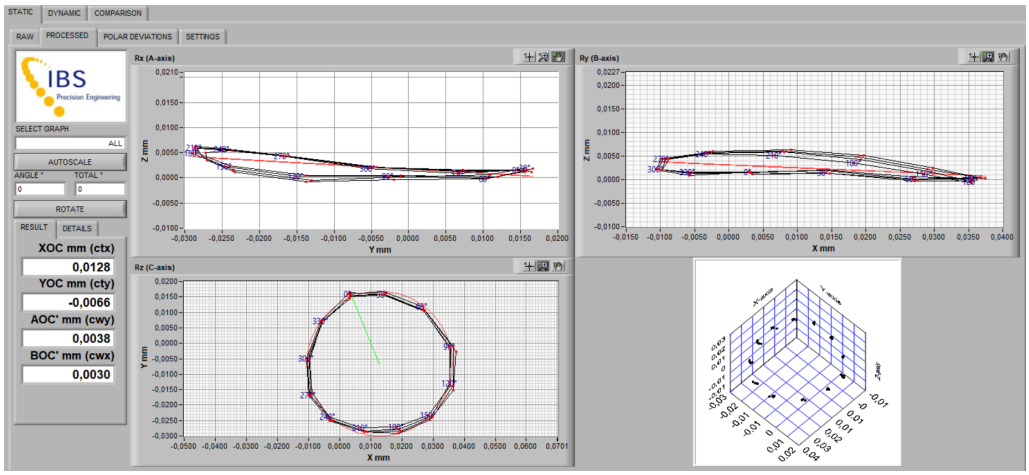


Fig. 3. Example error results for the monoBLOCK 65 machine tool using a R = 75 mm measuring radius with the results shown against an ideal circle and a straight line fit to the data. figura jak w oryginale

Figure 4 above presents the position errors of the kinematic centre of rotary C axis in planes X–Y, X–Z and Y–Z. The measured offsets are XOC = 12.8 µm and YOC = −6.6 µm, which are used as controller offset corrections to improve positioning accuracy. The straightness errors relative to axes A and B are AOC = 3.8 µm and BOC = 3.0 µm, determined using the least squares method. For an ideal machine, all these values should be zero, so the results indicate a slight displacement of the C-axis kinematic centre.

Additionally, the displacement of the calibration sphere during rotary motion over the full 0–360° range is shown as amplitude in planes X–Y, X–Z and Y–Z. Based on these data, position errors of the rotary axes were determined for different measuring radii and feed rates; exemplary results for the monoBLOCK 65 machine tool with R = 75 mm are presented.

In addition, a statistical analysis of the results of CNC machine tool measurements was carried out. These data were collected in the form of measurement radius R, feed rate v_f, motion direction of D and table rotation angle α. Then models were developed to estimate of the position errors

of the table's centre of rotation in three linear axes designated X', Y', Z'. Statistical analysis was carried out to determine the overall difficulty of the problem. This allowed the selection of appropriate data modelling methods. To model the relationships between the described variables, 3 approaches based on *multilayer perceptron* (MLP), *Kolmogorov-Arnold network* (KAN) and *multi-output Gaussian process* (MOGP) were considered. The models were compared with each other using *mean percentage error* (MPE) metrics and R^2 coefficient of determination. The metrics in question are represented by (1):

$$L_{\mathrm{MPE}}(y,\widehat{y}) = \frac{100\%}{n}\sum_{i=1}^{n}\frac{y_i-\widehat{y}_i}{y_i}, \quad L_{R^2}(y,\widehat{y}) = 1 - \frac{\sum_{i=1}^{n} y_i - \widehat{y}_l}{\sum_{i=1}^{n}(y_i-\overline{y})^2}, \tag{1}$$

where: n – number of samples, y_i – i-th ground truth value, $\widehat{y}_i$ – i-th estimated value, $\overline{y}$ – mean of ground truth values.

Based on the value of these metrics, a model was selected that is best able to estimate the value of deviations. Based on the diagram, a predictive model equation was determined for the position errors of kinematic centres of rotation of CNC machine tools (2):

$$\widehat{x} = \beta_0 + \sum_{j=1}^{k}\beta_j \cdot x_j, \tag{2}$$

where: $\widehat{x}$ – an error in the position of the kinematic centre of the rotating axis of a CNC machine tool, β_0 – model displacement, β_j – weights (parameters) assigned to each feature, x_j – j-th feature (position of rotary axes A, B, C), k – number of input parameters.

The entire methodology of the study is shown in Fig. 4.

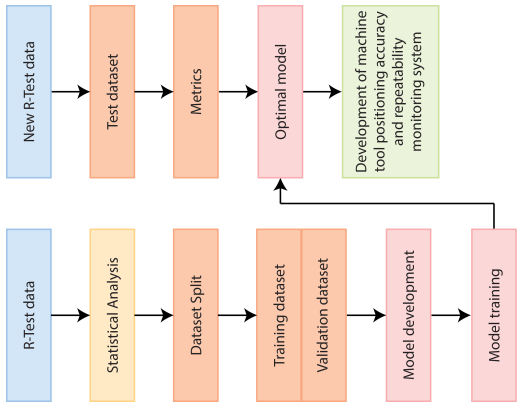


Fig. 4. Methodology of the study.

3.1. Dataset analysis

The developed dataset consisted of 1568 measurements of deviations of parameters of mean deviation of the centre point of the reference sphere in the three linear axes designated X', Y', Z' with the corresponding input values R, v_f, D, and α. In the first stage of statistical analysis,

mean values, standard deviation, maximum and minimum values, as well as percentiles of 25%, 50% and 75% were determined. Based on the analysis, it can be concluded that the X' and Y' parameters had the highest variability, while the Z' parameter had the smallest standard deviation, suggesting less dispersion around the mean value. In addition, parameters X' and Y' had a larger scatter than parameter Z'. In the case of parameter X', based on the 50% percentile, it can be concluded that larger values are in the upper part of the distribution. A summary of the results of the statistical analysis is shown in Table 3.

Table 3. Summary of the results of the statistical analysis.

	Numerical controlled axis		
Parameter	X'	Y'	Z'
mean, mm	0.0110	0.0062	–0.0001
std.	0.0189	0.0181	0.0071
min, mm	–0.0284	–0.0293	–0.0217
percentile 25%	–0.0024	–0.0060	–0.0030
percentile 50%	0.0109	0.00223	0.0007
percentile 75%	0.0216	0.01732	0.0042
max, mm	0.0582	0.0473	0.0174

In order to further illustrate the occurrence of outliers in the output parameters, a boxplot was created – Fig. 5a. It can be seen that there are outliers in the data set, and they relate to parameters X' and Z'. Parameter X' is characterized by a small number of outliers, while for parameter Z' the outliers are quite numerous. Parameter Y' did not show the presence of outliers.

In the next step of the statistical analysis, Spearman's rank correlation was examined, due to the fact that the data does not have a normal distribution. Based on this analysis, it was found that there is no significant correlation between the input and output characteristics. Parameter X' is moderately positively correlated with the α parameter. There is a negative correlation between parameter Z' and parameters X' and Y', which is -0.45 and -0.35 respectively. In other cases, no linear relationship was found. The correlation matrix is shown in Fig. 5b.

The data set presented for model selection was divided into a training set, a validation set and a test set in a ratio 7:2:1.

3.2. Numerical analysis using machine learning algorithms

Due to the very nature of the data and the possible presence of nonlinear relationships, it was decided to develop machine learning models based on MLP, KAN, and MOGP. Machine learning-based solutions were also chosen because they perform very well in areas related to machine tools [29–31]. MLP is a layered artificial neural network architecture, consisting of at least one hidden layer, which allows modelling complex nonlinear relationships. This type of model uses layers of linear transformations (weight matrices) and nonlinear activation functions [31]. The principle of a single neuron in the MLP layer is shown in (3):

$$y^{(1)}(x) = \phi\left(\sum_{i=1}^{n} \omega_i^{(1)} x_i + b^{(1)}\right), \tag{3}$$

where: x_i – i-th element of input parameters (R, v_f, movement direction D), $\phi(\cdot)$ – activation function, ω_i – i-th weight, b – bias.

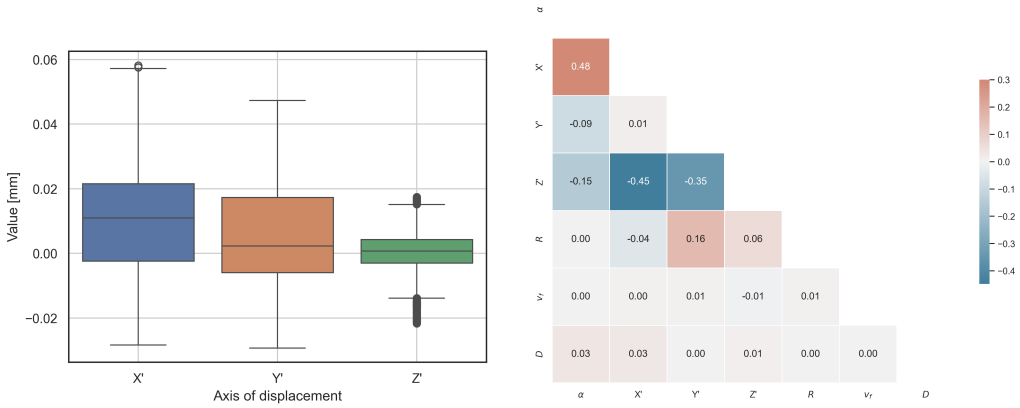


Fig. 5. Boxplot of the parameters of the mean deviation for the reference sphere centre point (a) and the correlation matrix (b).

Unlike MLP, solutions based on the KAN architecture do not use linear transformations and nonlinear activation functions, but are based on the transformation of input values by adaptive basis functions. The basis of this solution is the Kolmogorov-Arnold approximation theory [32]. The theory states that any continuous n-variable function that is defined on an n-dimensional cube can be represented by using the assembly of continuous functions of one variable and their addition. Let be $f : [0, 1]^n \to \mathbb{R}$ any n-dimensional continuous function. Then there are continuous one-dimensional functions $\Phi_q : \mathbb{R} \to \mathbb{R}$ and $\phi_{q,p} : [0, 1] \to \mathbb{R}$ for indices q and p, satisfying (4):

$$f(x_1, \ldots, x_n) = \sum_{q=1}^{2n+1} \Phi_q \left(\sum_{p=1}^{n} \phi_{q,p}(x_p) \right). \tag{4}$$

The process of function selection involves transforming functions with appropriate spline which are a group of functions defined on intervals that provide a smooth transition between node points. The selection of nodes, which are a set of points that define the boundaries of the intervals, is the first step of the process. Then, to obtain the value of the function at any point, a recursive Cox-de Boor formula is used to efficiently calculate the B-spline value for each point based on the nodes and their coefficients [32]. The principle of B-spline is described by (5):

$$\phi_{q,p}(x_p) = \sum_{i=0}^{n} K_{q,p,i} \cdot B_{i,k}(x_p), \tag{5}$$

where: $B_{i,k}(x_p)$ – B-spline of k-degree, $K_{q,p,i}$ – control factor (q – function, p – variable), n – nodes number.

A Gaussian process (GP) is a nonlinear probabilistic model used for function modelling and value estimation. GP extends the multivariate normal distribution to infinitely many dimensions by which it is possible to define probability distributions over functions. Let a measurable function $f_{\mathrm{GP}}(x)$ be defined on a sample space with index $x \in X \subseteq \mathbb{R}^{n_x}$ which is a random variable on that space [33]. The GP prior can be described by the mean function and covariance according to (6):

$$f_{\mathrm{GP}}(x) = \mathrm{GP}(m(x), k(x, x')). \tag{6}$$

For tasks of estimating multiple output parameters, an extension of GP is the *Multi-Output Gaussian Process* (MOGP). This method allows to find similarities in output values which are estimated based on the same parameters [34]. For this purpose, the covariance function described by (7) is used. This function is based on input features x and x', as well as output values i and j.

$$k\left([x,i],[x',j]\right) = k_{\mathrm{RBF}}\left(x,x'\right) * k_{\mathrm{out}}(i,j), \tag{7}$$

where: k_{RBF} – Radial Basis Function, k_{out} – lookup table containing inter-output covariance.

First, an artificial neural network model was developed which is based on MLP layers. The model consists of 4 inputs, 2 dense hidden layers consisting of 8 and 16 neurons, respectively, and 3 outputs. The activation function used was a rectified linear unit. The architecture of the MLP model is shown in Figure 6.

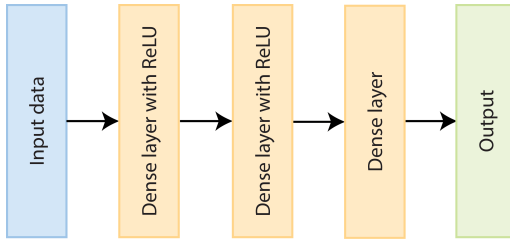
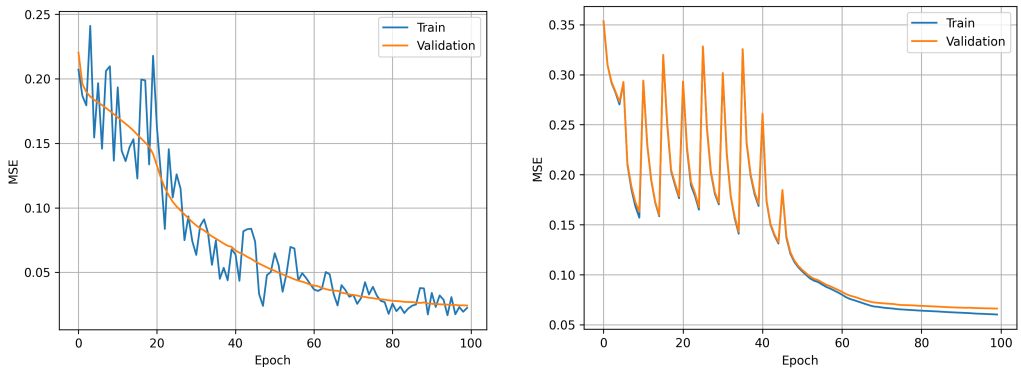


Fig. 6. Architecture of the MLP model.

In the optimization process, such an architecture achieved the best values of the metrics. MSE was adopted as the loss function, while rectified linear unit was used as the activation function. The developed model was trained for 100 epochs using the ADAM algorithm with $\beta_1 = 0.9$, $\beta_2 = 0.999$, $\epsilon = 10^{-8}$ according to [35]. The upper limit of the learning rate parameter was set as $\iota = 0.1$. The course of the loss function due to epoch is shown in Fig. 7a.

The model achieved an MSE metric value of 0.0243 for the validation set, while for the training set the metric value was 0.0225. These metrics were achieved for epoch 100. For the test set, the model achieved an MPE metric value of 12.18% and an average score of $R^2 = 0.8373$.

Its next model developed an architecture based on KAN assumptions. Initially, the architecture consisted of a single summation node but the overall generalization capability of the model was



(a) (b)

Fig. 7. Course of the loss function due to epoch: MLP (a) and KAN (b).

poor, and for this reason architectures with different numbers of layers and summation nodes were considered. In the optimization process, an architecture consisting of a single hidden layer with 2 summation nodes was finally created. The data were normalized, and the data space was defined as $\{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$. The input data space was divided into 24 nodes, therefore the function was approximated based on points. The number of points used for approximation was 12. A *Limited Memory Broyden–Fletcher–Goldfarb–Shanno* (LBFGS) algorithm [36] was adopted as the optimizer. This model was also trained for 100 epochs, and the MSE function was used as the metric. The model achieved the best metrics at epoch 100, where the MSE value for the validation set was 0.6609 and for the training set was 0.0602. The course of the loss function due to epoch is shown in Fig. 7b.

For the test data, the model in question achieved an MPE of 4.53%, and the average R^2 score was 0.9791. The visualization of the developed model is shown by Fig. 8a. In the developed KAN architecture, it is noticeable that the parameter v_f is assigned a negligible weight value, by which the contribution of this parameter in the overall prediction is very small. In order to better visualize the impact of individual features on the outputs, a *Feature Attribution Score* (FAS) analysis was conducted which determines the weight of each feature. A visualization of the value is shown in Fig. 8b.

From Fig. 8, it is noticeable that the rotation direction of the machine tool table (D) is quite distinguished, however, due to the fact that the values of this feature are binary and the model is assigned to a linear function, it can be considered that this feature is not so important in the overall prediction. The second most important parameter is the measurement stroke (α), which was actually an apt choice on the part of the model since this angle can significantly affect the occurrence of deviations. In the model, the measurement radius (R) was assigned little weight. However, from the theoretical point of view, this parameter was quite important - the larger the measuring radius, the higher should be the position error values of the CNC machine tool's rotary axes.

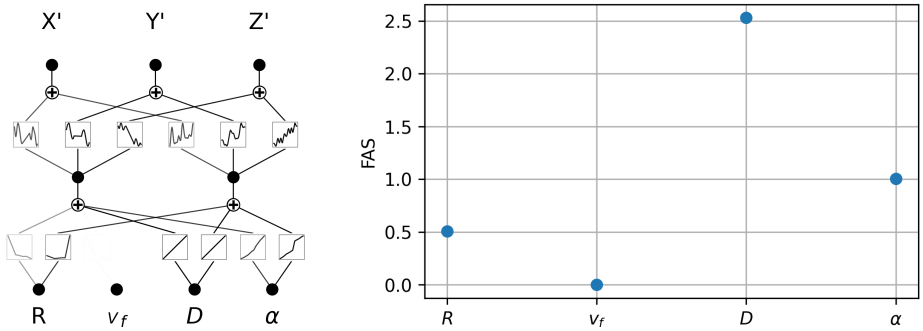
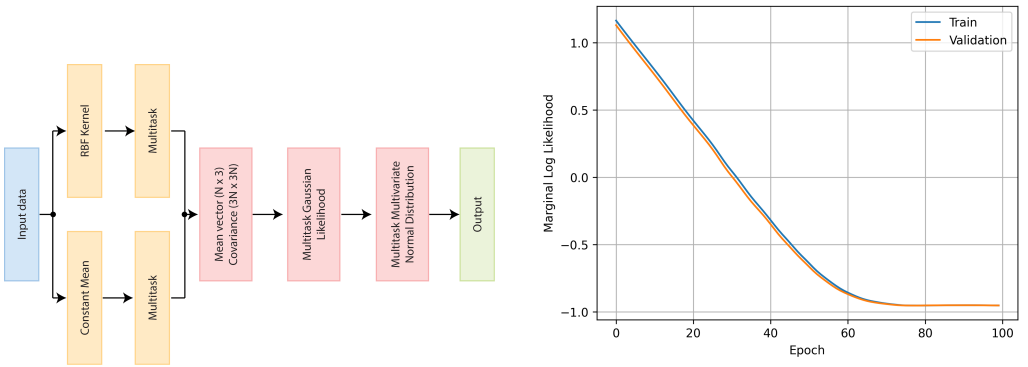


Fig. 8. Developed KAN architecture (a) and feature attribution score for KAN inputs (b).

The last of the models studied is the MOGP model. For this model, an exact MOGP model was developed which consists of a single layer. The ADAM algorithm was used to train the model with the same parameters as for MLP. *Marginal Log Likelihood* (MLL) was chosen as the loss function. Given the *a priori* distribution over GP functions and the probability of the output data, this function integrates over all possible realizations of the function. The function in question is represented by (8):

$$L_{\mathrm{MLL}} = p_f\,(y|x) = \int p\,(y \vee f_{\mathrm{GP}}(x))\; p\,(f_{\mathrm{GP}}(x) \vee x)\,\mathrm{d}f. \tag{8}$$

The architecture of the developed MOGP model is presented in Fig. 9a. The model in question was trained for 100 epochs. At epoch 100, the model achieved the best values of the metric which are –0.9515 for the training set and –9517 for the validation set. The course of the loss function due to epoch is shown in Fig. 9b.



(a) (b)

Fig. 9. Architecture of the developed MOGP model (a) and course of the loss function due to epoch (MOGP) (b).

This model on the test set settled on an average score of 0.9906 and an MPE of 2.79%. On the basis of the tests carried out, it was found that the model based on the MOGP solution performed best in the estimation of deviation parameters. It achieved the best values for all metrics compared to the other models tested. The exact set of all obtained values of metrics for each model is shown in Table 4.

Table 4. Summary metrics of the developed models.

Metrics		**MLP**	**KAN**	**MOGP**
R^2	X'	0.8465	0.9849	0.9918
	Y'	0.8288	0.9781	0.9904
	Z'	0.8366	0.9741	0.9896
	average	0.8373	0.9791	0.9906
MPE, %		12.1834	4.5308	2.7977

In order to better visualize the results obtained by each of the developed models, graphs of the dependence of the estimated value on the measured value were plotted. The graphs take into account the values from the test set. The graphs in question are shown in Fig. 10a. This figure shows that the largest scatter of the measured value around the estimated value was recorded for errors appearing in the Y-axis. The smallest deviations were recorded for errors appearing in the Z-axis direction. Deviations in the Y-axis were due to the appearance of mechanical backlash which manifested itself in sudden changes in the error values at times when the direction of axis movement changed. Deviations in the Z-axis, on the other hand, were due to thermal drift which caused heating of the sonics and drive components of the CNC machine tools. Taking into account the frequency of operation of the monoBLOCK 65 machine tool, the errors were due to natural wear, *i.e.* occurring under normal operating conditions (proper operation and correct performance of necessary inter-overhaul maintenance procedures). Whereas, taking into account

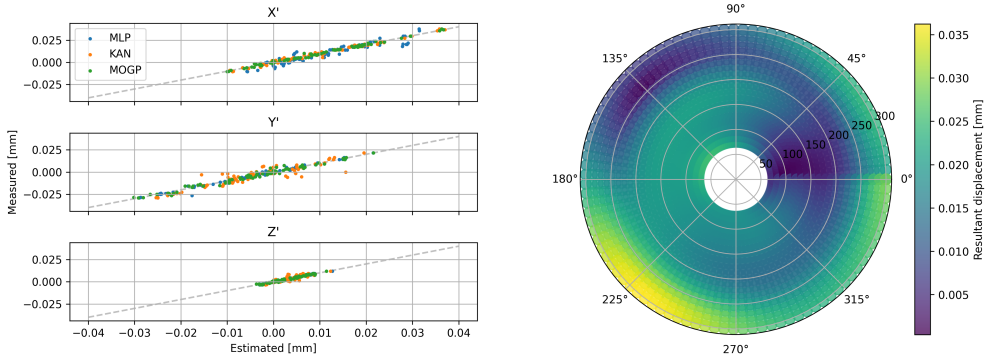
the frequency of operation of the Lasertec 65 machine tool, the deviations were due to the lack of proper operation of the machine (too much downtime, lack of continuous operation of the machine tool, lack of regular maintenance and servicing of the machine).

3.3. Machine tool table map

In the final stage of the study, a table map was made taking into account the deviations at the given points. In order to clearly visualize the results obtained, the deviations for each of the three axes were generalized to resultant displacement according to (9):

$$R_d = \sqrt{(X')^2 + (Y')^2 + (Z')^2}. \tag{9}$$

The example map was created with fixed values of direction (right) and with feed rate $v_f = 500$ mm/min. The machine tool table map is shown in Fig. 10b.



(a) (b)

Fig. 10. Comparison of estimated and measured values (a) and sample resultant displacement map of the worktable (b).

Modern machining requires high precision and repeatability of CNC machine tools as even small positioning errors can increase costs and reduce product quality. To address this, a system for monitoring positioning accuracy and repeatability of CNC machines was developed. The system enables identification and analysis of error sources based on knowledge derived from experimental tests on five-axis CNC machines, forming the basis of the diagnostic application core.

In the final stage, a graphical user interface was implemented to enable user interaction with the system, allowing operators to submit queries and receive diagnostic results with explanations. The solution was developed as a desktop application in Python using the Tkinter library, providing an intuitive GUI (Fig. 11).

The system was designed for industrial use and offers a bilingual (Polish/English) interface, diagnostic support tools, and a modular architecture consisting of three main modules: data import and preprocessing, analytical-diagnostic core, and results visualization with report generation. The analytical module detects typical CNC machine tool errors, including mechanical backlash, thermal drift, and geometric errors, using methods such as derivative analysis, correlation analysis, and *Fast Fourier Transform* (FFT). It also enables recognition of characteristic patterns indicating failures of drives or encoders. The system provides graphical visualization of results, including error plots versus rotation angle and two-dimensional error maps of the workspace, facilitating quick interpretation of machine condition. Additionally, it automatically generates PDF reports containing

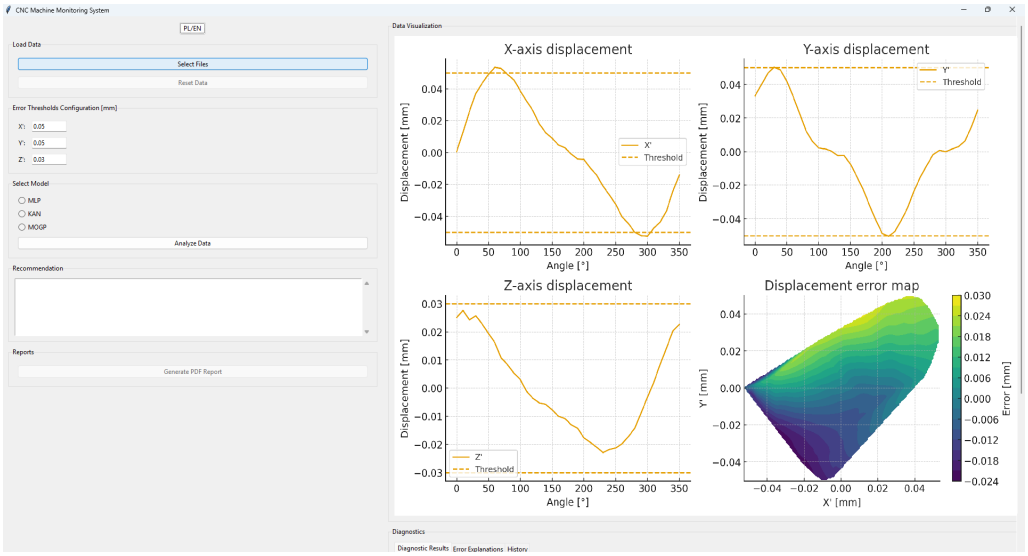


Fig. 11. View of the main interface of the CNC machine tool positioning accuracy and repeatability monitoring system.

summaries of results, identified anomalies, and service recommendations. The system was validated using real measurement data from five-axis CNC machine tools, confirming its effectiveness in industrial conditions. Future development includes integration with SCADA systems for real-time data acquisition and the implementation of machine learning methods for wear prediction and anomaly detection. Thanks to its modular structure, the system constitutes a flexible tool supporting maintenance and quality management in modern CNC-based manufacturing environments.

4. Conclusions

Based on the experimental study results, the following conclusions were drawn:

- statistical analysis of the measurement results showed that for the Lasertec 65 machine tool the largest error was recorded for rotary axis A and for a measurement radius $R = 150$ mm. The smallest error was recorded for rotary axis C, for the same radius,
- for the monoBLOCK 65 machine tool, the largest error was recorded for rotary axis B for a maximum measuring radius $R = 300$ mm. The smallest error was recorded for rotary axis A for measuring radius $R = 75$ mm,
- the table rotation direction parameter (D) and the measuring pitch parameter (α) had the greatest influence on the value of errors of the CNC machine tool rotary axes. Numerical analysis shows that the measurement radius (R) did not have much influence on the error values. However, from the theoretical point of view, this parameter was quite important – the larger the measuring radius, the higher should be the error values of the CNC machine tool's rotary axis position,
- for the KAN model, the MPE metric was 4.86% and the MSE was 0.004. For the MPL model, the MPE metric was 14.68% and the MSE was 0.032. Of the models used in the numerical analysis, the best results from all metrics were obtained for the MOGP model. This model achieved the best values of the metrics which are −0.9515 for the training set and −9517 for the validation set. In addition, it achieved an average R^2 score of 0.9906 and an MPE of 2.79%,

– the smallest metric value recorded for the MLP model for the Y-axis measurement was 0.8288,
– the highest results correlation recorded between X' parameter and the constant angle of rotation of the CNC machine tool table (α). The value of this correlation equalled −0.48. The lowest results correlation was recorded between parameter Z' and parameter X' (−0.45).

An integrated system that monitors the accuracy and positioning repeatability of the multi-axis CNC machine tools using machine learning techniques has proved to be a very good tool for diagnostic testing of a CNC machine tools used in a laboratory environment. The system has 3 modules that are responsible for processing data, detecting anomalies in the operation of a given machine, and generating diagnostic reports. An additional benefit of building the CNC machine tool accuracy monitoring system is that it can be used in an industrial environment. The extensive expert system will allow to predict the wear and tear of individual machine components.

A major advantage of the developed expert system is its practical relevance, as it bridges the gap between research and industrial application and supports maintenance and quality management. However, the methodology has limitations, including the lack of recording of direct data from the R-Test system in the CNC controller, its applicability mainly to five-axis machines, and the absence of an extended knowledge base with learning mechanisms. Future work will focus on integrating data from multiple sensors, including inertial measurement units [37], to enable a more comprehensive assessment of machine behaviour, long-term condition monitoring, and evaluation of reliability and degradation.

Previous studies have mainly addressed the identification of rotary axis kinematic centre errors using measurement systems and basic analysis methods, while no comprehensive expert system for assessing accuracy and repeatability of rotary axes has been reported so far. Therefore, the proposed database and expert approach constitute a novel contribution. Although many diagnostic systems exist, the R-Test system is increasingly used for multi-axis machines, and its combination with machine learning enables trend analysis, error prediction, and forecasting of component wear, leading to improved manufacturing quality.

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Nomenclature

YOA – the A-axis position error in the Y-axis direction,
ZOA – the A-axis position error in the Z-axis direction,
BOA – the A-axis perpendicularity error relative to the B-axis,
COA – the A-axis perpendicularity error relative to the C-axis,
XOB – the B-axis position error in the X-axis direction,
ZOB – the B-axis position error in the Z-axis direction,
AOB – the B-axis perpendicularity error relative to the A-axis,
COB – the B-axis perpendicularity error relative to the C-axis,
XOC – the C-axis position error in the X-axis direction,
YOC – the C-axis position error in the Y-axis direction,
AOC – the C-axis perpendicularity error relative to the A-axis,
BOC – the C-axis perpendicularity error relative to the B-axis.

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