

From inverse error function to resolution enhancement: hardware-efficient virtual subdivision in single photon counting detectors

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Abstract

Single-photon counting hybrid pixel X-ray detectors combine a wide dynamic range with effective noise suppression, but their spatial resolution is degraded by charge sharing, which intensifies as the pixel pitch shrinks. The charge fractions registered by neighbouring pixels can, however, be exploited to estimate the photon interaction position with subpixel precision. This work presents a fully digital, in-pixel algorithm that reconstructs the interaction position from the digitised readings of a 3×3 pixel neighbourhood, assigning each event to one of 2×2 or 3×3 virtual subdivisions. An exact reconstruction based on the inverse error function is reformulated into the ratio-based virtual subdivision allocation (RVSA) algorithm, which replaces the per-event division and erf^{-1} evaluation with a single integer ratio test. Simulations against a behavioural front-end model with noise and converter non-idealities show that RVSA matches the floating-point reference, reducing the mean allocation error from 0.38 of the pixel pitch for conventional single-photon counting to 0.14 in the 3×3 mode. Synthesized in a 40 nm CMOS process, the RVSA datapath occupies about $1740 \mu\text{m}^2$ and dissipates $1.42 \mu\text{W}$ per pixel, fitting within a $50 \times 50 \mu\text{m}^2$ pixel alongside the analogue front-end.

1. Introduction

Semiconductor pixel detectors are widely used in X-ray instrumentation, including spectroscopy, synchrotron experiments, and medical imaging. Progressive CMOS scaling and the co-integration of analogue and digital circuitry allow a complete signal processing chain within pixels measuring tens of micrometres [1]; the hybrid architecture additionally decouples the sensor from the readout chip, so the sensor material can be matched to the photon energy range of interest.

In the single-photon counting mode, each absorbed photon generates electron–hole pairs that are collected by the electrodes and processed by the readout chain of the corresponding pixel. Compared with integrating detectors, this mode offers a wide dynamic range, inherent noise discrimination, and multiple energy thresholds, permitting, for instance, simultaneous imaging of several contrast agents in medical examinations [2].

As the pixel pitch is scaled down to improve spatial resolution, charge sharing — the splitting of a single-photon event into fractional signals in several neighbouring channels — becomes the dominant limitation, degrading both spatial resolution and spectroscopic performance [3]. Over the past two decades, the way detectors handle this effect has shifted from suppressing it in hardware towards exploiting it as a carrier of subpixel position information. The relative charge collected by neighbouring pixels is a monotonic function of the lateral distance separating the interaction point from the pixel boundaries and can therefore be inverted to localise the photon with a precision finer than the physical pitch [4].

This work develops a digital algorithm that performs this inversion inside the pixel itself. Operating on the analogue-to-digital converter (ADC) readings of a 3×3 neighbourhood, it groups the surrounding signals into directional cumulative sums and maps their normalised ratios to virtual subdivision coordinates through the inverse error function (erf^{-1}). It extends a previously published approach [5] from 2×2 to 3×3 virtual subdivisions within a single scalable scheme, and adds an arbitration mechanism that prevents double counting when

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electronic noise is present. Three steps take the algorithm from concept to synthesizable in-pixel logic. A floating-point reference model first fixes the exact mathematics and acts as a golden reference [6]. A synthesizable fixed-point version then realises it at the cost of look-up tables for the reciprocal and the erf^{-1} . Finally, recognising that each subdivision boundary corresponds to a fixed charge-fraction ratio, a simplified form replaces the coordinate computation with a single ratio test. The division folds into a constant and the erf^{-1} into a precomputed threshold, removing the look-up tables and compacting the datapath to fit inside a $50 \times 50 \mu\text{m}^2$ pixel within a microwatt power budget. Each variant is verified against realistic stimuli produced by a behavioural analogue front-end (AFE) model.

2. Charge-sharing effect in hybrid pixel detectors

In a hybrid pixel detector, the sensor is bump-bonded to a readout application specific integrated circuit (ASIC), so that every sensor electrode maps onto exactly one readout channel (Fig. 1). The electron-hole pairs created by an absorbed X-ray photon drift towards the collecting electrodes under the bias voltage, and the charge cloud broadens along the way through diffusion and Coulomb repulsion. At the photon energies considered here, repulsion remains negligible compared to diffusion, so purely diffusive spreading is assumed in the simulation model. When the cloud straddles a pixel boundary — which occurs increasingly often as the pitch shrinks — several channels register fractional signals, an effect referred to as charge sharing [3].

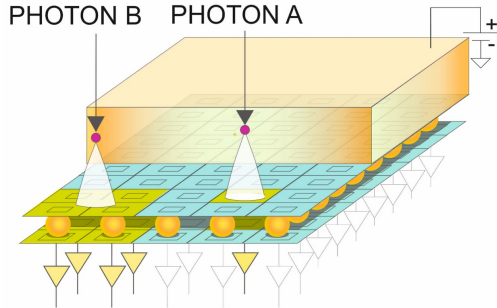


Fig. 1. Structure of a hybrid pixel detector: a sensor layer bump-bonded to a readout ASIC. For a photon absorbed close to a pixel boundary, the charge cloud spreads over adjacent electrodes, producing fractional signals in several readout channels. Figure from [7].

Charge sharing is detrimental in two respects: spectroscopically, each channel registers only part of the deposited energy, distorting the measured spectrum; spatially, the same photon may be counted by several pixels at once (overcounting) or, if the detection threshold is raised to suppress this effect, missed entirely near pixel borders (undercounting) [5].

The first response to charge sharing was to eliminate or compensate it in hardware, restoring one photon to one count in the correct pixel with its full energy [8]. On-chip strategies assign each event to the pixel with the largest signal, reconstruct the cluster charge through interpixel summing, or estimate the cluster centre of gravity [3, 9]. At AGH University of Krakow, this motivated a line of

pattern-recognition and charge-summing ASICs, including the synchrotron-verified CHASE Jr. [7] and the $50 \mu\text{m}$ -pitch FRIC [10]. Comparable analogue charge summing was implemented in Medipix3RX [11], while for thick CdTe sensors, HEXITEC corrects the spectrum off-chip [12]. All recover the correct count and energy, but none reconstructs position beyond the physical pixel pitch.

A complementary view regards charge sharing as a carrier of positional information rather than a nuisance. Since the charge fractions collected by neighbouring channels depend on the lateral offset of the interaction point from the pixel boundaries, the measured fraction (the η function) can be mapped to the true impact coordinate [13]. Under a Gaussian model of the charge cloud, this dependence is monotonic and can be inverted analytically to recover the hit position with subpixel precision [4]. Several groups exploit this off-chip: event-driven Timepix3 detectors weight a cluster centroid by time-over-threshold to reach subpixel resolution in CdTe [14], and the charge-integrating MÖNCH detector interpolates to the order of $1 \mu\text{m}$ at a $25 \mu\text{m}$ pitch [15]; related efforts pursue learned reconstruction and charge-sharing-tolerant sensor geometries [16, 17], and coincidence-counting schemes localise boundary events in medical photon-counting systems [18]. Within our group, the SPHIRD chip demonstrated real-time interpixel relocation into 2×2 and 3×3 subpixels, validated at the European Synchrotron Radiation Facility (ESRF) [19], followed by a fully digital subpixel allocation algorithm [4]. In these amplitude-comparison schemes, however, the virtual subdivisions are neither square nor uniform: their size and shape depend on the charge-cloud extent and, among other factors, on the photon energy.

Off-chip processing admits nonlinear or learned reconstruction but is bound by readout bandwidth and requires a low photon flux to keep clusters separated — frame-based centroiding of soft X-ray CCD images, for instance, reaches micrometre-level localisation, yet only for isolated single-photon spots and a slow, low-noise frame readout [20]; in-pixel digital logic instead reduces data at the source and sustains high count rates, within a tight area and power budget. The present work follows the latter, in-pixel route; unlike the amplitude-comparison schemes above, normalising the directional sums removes the energy dependence and restores uniform, square subdivisions.

3. Mathematics of the virtual subpixel methods

The algorithm operates on the digitized peak ADC values of a 3×3 pixel neighbourhood. The eight readings surrounding the central pixel are grouped into four directional sums (Σ_L , Σ_R , Σ_B , Σ_T), and the total neighbourhood charge Σ_{tot} normalises them into fractional shares $P_k = \Sigma_k / \Sigma_{\text{tot}}$ (Fig. 2). Under a Gaussian charge-cloud model with standard deviation σ , each fraction varies monotonically with the true hit position, so the relation can be inverted to recover the intra-pixel coordinate [5]:

$$x_a = \begin{cases} -\sigma\sqrt{2}\text{erf}^{-1}(2P_L - 1), & P_L \geq P_R, \\ \sigma\sqrt{2}\text{erf}^{-1}(2P_R - 1) + a, & P_R > P_L, \end{cases} \quad (1)$$

where a is the pixel pitch; the vertical coordinate y_a follows analogously from P_B and P_T . Normalising by Σ_{tot}

keeps the virtual subdivisions uniform and square regardless of photon energy. In what follows, the charge-cloud size is expressed by the radius $r = \sigma\sqrt{2}$ — the natural scale of the erf argument — normalised to the pixel pitch a .

Statistically, equation (1) is a deterministic inversion of the expected, noise-free charge-sharing response — the two-dimensional analogue of the η -function inversion used in strip detectors [13] — rather than a probabilistic estimator. Since each fraction is strictly monotonic in the hit position, thresholding a measured fraction is equivalent to comparing the reconstructed coordinate with a subdivision boundary, so in the noise-free limit and under uniform illumination the resulting classification coincides with the maximum *a priori* assignment; electronic noise blurs, but does not systematically displace, the decision boundaries (cf. Fig. 6).

This inversion captures only the charge sharing of a single Gaussian cloud, omitting secondary effects (fluorescence, scattering, carrier trapping, field non-uniformities) that are significant in high- Z sensors such as CdTe [12,21] but limited in the silicon sensors targeted here, where K-fluorescence plays a minor role [22].

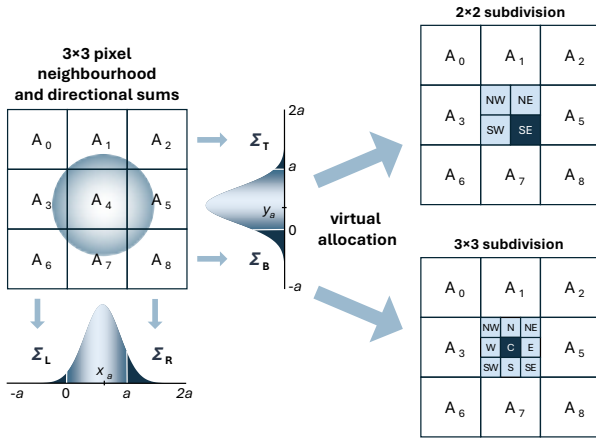


Fig. 2. A 3×3 cluster of physical pixels with the directional sums Σ_L , Σ_R , Σ_B , Σ_T (left), and the 2×2 (top right) and 3×3 (bottom right) virtual subdivision schemes of the central pixel. The central pixel A_4 contributes to Σ_{tot} but not to the directional sums.

The continuous coordinates (x_a, y_a) are then classified into $N \times N$ equal virtual subdivisions ($N = 2$ or $N = 3$), with boundaries at $a/2$, or at $a/3$ and $2a/3$, respectively. Because the estimator is continuous, higher subdivision orders require only additional boundary comparisons, although the attainable gain saturates once the subdivision pitch approaches the noise- and quantisation-limited blur of the boundaries; the trade-off between the subdivision count and the reconstruction error was studied in [23]. At a boundary, where $\Sigma_L = \Sigma_R$ or $\Sigma_T = \Sigma_B$, the coordinate is ambiguous: in the 2×2 mode, a per-axis toggle register resolves successive tied events by alternating between the two adjacent subdivisions, which removes systematic bias, while in the 3×3 mode, a tie defaults to the centre index. To keep a single event from being counted by more than one pixel, exactly one pixel of the cluster is selected as the *winner* by the arbitration mechanism described in section 4, alongside the in-pixel datapath that implements it.

Equation (1) defines the floating-point reference model — the enhanced virtual subdivision allocation reference model (EVSA_{REF}) [6] — which fixes the exact mathematics and serves as a golden reference. A fixed-point realisation (EVSA) was derived from it by quantising the datapath and implementing the two nonlinear operations as look-up tables: the reciprocal required for the normalisation $P = \Sigma/\Sigma_{\text{tot}}$ and the inverse error function erf^{-1} [5]. Although a faithful quantised version of the reference, its reciprocal multiplier and erf^{-1} look-up table make the 3×3 datapath too large to fit inside the pixel, which motivates the reformulation presented next.

4. From the reference model to an in-pixel realisation

Bringing the reconstruction inside the pixel proceeds in three steps of decreasing arithmetic exactness and increasing hardware efficiency (Fig. 3). The floating-point EVSA_{REF} model is exact but not synthesizable. Quantising it yields the fixed-point EVSA model, which is synthesizable but, with its reciprocal and erf^{-1} look-up tables, occupies about $4160 \mu\text{m}^2$ in the 3×3 mode — larger than the entire $50 \times 50 \mu\text{m}^2$ pixel cell. The third step reformulates the classification so that no coordinate is ever computed, yielding the ratio-based virtual subdivision allocation (RVSA) algorithm.

The reformulation rests on a single observation: the allocation logic does not need the coordinate x_a , only the index of the subdivision it falls into. Each subdivision boundary corresponds to a fixed value of the reconstructed coordinate (e.g. $a/3$), which through (1) maps to a fixed threshold value P_{th} of the dominant charge fraction $P_{\text{max}} = \Sigma_{\text{max}}/\Sigma_{\text{tot}}$, where Σ_{max} denotes the larger of the two opposing directional sums — $\max(\Sigma_L, \Sigma_R)$ for the horizontal axis and $\max(\Sigma_B, \Sigma_T)$ for the vertical; the threshold depends only on the operating point r/a . Approximating the threshold as a dyadic fraction $P_{\text{th}} = n/d$, with integer numerator n and denominator $d = 2^{10} = 1024$ — chosen so that the quantisation error remains negligible across the whole calibrated charge-cloud range $r/a = 0.2\text{--}0.9$ — collapses the per-event division and erf^{-1} into a single comparison:

$$P_{\text{max}} \geq P_{\text{th}} \iff \Sigma_{\text{max}} \cdot d \geq \Sigma_{\text{tot}} \cdot n. \quad (2)$$

The multiplication by d is a 10-bit shift and that by n a short shift-add chain, so each axis needs only one comparison. The numerator n is precomputed for the chosen r/a . Setting the reconstructed coordinate in (1) equal to a subdivision boundary x_b and solving for the fraction gives

$$P_{\text{th}} = \frac{1}{2} \left[1 - \text{erf}\left(\frac{x_b}{r}\right) \right], \quad (3)$$

so the inverse error function of the reference model survives only inside this constant, evaluated once at design time. A mismatch between the calibrated and actual cloud radius does not corrupt the classification but rescales the effective boundary linearly, $x_{b,\text{eff}} = (r/r_0) x_b$, so the allocation degrades gracefully with calibration error. The larger sum is used because the smaller of the two opposing sums can fall to the level of the electronic noise, so the test always evaluates the better-measured quantity. The selected sum plays a double role: its fraction of Σ_{tot} , tested by (2), decides

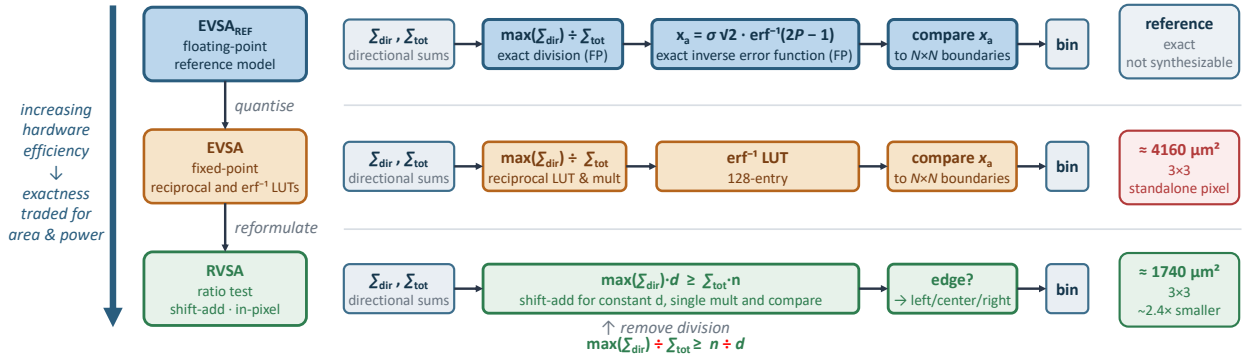


Fig. 3. Three-step evolution of the allocation algorithm: the exact floating-point reference (EVSA_{REF}, not synthesizable), its synthesizable fixed-point realisation (EVSA), and the reformulated in-pixel ratio test (RVSA). Annotated areas are post-synthesis values for the 3 × 3 mode.

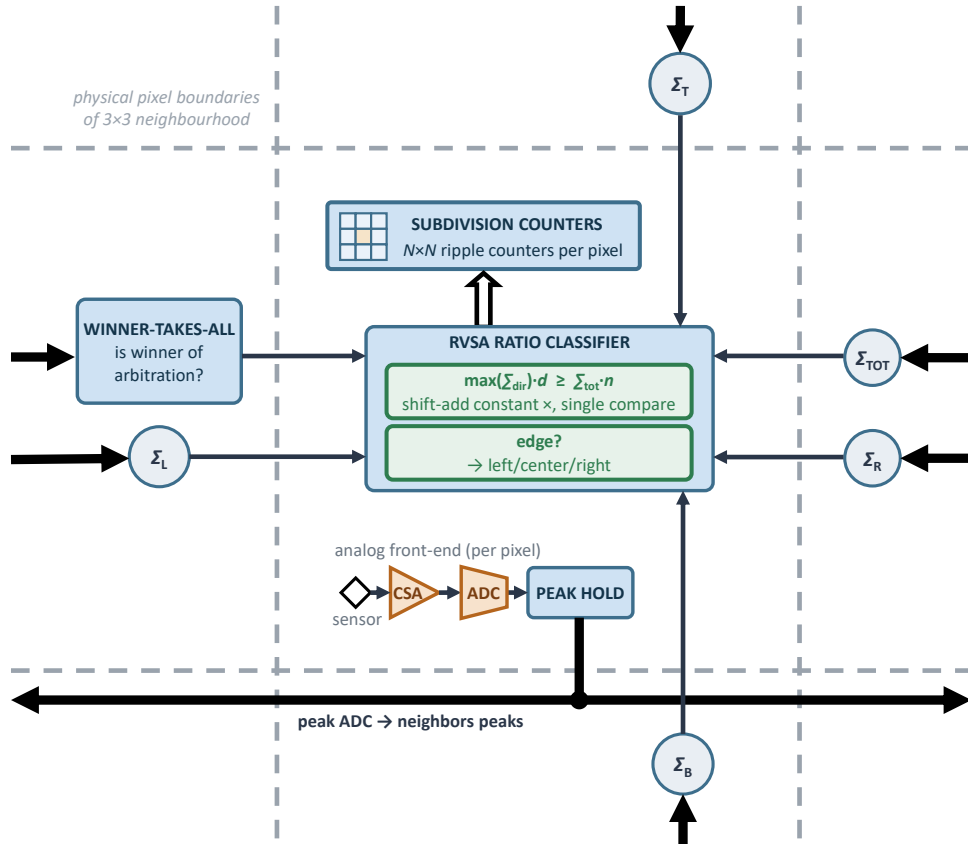


Fig. 4. Per-pixel RVSA datapath within the 3 × 3 neighbourhood: the analogue front-end and peak-hold, the peak broadcast to neighbours, the four directional line sums and total Σ_{tot} , the ratio classifier, and the $N \times N$ subdivision counters gated by the winner-takes-all arbitration.

whether the hit belongs to a side subdivision or the centre, and which of the two sums is larger selects the side — so a single ratio test per axis resolves the full left/centre/right (and top/centre/bottom) decision, replacing the coordinate computation and boundary comparison of the reference model.

The complete per-pixel datapath is shown in Fig. 4. The analogue front-end [a charge-sensitive amplifier (CSA) followed by an ADC] feeds a peak-hold stage capturing the largest ADC sample of the event; each pixel broadcasts its peak to its neighbours, from which the directional line sums and Σ_{tot} are assembled locally. The RVSA ratio classifier

applies (2) and increments one of the $N \times N$ subdivision counters, provided the pixel won the arbitration and its centre crossed the activation threshold.

To prevent double counting in the presence of electronic noise, a *winner-takes-all* arbitration selects exactly one pixel per event. The central pixel claims the event only if its peak ADC value is strictly greater than those of its top, top-left, left, and bottom-left neighbours, and greater than or equal to those of its bottom, bottom-right, right, and top-right neighbours. This asymmetric priority resolves ties deterministically; the same arbitration is shared by all three model variants.

Removing the look-up tables and the reciprocal multiplier shrinks the 3×3 allocation logic — the datapath excluding the subdivision counters, which are identical in both variants — to roughly a quarter of that of the exact fixed-point EVSA implementation, small enough to place the complete pixel inside the $50 \times 50 \mu\text{m}^2$ cell alongside the analogue front-end and readout.

5. Simulation environment

The algorithm was evaluated in a simulation environment written entirely in SystemVerilog and run in the Cadence Xcelium simulator. The framework pairs a behavioural AFE model, which generates digital stimuli reproducing the principal front-end non-idealities, with the subpixel allocation logic under test.

The AFE model represents a conventional readout channel: the CSA is modelled as a triangular output pulse with a configurable gain, rise rate, short plateau, and fall rate, plus Gaussian noise corresponding to the equivalent noise charge (ENC), with device-to-device dispersion of these parameters applied across the pixel array; the ADC samples this output at a fixed rate with a configurable resolution, full-scale range, and integral nonlinearity (INL). An example of the resulting waveforms is shown in Fig. 5.

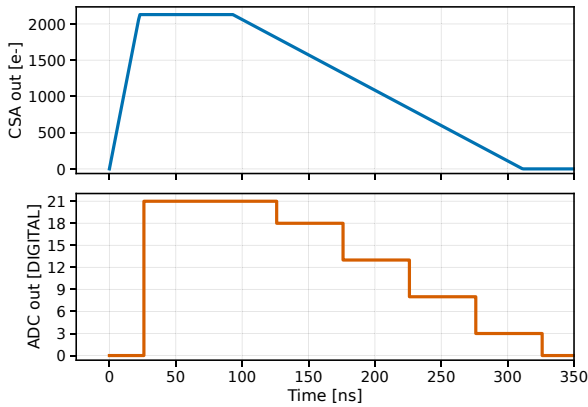


Fig. 5. Simulated CSA output (top) and the corresponding digitised ADC output (bottom) for a representative hit with the parameters of Table 1.

Table 1

Parameters used in the behavioural AFE simulations.

Parameter	Value
Charge	3000 e^-
Charge-cloud radius ($r = \sigma\sqrt{2}$)	50% pixel pitch
Equivalent noise charge (ENC)	80 e^-
CSA rise rate	$100 \text{ e}^-/\text{ns}$
CSA fall rate	$8 \text{ e}^-/\text{ns}$
ADC range	3200 e^-
ADC resolution	5 bits
ADC sample rate	20 MS/s
ADC integral nonlinearity (INL)	0.25 LSB
Activation threshold	400 e^-

Charge sharing was emulated by depositing a Gaussian charge cloud at a chosen position. Pixels in a 3×3 neighbourhood received charge fractions determined by their distance from the interaction point, following the model of [4]; the resulting per-pixel charges, processed by the AFE with the parameters of Table 1, form the digital inputs of the allocation logic.

The testbench instantiates a 3×3 array of identical pixel cells — the minimum configuration providing the central pixel with a complete neighbourhood. In a positional scan, charge is deposited on a grid of locations covering the central pixel together with the adjoining edges and corners of its eight neighbours; repeated depositions at every grid point accumulate the counts registered by each virtual subdivision of the central pixel, for both the 2×2 and 3×3 configurations. An ideal Gaussian charge cloud is assumed; the secondary transport effects discussed in section 3, together with charge repulsion during transport [24], are not modelled.

6. Simulation results

The positional scan covers a 240×240 grid of interaction positions spanning a $2a \times 2a$ region of the 3×3 array, centred on the central pixel, with 100 photon events deposited at each grid point; the maps reported here are produced by the synthesizable in-pixel RVSA algorithm. For the 3×3 configuration (Fig. 6), the nine panels present the counts collected by the individual subdivisions of the central pixel. Every subdivision responds within a localised region, confirming a partition of the pixel area into nine distinct zones.

The SUM panel, aggregating all subdivisions, is spatially uniform: each hit lands in exactly one subdivision, none lost or counted twice. Multi-pixel counting is thus suppressed by the asymmetric winner-takes-all arbitration over charge-cloud radii $r/a = 0.2\text{--}0.9$, despite electronic noise and ADC nonlinearity.

The COMBINED panel indicates, at each position, which subdivision dominates. Its boundaries depart from the ideal straight lines at $a/3$ and $2a/3$ in two ways: they are warped rather than rectilinear — a systematic distortion caused by the ADC INL (0.25 least significant bit, LSB) biasing the directional sums — and blurred over a finite transition width, a random spread introduced by the electronic noise (ENC 80 e^-) that makes the allocation of hits landing close to a boundary probabilistic.

The accuracy of the scheme is compared directly with the baseline in the relative allocation-error maps of Fig. 7, which contrast conventional single-photon counting (SPC) with the RVSA 2×2 and 3×3 modes on a shared colour scale. For SPC — an idealised baseline registering every event once, in the correct pixel, at its centre — the error peaks at 0.70 of the pitch in the corners; virtual subdivision flattens it across the pixel area. The mean allocation error — the Euclidean distance between the true hit position and the centre of the assigned subdivision, normalised by the pixel pitch — drops from 0.38 for SPC to 0.20 for RVSA 2×2 and 0.14 for RVSA 3×3 . The 3×3 mode therefore reduces the mean allocation error about $2.7\times$ relative to conventional SPC. The 2×2 figure matches the mean error of about 0.20 reported for the interpolator-based in-pixel implementation [5], which the 3×3 mode improves upon without any erf^{-1} arithmetic.

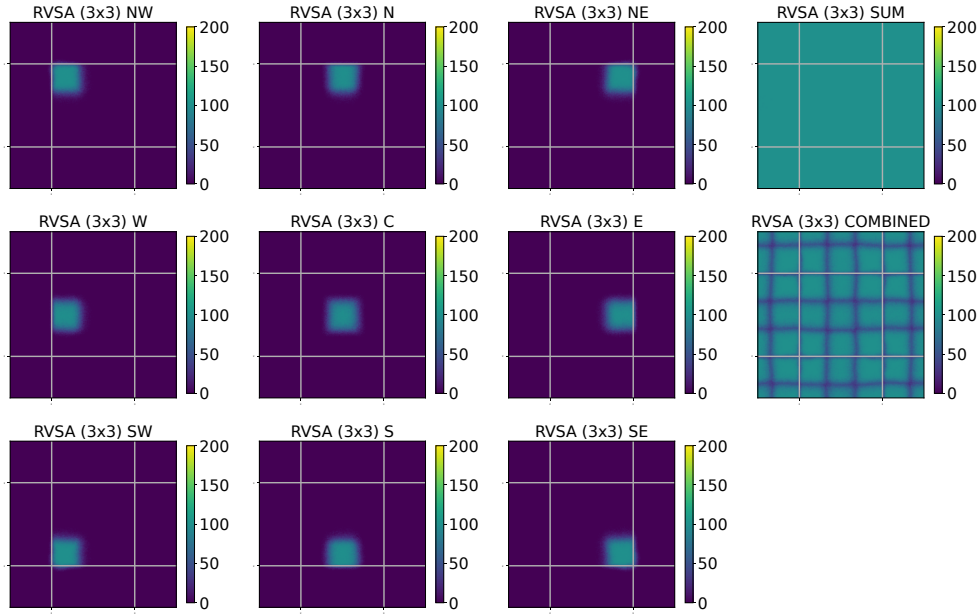


Fig. 6. In-pixel RVSA algorithm, 3×3 virtual subdivision. Panels NW–SE map the events allocated to each subdivision of the central pixel vs. the true hit position. The SUM panel is the total over all nine subdivisions, while COMBINED shows which subdivision collects the most counts at each position. Physical pixel boundaries are marked with thin white lines.

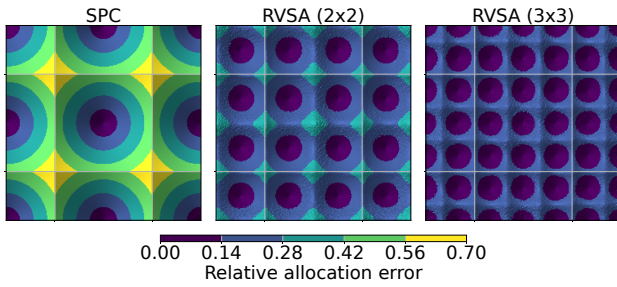


Fig. 7. Relative allocation-error maps over the pixel area for conventional SPC (left), RVSA 2×2 (centre) and RVSA 3×3 (right), on a shared colour scale normalised to the pixel pitch. Thin white lines indicate physical pixel boundaries.

The dependence on the charge-cloud size is shown in Fig. 8, which plots the mean allocation error against r/a for the floating-point reference and RVSA in both subdivision modes. The two overlap almost exactly across the whole range: folding the division and erf^{-1} into a single ratio test leaves the reconstruction accuracy unchanged. Both modes share an optimal charge-cloud size, with the error minimal near $r/a = 0.5$ ($\sigma/a \approx 0.35$), consistent with the optimum reported in [23]. For small clouds ($r/a < 0.3$), most of the charge stays within the central pixel, so the directional sums approach zero, ties become frequent, and tie-breaking dominates over physics-driven allocation. For large clouds ($r/a > 0.7$), the per-pixel signal-to-noise ratio deteriorates as the charge spreads over a wider area. The operating point $r/a = 0.5$ was therefore used for all other simulations.

Figure 9 examines the effect of ADC quantisation: increasing the converter resolution initially improves the allocation, but above 5 bits the curves plateau (relative improvements below 2%) and RVSA follows the reference throughout. A 5-bit ADC was therefore retained for the nominal configuration.

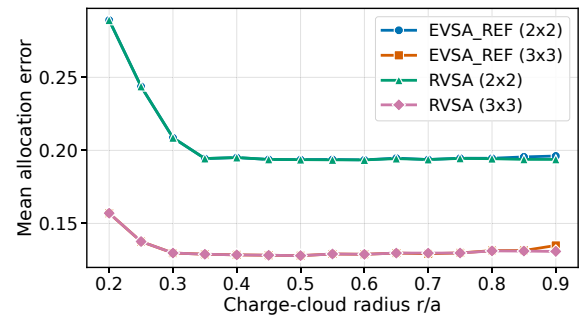


Fig. 8. Mean normalised allocation error vs. relative charge-cloud radius r/a . Floating-point reference and RVSA, in the 2×2 and 3×3 modes.

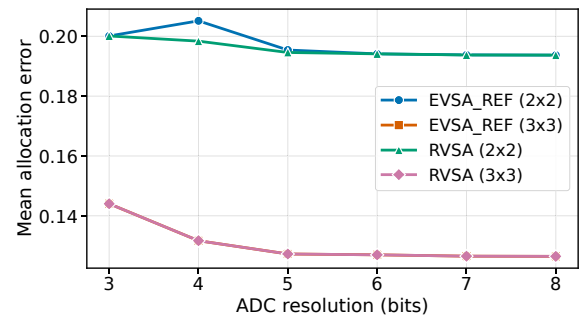


Fig. 9. Mean normalised allocation error as a function of ADC resolution for EVSA_{REF} and RVSA, evaluated in both virtual-subdivision configurations.

7. Synthesis results

With RVSA matching the reference accuracy, the hardware cost of the two synthesizable variants was quantified. Both designs were synthesized in a 40 nm CMOS process with

Cadence Genus at a high effort setting. The flow stops before place-and-route, so the comparison reflects the logic itself rather than a particular layout. Module boundaries were preserved during synthesis (automatic ungrouping disabled) to obtain the per-block breakdown of Table 2, which changes the totals only marginally. Part of the RVSA logic — the directional sums and the inter-pixel comparators — is shared between neighbouring pixels, so its area is reported as the average over a synthesized 8×8 array.

The key result is summarised in Fig. 10: the exact fixed-point EVSA pixel requires about $4160 \mu\text{m}^2$ and exceeds the area of the $50 \times 50 \mu\text{m}^2$ ($2500 \mu\text{m}^2$) cell, whereas RVSA occupies about $1740 \mu\text{m}^2$ — roughly 70% of the cell, leaving about $760 \mu\text{m}^2$ for the analogue front-end. Discarding the erf^{-1} and reciprocal look-up tables, which alone occupy nearly half of the EVSA pixel, renders RVSA about 2.4× smaller and 1.9× lower in power (1.42 against $2.72 \mu\text{W}$). The 2×2 variants follow the same pattern — about $3630 \mu\text{m}^2$ for EVSA against $830 \mu\text{m}^2$ for RVSA — since the look-up tables do not scale with the subdivision grid. Both the area and power figures are pre-layout synthesis estimates, with power evaluated at a clock frequency equal to the ADC sampling rate of 20 MHz.

Table 2 details the distribution of the RVSA logic: the nine subdivision counters — one ripple counter per virtual subpixel — occupy about 58% of the per-pixel datapath, more than all the allocation logic combined, so most of the area is event storage rather than arithmetic. The datapath total ($1316 \mu\text{m}^2$) is below the $1740 \mu\text{m}^2$ per-pixel figure because the latter, derived from the full array, includes the per-pixel share of the common inter-pixel logic.

Table 2

Per-pixel area breakdown of the RVSA 3×3 datapath with 16-bit counters. Shares are relative to the per-pixel datapath total.

Block	Area (μm^2)	Share
Subdivision counters (ripple)	767	58 %
Ratio classifier	213	16 %
Control finite-state machine (FSM)	166	13 %
Peak-hold	92	7 %
Directional sums Σ	78	6 %
Per-pixel datapath	1316	100 %

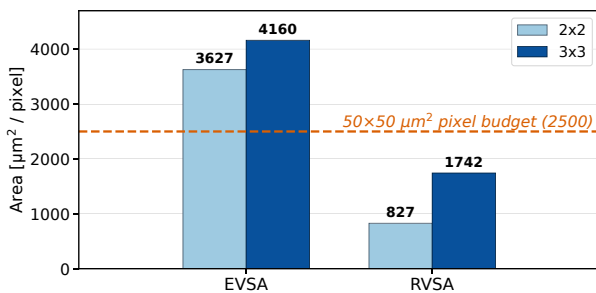


Fig. 10. Post-synthesis per-pixel area of the exact fixed-point EVSA pixel and the simplified RVSA (array average) in the 2×2 and 3×3 modes with 16-bit counters.

8. Conclusions

This work has developed a virtual subdivision allocation algorithm, progressing from an exact floating-point reference to synthesizable in-pixel logic that reconstructs the photon interaction position within a 3×3 pixel neighbourhood. Simulations with a behavioural front-end model confirmed that the asymmetric winner-takes-all arbitration provides complete spatial coverage without double counting, and that the simplified in-pixel RVSA reproduces the accuracy of the floating-point reference across charge-cloud size and ADC resolution. In the 3×3 mode, the algorithm reaches a mean allocation error of about 0.14 (normalised to the pixel pitch), a nearly threefold reduction relative to baseline single-photon counting.

Removing the division and erf^{-1} makes the RVSA 3×3 datapath fit within the $50 \times 50 \mu\text{m}^2$ pixel in a 40 nm CMOS process — about $1740 \mu\text{m}^2$, some 2.4× smaller than the exact fixed-point EVSA — at a power of about $1.42 \mu\text{W}$, leaving room for the analogue front-end and readout. The nine counters per pixel nevertheless require a dedicated readout architecture, and the residual ADC-INL influence on the subdivision boundaries calls for improved converter linearity or a digital correction within the allocation logic.

The simulation-based validation is model-matched — the same Gaussian cloud generates the stimuli and calibrates the threshold — and therefore demonstrates the fidelity of the hardware reformulation rather than the accuracy of the transport model itself. The Gaussian model enters the synthesized logic only through the precomputed threshold constant, and the classification requires only that the directional charge fraction remain monotonic in the interaction position; thresholds derived from a more realistic charge-transport model, or directly from measured charge-fraction curves, can therefore be substituted without altering the classifier structure. Experimental validation is planned following the pencil-beam methodology previously applied to the SPHIRD chip [19]: raster-scanning a focused beam with a spot smaller than the virtual subdivision measures the localisation accuracy and the effective subdivision boundaries, and yields measured charge-fraction curves for threshold correction.

The analytical classifier is also the first step in a progression of reconstruction methods of increasing fidelity: a Monte Carlo-derived look-up table capturing the full two-dimensional charge-sharing response, a maximum a posteriori estimator incorporating an explicit noise model, or a learning-based regressor free of the Gaussian assumption. Readout architectures preserving per-pixel amplitude information — such as the IntelPixel chip with an in-pixel neural network [25] and the SPECTRUM 1k chip with in-pixel energy histogramming [26], both developed at AGH University of Krakow — provide platforms on which these estimators can be applied offline to the same recorded events and compared directly with the analytical classifier.

The scheme targets fine-pitch silicon detectors operated at photon fluxes that require counting-mode readout, with prospective applications in dispersive soft X-ray spectroscopy, X-ray photon correlation spectroscopy [27], and high-flux synchrotron imaging and diffraction, where an effective pixel smaller than the physical pitch translates directly into instrument resolution. Furthermore, since Σ_{tot} is already

assembled by the algorithm, the scheme could be extended to energy-resolved (colour) imaging by discriminating this sum into several energy bins, at the cost of replicating the counters per bin.

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Declaration on the use of AI

During the preparation of this manuscript, the authors used an LLM for language editing and grammatical refinement. All scientific content, results, and conclusions are the authors' own; the authors reviewed and edited the text after using the tool and take full responsibility for the content of the publication.

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