

SUPPRESSION OF PERIODIC INTERFERENCE IN THE INPUT SIGNAL OF A PRECISION PULSE WIDTH MODULATOR USING AN S&H CIRCUIT AND WEIGHTED AVERAGING

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Abstract

The results of theoretical and simulation studies concerning the occurrence of nonlinear distortions and the reduction of their impact during pulse width modulation of variable measurement signals are presented in this article. It has been shown that when input signal is variable, the classic PWM method may be inaccurate due to addition intense modulation distortion, which may be caused by noise or interference (e.g., power lines). The level of these distortions is dependent on the level and speed of the input signal changes, as well as the modulation frequency. In order to reduce the impact of those distortions, the use of the sampling and holding circuit at the input of the pulse width modulator, and weight averaging of the output pulses have been proposed. The effectiveness of the suppression of the periodic interferences in both narrow and wide bands was investigated through theoretical research and through simulations conducted in the LTSpice software using uniform, triangular, Hamming, and Blackman weighting functions for the classic PWM, and for one with the addition of the S&H circuit.

Keywords: pulse width modulator, PWM, interference, weight function, normal-mode rejection ratio, NMRR.

1. Introduction

The time domain and frequency-modulated pulses are among the three main forms of measurement and control information, the other two being analogue and digital [1]. The analogue form of information is usually considered to be primary or natural, while microcontrollers with digital, pulse, and analogue output signals are used to control various devices. Compared to analogue signals, the use of modulated pulses and digital signals provides significantly higher accuracy and remarkable resistance to disruptions caused by noise or interference in the transmission channel. Most modern universal *data acquisition* (DAQ) modules provide both input processing and output forming of these three types of signals, as they are equipped with analogue, pulse,

and digital inputs and outputs [2]. For mutual conversion of information, appropriate converters are used: *analog-to-digital converters* (ADCs), *digital-to-analog converters* (DACs), as well as pulse modulators and demodulators [3]. In addition to pulse frequency modulation (seen in voltage-to-frequency converters), *pulse width modulation* (PWM) is also widely used in forming, processing and transmitting signals. In essence, PWM involves generation of periodic pulses with constant amplitude, the width of which corresponds to the value of the input signal [4,5].

The inverse operation of generating the analogue output signal according to the width of the input pulses is called demodulation [6]. Pulse width demodulation circuits are used in so-called inverters which generate alternating voltages by switching the DC voltage [7]. Using a simple low-pass filter and averaging, it is possible to generate an analogue signal from the PWM input signal in controllers with such input signals [8].

Due to the simplicity of implementing PWM based on low-cost electronic circuits, and the simplicity of the processing output pulses, devices with PWM are manufactured by many electronics companies [9–13]. For example, the popular IC 555 (LM555) can easily be used to generate a PWM signal [12,13]. Advanced modules, such as Arduino [14–16], and modules paired with LabVIEW software [17] can also be used to implement PWM.

Other applications of PWM signals are found in telecommunication systems [18], amplifiers of audio signals [19,20] and power devices [21,22]. PWM signals are also used in isolated amplifiers to transmit measured signals from the input to the output [23], to precisely form output calibration analogue signal in the output of a DAC [24], and so on. Additionally, it is important to mention that the first digital voltmeters were built on the PWM principle, *i.e.*, converting voltage into pulse width.

It should be noted that, in general, PWM applications related to controlling various objects, the speed of motors, the output voltage or the current in switching power supplies, *etc.*, do not require very high linearity and accuracy.

However, the requirements for PWM accuracy and linearity are much higher when such units are used in measurement systems. This article discusses, analyses, and studies systems that should meet these two basic conditions. In addition, since the modulated pulse signal in the above-mentioned applications is subjected to subsequent demodulation, the additional prerequisite is the reduction of modulation distortion caused by the input signal of periodic interference.

The aims of the following research are: (i) theoretical and simulation study of non-linear distortions caused by periodic interference present in PWM input signal when weighted averaging of PWM output pulses is used, (ii) to propose the usage of *sampling and holding* (S&H) circuits to reduce such distortions, and (iii) to investigate the effectiveness of the proposed methods by theoretical and simulation study.

2. Investigation of non-linear effects in classical PWM

2.1. General description of the distortion problems

In the practice of studying the properties of PWM modulator circuits, the most commonly used circuits are those with processing based on the comparison of the input signal with single-edge or double-edge saw-tooth signals (Fig. 1). Such a PWM modulator will be henceforth referred to as classic or conventional. Note that according to the processing principle, in such a device, the persistence of the output pulse depends on the value of the input signal at the moment of its comparison with the saw-tooth signal (V_s). In the case of a double-edge saw-tooth signal (Fig. 1), this effect is analysed below.

In such modulation, the input signal $v_{in}(t)$ is directly compared with the double-edge saw-tooth reference signals:

$$V_{S1} = -S \cdot (t - t_{i0}), \quad V_{S2} = S \cdot (t - t_{i0}), \quad (1)$$

where: $S = \frac{2V_K}{T_m}$ is a slope of reference signals, $t_{i0} = (i + 0.5) \cdot T_m$ is a centre point of an i -th modulation interval of period T_m . The time moments t'_i and t''_i of the comparisons are the solution of two equations:

$$v_{in}(t'_i) = -S \cdot (t'_i - t_{i0}), \quad v_{in}(t''_i) = S \cdot (t''_i - t_{i0}). \quad (2)$$

From (2), the two informative time intervals are equals:

$$\tau'_i = t_{i0} - t'_i = \frac{v_{in}(t'_i)}{S}, \quad \tau''_i = t''_i - t_{i0} = v_{in}(t''_i) / S, \quad (3)$$

which determines the output pulse of duration τ_i in two-edge PWM:

$$\tau_i = t''_i - t'_i = \tau'_i + \tau''_i. \quad (4)$$

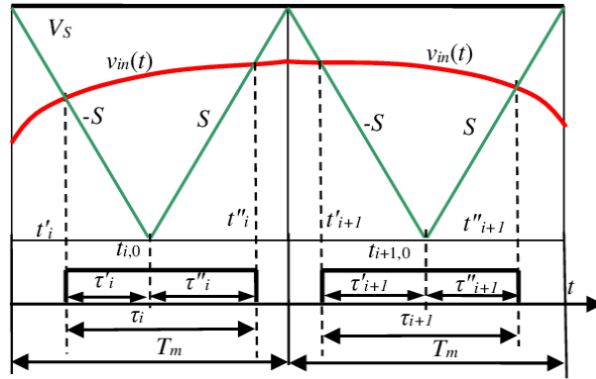


Fig. 1. Essences of the classical double-edge PWM.

Correct analysis of the effects resulting from the modulation of a variable input signal is based on the direct dependence of the pulse duration on the input voltage value. From (3) and Fig. 1, the values of the time moments are: $t'_i = t_{i0} - \tau'_i$, $t''_i = t_{i0} + \tau''_i$, therefore the informative time intervals in (3) transform into:

$$\tau'_i = \frac{v_{in}(t_{i0} - \tau'_i)}{S}, \quad \tau''_i = v_{in}(t_{i0} + \tau''_i) / S. \quad (5)$$

Assuming that during modulation period T_m the informative part of the input signal is practically stable, $V_0 \approx \text{const}$ and the variable part $v(t)$ is a harmonic interference of the amplitude V_m , frequency f_h , and the initial phase ϕ , which can be presented in the classical form as $v_h(t) = V_m \cos(2\pi f_h t + \phi)$. So, the model of input signal model takes the form:

$$v_{in}(t) = V_0 + v(t) = V_0 + V_m \cos(2\pi f_h t + \phi). \quad (6)$$

Then, according to (5), (6) and Fig. 2a, the informative time intervals are determined according to the equations:

$$\tau'_i = \frac{V_0}{S} + \frac{V_m}{S} \cos(2\pi f_h(t_{i0} - \tau'_i) + \phi), \quad \tau''_i = \frac{V_0}{S} + \frac{V_m}{S} \cos(2\pi f_h(t_{i0} + \tau''_i) + \phi). \quad (7)$$

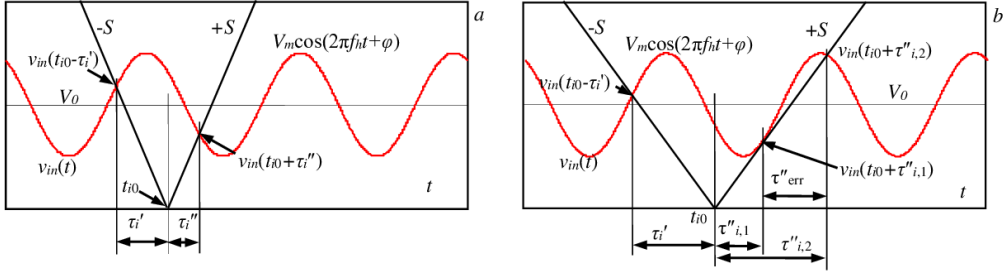


Fig. 2. Issues with the correct formation of output width modulated pulses in classical PWM: $v_{in}(t)$ is input signal, $\pm S$ is a slope of the reference signal, τ' , τ'' are the informative time intervals. Case of $S > |v'_{in}(t)|$ (a), case of $S < |v'_{in}(t)|$ (b). Within the time interval τ''_{err} , it is not possible to correctly compare the input signal and the reference signal.

It can be seen in (7) that both informative time intervals τ'_i and τ''_i on the left side of these equations are simultaneously arguments of the corresponding values of the input signal on the right. As a result, these equations cannot be solved directly with respect to the information intervals τ'_i and τ''_i . Only when the signal value is constant ($v_{in}(t) = V_0 = \text{const}$), i.e., the harmonic component is absent, the direct solutions of (7) are: $\tau' = \tau'' = V_0/S$.

Hence, in the general case, if the signal value is not constant, $v_{in}(t) \neq \text{const}$, (5) and (7) are recursive. This means that when using classic PWM, non-linear distortions occur. The result of this effect is the additional change in the widths of the output modulated pulses. Those changes depend on the parameters of the variable component of the input signal and slope S . In spectral domain, this effect results in the higher harmonics which are not present in the input signal. Those harmonics are not directly related to the modulation frequency $f_m = 1/T_m$.

In practice, the cause of distortions is the presence of interference and random noise in the input signal. Those kinds of conditions occur more frequently during signal processing in industrial measurements, when there is a high level of such influences, especially power line interference [24–26]. The level of power line interference can be even tens of times greater than the level of random noise. The analysis of the distortion that occurs in a classical PWM with a harmonic input signal was carried out in [22], and, in the case of a multi-harmonic input signal, presented in [27].

It should be noted that when using classical PWM in the case of a variable part of the input signal $v(t)$, correct conversion of this signal into the time intervals is possible under the following condition: $|v'_{in}(t)| \leq S$, i.e., when the maximum signal change rate does not exceed the slope of the reference signal. This condition is also applied to (7). In the case shown in Fig. 2a, at the given initial phase ϕ of interference and the given time position t_{i0} , this condition is satisfied. On the other hand, in the case shown in Fig. 2b, it is not. Because if $|v'_{in}(t)| > S$, a small change (dt_{i0}) in the time position t_{i0} brings about a step change τ''_{err} in the second part of the comparison between the reference ($+S$) and the input signals. This causes the second time interval to change abruptly from $\tau''_{i,1}$ to $\tau''_{i,2}$ when the input signal changes continuously, i.e., an incorrect time step τ''_{err} occurs. That is, in this instance, an output pulse signal with an erroneous persistence is formed. At the appropriate initial phase ϕ of interference and the time position t_{i0} , this may cause a similar effect in the generation width τ' of the first pulse. The presence of random noise can also cause a similar effect.

Traditionally, filtering and averaging (including weighting, [26, 28]) are used to suppress the impact of the periodic interferences. The general requirement in such cases is to reduce the influence of such disturbances to a sufficiently high degree. For example, software filters, which are described in the Advanced Analysis Toolkit for LabVIEW [25], can be used for this purpose. However, the use of filters is associated with an increase in the signal processing time, especially in multi-channel systems. The time taken to set the filter output signal when switching channels increases the measurement time. Averaging is more effective than filtering, because it requires significantly less time to suppress interference [26, 28]. A detailed analysis of modulation distortion caused by the influence of interference when performing the weight averaging of modulated pulses will be carried out in the following section.

2.2. Theoretical analysis of the level of distortion impact when using weight averaging

It is known [29, 30] that a series of pulses with amplitude C_0 formed at the output of a PWM with double-edged modulation of the variable signal $v(t)$ can be represented by a Fourier series *i.e.*, in the form:

$$u_{\text{PWM}}(t) = \frac{C_0}{T_m} \tau_0 + \frac{2C_0}{T_m} \frac{v(t)}{S} + \sum_{k=1}^{\infty} \frac{2C_0}{k\pi} \sin\left(\frac{k\pi}{T_m} \tau_0 + \frac{k\pi}{T_m} \frac{v(t)}{S}\right) \cos\left(\frac{2\pi k}{T_m} t - \frac{2\pi k}{T_m} t_0 - \frac{3k\pi}{T_m} \tau_0\right), \quad (8)$$

where t_0 is the initial pulse delay and τ_0 is the mean width of pulses.

For the model (6), the mean width of pulses is $\tau_0 = \frac{V_0}{S}$. Assuming zero delay of the initial pulse ($t_0 = 0$) and also taking into account that $S = \frac{2V_R}{T_m}$, after substituting (6) in (8), the series of the PWM signal can be described in the form:

$$u_{\text{PWM}}(t) = C_0 \frac{V_0}{V_R} + C_0 \frac{V_m}{V_R} \cos(2\pi f_h t + \phi) + \sum_{k=1}^{\infty} \frac{2C_0}{k\pi} \sin\left(k\pi \frac{V_0}{V_R} + k\pi \frac{V_m}{V_R} \cos(2\pi f_h t + \phi)\right) \cos\left(\frac{2\pi k}{T_m} t - \frac{3k\pi}{2} \frac{V_0}{V_R}\right). \quad (9)$$

The sine of a complex argument with the cosine function in the sum component in (10) can be presented as a series using a first order Bessel function $J_i(x)$ [31]:

$$\sin\left(k\pi \frac{V_0}{V_R} + k\pi \frac{V_m}{V_R} \cos(2\pi f_h t + \phi)\right) = \sum_{i=-\infty}^{\infty} J_i\left(\frac{\pi k V_m}{V_R}\right) \cdot \sin\left(\frac{\pi k V_0}{V_R} + i \frac{\pi}{2}\right) \cdot \cos(i(2\pi f_h t + \phi)). \quad (10)$$

Therefore, (9) takes the form:

$$v_{\text{PWM}}(t) = \frac{C_0}{V_R} (V_0 + V_m \cos(2\pi f_h t + \phi)) + \frac{2C_0}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sum_{j=-\infty}^{\infty} J_j\left(\frac{\pi k V_m}{V_R}\right) \cdot \sin\left(\frac{\pi k V_0}{V_R} + j \frac{\pi}{2}\right) \cdot \cos(j(2\pi f_h t + \phi)) \cos\left(2\pi k f_m t - \frac{3k\pi}{2} \frac{V_0}{V_R}\right). \quad (11)$$

In (11), the first part corresponds to the PWM input signal, while the second part contains harmonic components around multiples of the modulation frequency $k f_m$ ($k = 1, 2, \dots$), around each of which there are harmonics with a frequency $k f_m \pm j f_h$ ($j = 0, \pm 1, \pm 2, \dots$) as a multiple of the frequency f_h of the variable component of the input signal. The amplitudes of these harmonics are determined by the values of the Bessel function $J_j \left(\frac{\pi k V_m}{V_R} \right)$. These harmonic components are the result of a non-linear effect in such modulation.

The following section will analyse the level of non-linear distortion resulting from PWM signal demodulation through averaging. The use of averaging to suppress harmonic interference will be investigated not in the classical way, as used for digital data, but in relation to the pulse-width modulated pulses, as in the case of the frequency pulse modulated signal [28]. The principle of weight averaging of the PWM signal [30] is given in Fig. 3. For simplification, only the single-edge pulse width modulations are presented.

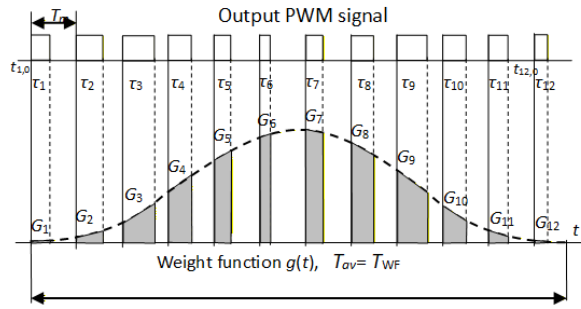


Fig. 3. Principle of weight averaging of the PWM signal ($n = 12$).

The results of periodic (M_x) or cyclic ($M_{x,j}$) averaging are described by the relations:

$$M_x = C_0 \sum_{i=1}^n \int_{t_{i,0}-\tau'_i}^{t_{i,0}+\tau''_i} g(t) dt = C_0 \sum_{i=1}^n G_i,$$

$$M_{x,j} = C_0 \sum_{i=1}^n \int_{t_{i+j,0}-\tau'_{i+j}}^{t_{i+j,0}+\tau''_{i+j}} g(t) dt = C_0 \sum_{i=1}^n G_{i+j}, \quad j = 1, 2, \dots, \quad (12)$$

where $g(t)$ is the weight function used (window) [26, 28, 32] of the duration $T_{WF} = T_{av}$, which determines the averaging time $T_{av} = T_{WF}$, C_0 – is the amplitude of pulses, and

$$G_i = \int_{t_{i,0}-\tau'_i}^{t_{i,0}+\tau''_i} g(t) dt \quad (13)$$

is the integral value of the weight function during the pulse width τ_i .

The amplitude C_0 of the pulses should be determined from the normalisation condition: the sum of the areas of the pulses when $V_0 = V_R$ and $V_m = 0$ should be equal to 1, i.e.: $C_0 = ST_m/2 = V_R$. Hence, applying the PWM signal (11) to (12), the result of weighted averaging of the width modulated pulses is equal to:

$$M_{\text{Wav}} = \int_{-T_{\text{av}}/2}^{T_{\text{av}}/2} v_{\text{PWM}}(t)g(t)dt = V_0 + V_m \cos(\phi) \cdot G(\pi f_h T_{\text{av}}) + \frac{V_R}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sum_{j=-\infty}^{\infty} J_j \left(\frac{\pi k V_m}{V_R} \right) \cdot \sin \left(\frac{\pi k V_0}{V_R} + j \frac{\pi}{2} \right) \cdot \cos \left(\phi \mp \frac{3k\pi}{2} \frac{V_0}{V_R} \right) G(\pi (k f_m \pm j f_h) T_{\text{av}}), \quad (14)$$

where $G(\pi f T_{\text{av}})$ is a spectral characteristic of the used weight function:

$$G(\pi f T_{\text{av}}) = \int_{-T_{\text{av}}/2}^{T_{\text{av}}/2} g(t) \cdot \cos(2\pi f t) dt. \quad (15)$$

The second part (14) reflects modulation distortions in the presence of harmonic interference in the signal.

The averaging time T_{av} is dependent on the width $v_{0,\text{WF}}$ of the main lobe of the used weight function and the minimal value of harmonic interference frequency $f_{h,\text{low}}$ [26, 28]:

$$T_{\text{av}} = \frac{v_{0,\text{WF}}}{f_{h,\text{low}}}. \quad (16)$$

For example, when the uniform weight function is used, then $v_{0,\text{U}} = 1$, as a minimum of 1 period of interference is needed to suppress it: $T_{\text{av}} = 1 \cdot T_{h,\text{low}}$. When triangle or Hamming weight functions (interference suppression level about 42 dB) are used, then $v_{0,\text{T}} = v_{0,\text{H}} = 2$, as a minimum of 2 periods is needed to suppress this interference: $T_{\text{av}} = 2 \cdot T_{h,\text{low}}$. For the Blackman weight function (suppression level about 70 dB), $v_{0,\text{B}} = 3$, meaning that a minimum of 3 periods are required to suppress interference $T_{\text{av}} = 3 \cdot T_{h,\text{low}}$.

On the other hand, when weight averaging is used, then the modulation frequency f_m depends on a high interference frequency $f_{h,\text{high}}$ [26, 28]:

$$f_m = f_{h,\text{high}} + f_{h,\text{low}}. \quad (17)$$

This choice of the modulation frequency will ensure the possibility of suppressing interference in a given frequency band through periodic or moving (including weighted) averaging of pulses at the output of the PWM unit. Therefore, since the averaging duration depends on the weighting function used, based on (16) and (17), the number of averaged pulses is determined as:

$$n_{\text{WF}} = f_m T_{\text{av}} = v_{0,\text{WF}}(D + 1). \quad (18)$$

where $D = \frac{f_{h,\text{high}}}{f_{h,\text{low}}}$ is the relative interference suppression frequency band.

When using uniform averaging, assuming the minimum number of averaged pulses in (18) as $n = D + 1$, the number of averaged pulses for other weighting functions is equal to:

$$n_{\text{WF}} = v_{0,\text{WF}} n. \quad (19)$$

For the triangle and Hamming weight functions, the number of pulses is $n_{\text{H}} = n_{\text{T}} = 2n$, and when the Blackman weight function is used, then $n_{\text{B}} = 3n$, and so on.

To analyse the level of the modulation distortions in (14), it is advantageous to use frequency normalisation with respect to the duration T_{av} of the weighting function: $v_h = f_h T_{av} = v_{0,Wf} r_h$, where $r_h = f_h / f_{h,low}$. Then, the components $\Delta_{mod}(v_h)$ describing the modulation distortions in (14) take the form:

$$\Delta_{mod}(v_h) = \frac{V_R}{\pi} \sum_{k=1}^{\infty} \frac{1}{k} \sum_{j=-\infty}^{\infty} J_j \left(\frac{\pi k V_m}{V_R} \right) \cdot \sin \left(\frac{\pi k V_0}{V_R} + j \frac{\pi}{2} \right) \cdot \cos \left(\phi \mp \frac{3k\pi}{2} \frac{V_0}{V_R} \right) G(\pi v_{0,Wf}(kn \pm jr_h)). \quad (20)$$

The level of distortion can be determined based on an approximate level of the amplitude envelope $A_{\Delta}(v_h)$ of the $\Delta_{mod}(v_h)$. As it follows from (20), the envelope value depends on the Bessel function parameters, especially on the V_m/V_R ratio and the k number.

The informative part V_x of the input signal may be unipolar or bipolar. In the first case, the input signal is positive: $0 \leq V_x \leq V_{Rx}$, where V_{Rx} is the range of the informative part. Assume that amplitude V_m of harmonic interference $v_h(t)$ is not exceeding the range of the informative part: $V_x \leq V_{Rx}$. Since the input signal must be positive to correct the PWM operation, the positive constant voltage shift $V_{sh} > V_{Rx}$ should be used. Therefore, the PWM range is: $V_R > 3V_{Rx}$. Assume $V_R = 3.5V_{Rx}$. When the input signal is bipolar, the range of V_{Rx} must be bigger, for example, $V_R = 4.5V_{Rx}$.

Therefore, in (20), the argument of the Bessel function is: $z = \frac{2\pi k V_m}{V_R} = \frac{k\pi V_m}{4.5V_{Rx}} \Big|_{\max} \approx \frac{k\pi}{4.5} \leq 0.7k$. The most important are the distortions that occur around the first modulation harmonic, i.e., for $k = 1$, in the range $f_{low} \leq f_h \leq f_m - f_{low}$ or $1 \leq r_h \leq n - 1$. This is due to the fact that the distortions that appear around the next harmonics, i.e., for $k = 2, 3 \dots$, are outside the interference suppression band, i.e., $f_h > f_m - f_{low}$, or $r_h > n - 1$.

The influence of the distortions appears at frequencies that fall into the width band $\pm v_{0,Wf}$ of the main lobe of the frequency characteristic $G(v)$ of the applied weighting function, i.e. when $|k f_m \pm j f_h| \leq f_{0,w}$ or $|k v_m \pm j v_h| = v_{0,Wf} |kn \pm jr_h| \leq v_{0,Wf}$, that is when: $|kn \pm jr_h| \leq 1$.

Hence, the index j in the Bessel function must be in range: $-(kn - 1)/r_h \leq j \leq (kn - 1)/r_h$. This means that $kn \gg 1$, even at the minimal value $k = 1$ and $n \gg 1$. Therefore, the number of the Bessel function index is $|j| \leq n/r_h$.

In (20), when $k = 1$, $z \leq 0.7k = 0.7$, the value of Bessel function can be presented as [31] $J_l(z) \approx \frac{1}{l!} \cdot \left(\frac{z}{2}\right)^l$. The factorial function can be presented by the Stirling approximation [33] $l! \approx \left(\frac{l}{e}\right)^l \sqrt{2\pi \cdot l}$. From this, the Bessel function (at $k = 1$) takes the value:

$$J_{\frac{n}{r_h}} \left(\frac{\pi}{4.5} \right) \approx \left(\frac{\pi e}{9n} \right)^{\frac{n}{r_h}} \cdot \sqrt{\frac{r_h}{2\pi n}} \approx \left(\frac{0.95}{n} \right)^{\frac{n}{r_h}} \cdot \sqrt{0.16 \frac{r_h}{n}}. \quad (21)$$

Therefore, without considering the sinus component in (20) taken into account in (21), the relative amplitude of the V_{Rx} envelope of the spectral distortion as a function of frequency has the approximate value:

$$A_{\delta}(v_h) = \frac{A_{\Delta}(v_h)}{V_{Rx}} \approx \frac{9}{\pi \sqrt{2\pi}} \left(\frac{\pi e}{9n} \frac{V_m}{V_{Rx}} \right)^{\frac{n}{r_h}} \sqrt{\frac{r_h}{n}} \approx 1.14 \left(\frac{0.95}{n} \frac{V_m}{V_{Rx}} \right)^{\frac{n}{r_h}} \sqrt{\frac{r_h}{n}}. \quad (22)$$

This relation is practically independent of the averaging function used and depends on interference amplitude V_m (ratio V_m/V_{Rx}), frequency (ratio $r_h = f_h/f_{h,low}$) and the number of averaging pulses (n). The relative envelope amplitude of the spectral distortions $A_\delta(v_h)$ (22) at $V_m/V_{Rx} = 1$, $f_{h,low} = 50$ Hz, $50 \text{ Hz} \leq f_h \leq 950$ Hz, $n = 20$, $f_m = 1000$ Hz, $S = 5$ V/1 ms is shown in the first row of Fig. 4 in the decibel scale (NMRR $_{A\delta}$).

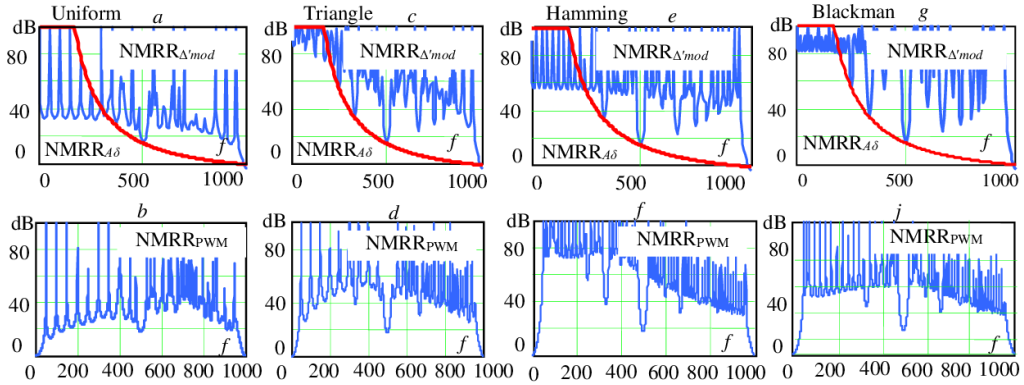


Fig. 4. Dependence (in dB scale) of the amplitude envelopes NMRR $_{A\delta}$ (red) and of the modulation distortions NMRR $_{\Delta'_{mod}}$ (blue) and NMRR $_{PWM}$ obtained in classical PWM for the averaging using weight functions: uniform (a, b), triangle (c, d), Hamming (e, f), Blackman (g, j).

3. Simulation studies of the impact of non-linear distortions

Simulation tests were conducted in LtSpice, a software designed for simulating electronic circuits [34]. A pulse width modulator circuit was designed with the ability to set the desired range and modulation frequency, as well as the ability to change the information signal level and the amplitude and frequency of interference at its input. PWM parameters were set as follows: modulation frequency $f_m = 1$ kHz, reference voltage slope $S = 4.5$ V/1 ms. The informative pulse widths are determined by (5), (6) and (7). An algorithm was developed for weighting the pulse widths at the modulator output through the use of appropriate weighting functions (uniform, triangular, Hamming, and Blackman) by (12) and (13). The spectral distortions normalised to V_m , determined for the same interference parameters, are shown in the second row of Fig. 4 in decibel scale (NMRR $_{PWM}$).

To compare, the spectral distortions determined directly from (20) after normalisation to V_m : $|\Delta'_{mod}(v_h)| = |\Delta_{mod}(v_h)|/V_m$ in decibel scale (NMRR $_{\Delta'_{mod}}$) are shown in the first row of Fig. 4 together with envelope NMRR $_{A\delta}$.

In all cases, the spectral distortions are determined using the following averaging functions: uniform, triangular, Hamming and Blackman.

From the curves shown in the first row of Fig. 4, it can be seen that when using classical PWM, the amplitude distortion envelopes are practically independent of the use of weighting functions. Therefore, in the frequency range about $\frac{n}{4} \leq r_h \leq n - 1$, the maximum levels of distortion are the same. This effect can be explained by the properties of the Bessel function described above,

due to which, during weighting, practically only one or two harmonic components of distortion falls within the frequency bandwidth of the main frequency band of the spectral characteristic of the weighted function. Comparing the NMRR_{PWM} plots calculated directly from the estimates achieved through simulation of the modulated pulse widths (second row in Fig. 4) and those calculated from the Fourier series expansion of the harmonic distortion by the Bessel functions ($\text{NMRR}_{\Delta'_{\text{mod}}}$, first row in Fig. 4), we can observe their good convergence. This means that the analysis of harmonic distortions resulting from classic pulse width modulation is quite accurate.

On the other hand, due to the high level of the harmonic distortions NMRR_{PWM} (second row in Fig. 4), especially in the frequency band approximately $\frac{n}{4} \leq r_h \leq n - 1$, it is not possible to ensure the required quality of width-modulated signal processing using classical PWM in the presence of harmonic interference in the input signal during modulation.

A similar situation occurs in the case of power line interference of nominal frequency 50 Hz. Due to the EN 50160 [35], the variations in the frequency of the power supply network are allowed within $\delta_f = \pm 1\%$, and short-term variations can be as high as -6% to $+4\%$ i.e., 47 Hz ... 52 Hz. Significantly greater frequency instabilities occur in the power supply networks of mobile vehicles or ships [36]. Simulation studies have shown that when the aforementioned weighting functions are applied to the pulse width modulated pulses formed by classical PWM in the frequency range 47 ... 53 Hz, the obtained NMRR levels (Fig. 5a) are practically equal to the theoretical levels (Fig. 5b).

However, according to this standard [35], higher harmonics with relatively high levels may be present in the supply voltage. Namely, the level of the first few odd harmonics may be approximately: $3h - 5\%$, $5h - 6\%$, $7h - 5\%$ [35]. For example, in the vicinity of the 5th harmonic (nominally 250 Hz), due to non-linear modulation effects, the NMRR levels during PWM pulse averaging (except for the use of uniform averaging) differ significantly from the theoretical values (Fig. 6). At higher frequencies, harmonic suppression is no longer effective. It should be noted that the effects of the reduced suppression level are already visible around the 7th harmonic (nominally 350 Hz) and around other high harmonics.

Based on the results obtained through theoretical study and simulations, it can be concluded that due to the high level of modulation distortion across a wide frequency band, the direct use of classical pulse width modulation of the input signal distorted by harmonic interference is not recommended in measurement applications.

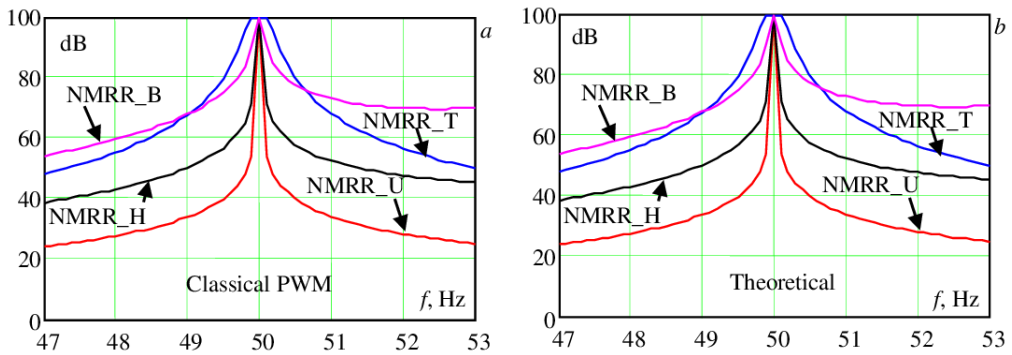


Fig. 5. NMRR in the frequency range 47 ... 53 Hz. Classical PWM ($n = 20$, $f_m = 1$ kHz) (a), theoretical (b).

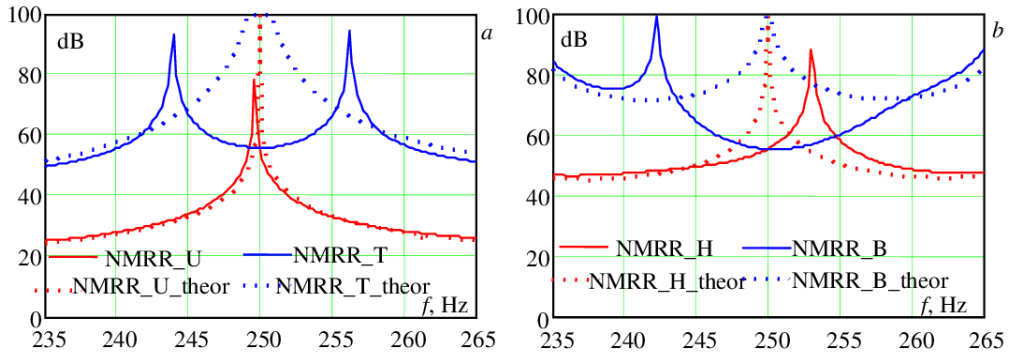


Fig. 6. Modulation of non-linear effects of NMRR around the 5th harmonic of interference (250 Hz) – solid lines, theoretical NMRR – dot lines. Averaging: uniform (U) and triangle (T) (a), Hamming (H) and Blackman (B) (b).

4. PWM combined with an S&H circuit

One possible solution to the modulation distortion problem discussed earlier is to use pulse width modulation combined with sampling and holding of the input signal ($PWM_{S\&H}$). This modulation is based on sampling and holding (S&H) of the input signal at periodic time intervals, followed by forming pulses with modulated output width by simply comparing the reference signals with the held signal (Fig. 7).

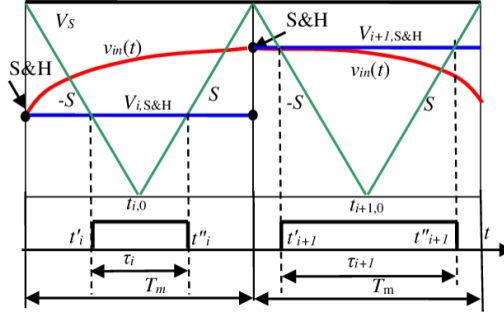


Fig. 7. Essences of double-edge PWM with an S&H circuit.

In the i -th modulated cycle, the signal value at the output of the S&H is equal to the input signal $V_{i,S\&H} = v_{in}\left(t_{i0} - \frac{T_m}{2}\right)$ taken at start time $t_{i0} - \frac{T_m}{2}$. Then, using the double-edge methods of pulse width modulation (Fig. 7), the same informational time intervals are obtained:

$$\tau_{i1} = \tau_{i2} = \tau_i = V_{i,S\&H}/S = v_{in}\left(t_{i0} - \frac{T_m}{2}\right)/S. \quad (23)$$

For the input signal (6) by (23), the width of modulated pulse is:

$$\tau_i = \tau_i\left(t_{i0} - \frac{T_m}{2}\right) = \frac{V_0 + V_m \cos\left(2\pi f_h\left(t_{i0} - \frac{T_m}{2}\right) + \phi\right)}{S}. \quad (24)$$

In this case, the pulses averaging result is:

$$N2_x = C_0 \sum_{i=1}^n \int_{\frac{-T_{av}}{2+i \cdot T_m}}^{\frac{-T_{av}}{2}+(i+1) \cdot T_m} \tau_i \cdot g(t) dt = V_0 + V_m \cos(\phi) \sum_{i=1}^n \cos\left(\frac{\pi(i-0.5)v}{n}\right) G_i, \quad (25)$$

where $G_i = \int_{t_{i,0}-\frac{\tau_i}{2}}^{t_{i,0}+\frac{\tau_i}{2}} g(t) dt$ is the integral value of the weighting function in the width interval $t_{i,0} - \frac{\tau_i}{2} \leq t \leq t_{i,0} + \frac{\tau_i}{2}$ of each output of PWM_{S&H} pulse output. Based on (24) and (25), the influence of the harmonic interference when sampling and holding operations are applied to the input signal before width modulation is performed is described by:

$$\Delta_{S\&H}(v_h) = V_m \sum_{i=1}^n \cos\left(\frac{\pi v_h}{n}(i-0.5)\right) G_i = V_m \sum_{i=1}^n \cos(\pi r_h(i-0.5)) G_i. \quad (26)$$

In (26), it is assumed that the maximal level of harmonic influence is obtained at $\cos(\phi) = 1$. Due to the symmetric integration limits, for relatively large values of n , *i.e.*, low values of T_m , the integral of the weighting function can be replaced by $g_i = g(t_{i,0})$, which is the value of the weighting function used at periodic mid-points of time $t_{i,0}$. Therefore

$$G_i \cong \tau_i g(t_{i,0}) = \tau_i g_i. \quad (27)$$

According to (27), the averaging is performed by multiplying the width τ_i of the pulse by the value g_i . Since there are no non-linear operations in (23), (24), the suppression of harmonic interference by pulse width averaging with sampling and holding of the input signal is, in practice, very close to the theoretical value.

5. Investigation of the PWM using the S&H circuit through simulation

The previously proposed method was tested using simulations. For this purpose, an S&H circuit was developed in the LtSpice software [34] at the input of the pulse width modulator with input range $V_R = 5$ V, modulation frequency $f_m = 1000$ Hz, modulation period $T_m = 1$ ms and slope $S = 5$ V/ms. Therefore, the output range $\tau_R = 1$ ms, *i.e.*, at the nominal PWM function, is $\tau_{nom} = 0.20000 \cdot v_{in}$. Since the nominal frequency of the power line interference $f_{p.l,nom} = 50$ Hz, then, at a modulation frequency of 1 kHz, the number of pulses at the PWM output during one interference period is $n = 20$. The averaging time and the number of averaged PWM pulses for the used weighting functions are 40 ms and 40 pulses for uniform, triangular and Hamming, and 60 ms and 60 pulses for Blackman. Higher harmonics of power line interference with frequencies equal to multiples of the fundamental frequency are suppressed in the bands: from 50 Hz to 950 Hz (19 harmonics) for triangular, Hamming and Blackman averaging. When using uniform averaging, the frequency step is equal to 25 Hz at $T_{av} = 40$ ms, therefore, the suppressed band is from 25 Hz to 975 Hz.

First, the actual static function of the modulator as the dependence of the output time interval τ (ms) on the input voltage v_i (V) was determined: $\tau = -0.00038 + 0.19949 \cdot v_{in}$. This function is used to convert voltage values into time interval values, and vice versa.

Subsequently, for constant input signal values of $V_x = 1, 2, 3, 4$ V and interference amplitude $V_m = 0.5$ V, simulation tests were performed to evaluate the effectiveness of this circuit in PWM in terms of suppressing the impact of interference present in the modulator input signal through weighted averaging. During the tests, the interference frequency was changed from 0 to $f_m = 1000$ Hz with a step of 0.5 Hz.

Figure 8a depicts the relationship between harmonic interference suppression (NMRR) in a wide frequency band up to the modulation frequency using the above-mentioned weighting functions. Figure 8b illustrates the theoretical dependence of the NMRR for these weighting functions. Comparing the NMRR values obtained in the PWM simulation with the S&H circuit (Fig. 8a) with the theoretical values (Fig. 8b) for all tested averaging functions, it can be concluded that they are very much in line with each other. In those tests, no significant non-linear distortions in the interference suppression level (NMRR) were observed, which proves the effectiveness of using an S&H circuit at the PWM input to reduce the impact of periodic interference present in the input signal by applying weighted averaging of output pulses.

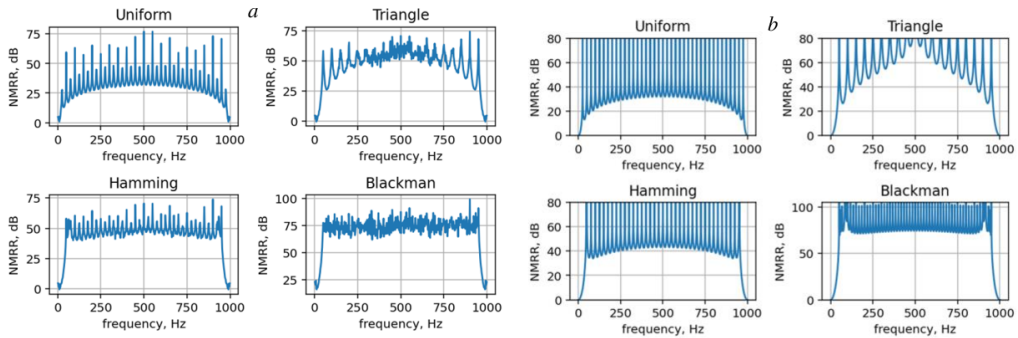


Fig. 8. Suppression of harmonic interference in the wide frequency band (0–1000 Hz, $T_m = 1$ ms $n = 20$) and the use of S&H circuit: a) simulation, b) theoretical.

It must be noted that interference with modulation frequency and its multiples, as in the theoretical case, is not suppressed. To suppress harmonics around modulation frequency, a low-pass filter can be used. As the modulation frequency is usually n times larger than the power line interference ($f_m \gg f_{pl}$), the requirements for such a filter are not rigorous.

6. Conclusions

Based on the results obtained from theoretical (Section 2) and simulation (Section 3) studies, it was determined that, in the case of periodic interference affecting the input signal, including power line interference, the use of the classic PWM method causes significant non-linear distortions in the formation of the output pulse width. A relationship was established between the level of pulse duration distortion and the level of interference, its frequency range, and the modulation parameters (mainly the modulation period and the reference signal slope).

It has been demonstrated that during demodulation (*e.g.*, by averaging to suppress the effect of periodic interferences), the present distortions cause significant limitations to both the level and the frequency band of suppression (NMRR) of these interferences. Such distortions mean, in particular, that the use of classic PWM transducers in measurement systems, where high accuracy and interference suppression are required, may be ineffective.

In order to reduce this type of non-linear distortions, the pulse-width modulation of the periodically sampled and held input signal during of the one modulation period, *i.e.*, using the S&H circuit, was proposed and investigated (Section 4). In such a modulator, the width of the output pulses depends on the values of the periodically sampled input signal. Therefore, thanks to this modulation method, the spectrum of the pulse-modulated signal repeats the spectrum of the sampled signal, taking into account zero-order interpolation within the modulation period interval.

The suppression (NMRR) of periodic interferences in both narrow and wide bands was investigated using weight averaging of output PWM_{S&H} pulses (Section 5). The effectiveness of suppressing the influence of interference on pulse width was examined using weight averaging based on four weighting functions or windows: uniform, triangular, Hamming and Blackman.

It has been shown that, in the case of power line interference (main and high harmonics), a sufficient level of suppression (up to 80 dB at 1% frequency instability) is achieved by using a triangular weighting function. However, in the case of interferences whose frequency varies over a wide band, higher-order functions such as Hamming (over 42 dB), Blackman (over 72 dB) and other should be used to suppress the effect.

Simulation tests performed in the LtSpice software confirmed the results obtained through theoretical analysis concerning distortion levels during weight averaging of modulated pulses. Based on the positive results obtained in theoretical and simulation studies, the next steps will involve the development of a prototype of a precise PWM converter based on the use of an S&H circuit, testing of the accuracy of the modulator's static characteristics, and the examination of the effectiveness of suppressing (NMRR) both narrowband and broadband interferences present in the input signal.

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