

S5. Alignment and calibration protocol (bulk-optics stochastic NOT)

Purpose. This protocol describes how to (i) align the bulk-optics NOT module, (ii) measure effective extinction ratios (ER) of PBS/analyser stages, (iii) set HWP angles (0° , 45° , 90°) used in the two architectures, (iv) verify the encoding $H/V \leftrightarrow 1/0$ and the transfer law $\hat{p}_{\text{out}} \approx 1 - \hat{p}_{\text{in}}$, and (v) calibrate the contrast-threshold decision settings (γ and optional erasure window). The measured values map directly to the Colab configuration parameters.

Scope. The steps apply to both implementations: Architecture A (folded PBS-based) and Architecture B ($3\times$ HWP chain). They are written for intensity-based operation (H/V discrimination) and can be executed with standard lab components (PBS cubes, HWPs, photodiodes/power meter).

S5.1 Equipment, notation, and measurement conventions

Recommended equipment: stable laser source, PBS cube(s), half-wave plates (HWP), rotation mounts with degree scale, mirrors (for folded A), polariser/analyser (or PBS used as analyser), photodetectors/power meter, and (optional) neutral-density filters to avoid saturation.

Notation: we denote measured intensities after the analyser as I_H and I_V . The decision contrast is $C = (I_H - I_V)/(I_H + I_V)$. The stochastic encoding uses $1 \equiv H$ and $0 \equiv V$.

S5.2 Measuring effective extinction ratio (ER) and leakage power

Definition: $ER_{\text{dB}} = 10 \log_{10}(P_{\text{pass}}/P_{\text{leak}})$, where P_{pass} is the power in the intended port and P_{leak} is the orthogonal leakage.

Procedure (PBS/analyser):

1. Prepare a known input polarisation (H). Measure powers at the analyser outputs: P_H (pass) and P_V (leak). Compute $ER_H(\text{dB}) = 10 \log_{10}(P_H/P_V)$.
2. Repeat for input V to obtain $ER_V(\text{dB}) = 10 \log_{10}(P_V/P_H)$.
3. If ER differs between H and V, record both values; for modelling you may use an average ER_{dB} or convert to leakage power $\ell = 10^{(-ER_{\text{dB}}/10)}$.
4. Practical note: repeat measurements at several power levels to exclude detector nonlinearity; keep detector below saturation.

S5.3 Setting HWP angles and verifying polarisation exchange

Goal: realise a swap $H \leftrightarrow V$ with maximal contrast at the analyser.

Procedure:

1. Insert the HWP under test and the analyser PBS. For input H, scan the HWP angle around the nominal exchange setting ($\approx 45^\circ$ relative to the H/V basis).
2. For each angle, measure I_H and I_V and compute C . The exchange setting corresponds to a strong sign flip of C (or maximal $|C|$ with inverted sign).
3. Lock the HWP at the angle that maximises $|C|$ for both logical inputs (repeat the scan for input V as a cross-check).
4. For Architecture B (0° – 45° – 90° chain): first set the 45° plate to achieve the swap; then set the 0° and 90° plates to their nominal orientations. If phase-normalisation is used (Supplement S6), fine-tuning is performed with a phase-sensitive diagnostic; intensity-domain metrics should remain unchanged.

S5.4 Verifying encoding and probability-level NOT behaviour

Generate a short calibration bitstream with several target probabilities p_{in} (e.g., $p_{\text{in}} \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$) by controlling the input polarisation state.

For each p_{in} , acquire N time slots (recommend $N \geq 10^4$), estimate $\hat{p}_{\text{in}}$ as the fraction of H-labelled slots at the input, and estimate $\hat{p}_{\text{out}}$ at the output.

Acceptance check (intensity domain): verify the transfer law $\hat{p}_{\text{out}}(p_{\text{in}}) \approx 1 - \hat{p}_{\text{in}}$ for all tested p_{in} within the chosen tolerance. If deviations are systematic, re-check HWP angles and ER values and ensure the analyser thresholds are not saturating the detector.

S5.5 Calibrating the decision settings (γ and optional erasure window)

Decision rule: classify a slot as H (1) if $C > +\gamma$, as V (0) if $C < -\gamma$. With erasure enabled, slots with $|C| < \gamma_{\text{erasure}}$ are discarded (throughput < 1).

Recommended calibration:

1. Measure C distributions for known inputs H and V (collect at least $N \geq 10^4$ samples per state).
2. Choose γ to balance false positives/negatives (BER) against robustness; a stricter γ reduces BER but may increase undecided slots if erasure is used.

3. If erasure is enabled, choose γ_{erasure} based on the overlap region of the two C distributions. Report the resulting throughput and BER.
4. Record the final $(\gamma, \gamma_{\text{erasure}})$ pair; use the same values in the harmonized comparison (S4a) when generating main-text sensitivity figures.

S5.6 Mapping measured values to Colab parameters

Measured quantity	Colab parameter name	Notes
PBS/analyser ER (dB)	pbs_ER_dB / analyzer_ER_dB	Use separate values if PBS1/PBS2/analyser differ; otherwise use an effective ER.
Leakage power	leak_power = $10^{(-\text{ER_dB}/10)}$	Convenient for shared axes and monotone scans.
HWP angle offset (deg)	hwp_theta_err_deg (A), dtheta_deg or hwp_theta_err_deg (B)	Use common-mode for manufacturing; per-plate for tolerance studies.
HWP retardance error (deg)	hwp_delta_deg / hwp_delta_err_deg	May include λ -detuning contribution in B.
Arm imbalance (dB)	imbalance_dB (A)	Measured as relative power ratio between arms.
Detector relative noise	detector_noise_rel	Estimate from repeated intensity measurements at fixed input.
Decision thresholds	gamma, gamma_erasure	Chosen from C distributions; report throughput if erasure is used.

Reporting. In the manuscript, report nominal component values and the calibrated decision settings, while detailed measurement steps and parameter-to-code mapping are provided here (Supplement S5).