

S6. Phase reference and compensation note (HWP-chain NOT)

Scope. This note follows the three-half-wave-plate (HWP) realisation of the NOT operation used in the manuscript. It clarifies why an intensity-correct polarisation swap may be accompanied by a π phase shift, and how a compact HWP sequence restores a common (baseline) phase reference. The phase discussion is relevant only when the NOT output is used in phase-sensitive blocks (coherent recombination, interferometric diagnostics, or phase-referenced cascading). For purely intensity-based H/V discrimination and thresholding, a global phase is immaterial.

Encoding and objective. We use the laboratory basis $\{|H\rangle, |V\rangle\}$ with the mapping $1 \equiv |H\rangle$ and $0 \equiv |V\rangle$. The optical NOT is implemented as a slot-by-slot polarisation exchange $H \leftrightarrow V$, while preserving a known baseline phase at the output plane for both logical states.

S6.1 Concept: three-HWP chain for swap with phase normalisation. A polarisation swap can be produced by passing each time slot through a HWP oriented at 45° with respect to the input polarisation axes. This first plate exchanges the logical states ($1 \leftrightarrow 0$), but—depending on the chosen Jones convention and physical plate realisation—both states can acquire a common additional π phase (180°) relative to the incident wave. To restore a common, predefined phase reference, two additional HWPs oriented at 0° and 90° are placed after the 45° plate. These plates do not change the already swapped polarisation logic, but act as a phase-normalisation pair: the second plate applies a π delay to one polarisation component, and the third plate applies a π delay to the orthogonal component, so that both output logical states share the same baseline phase.

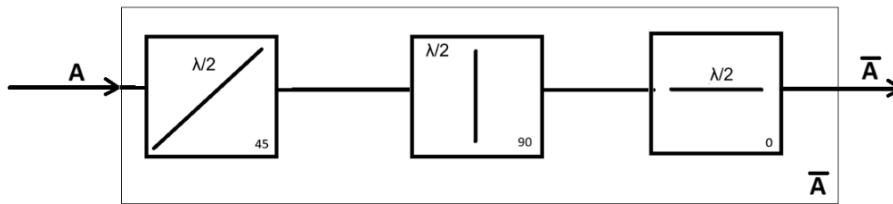


Fig. S6-1. Inverter structure implemented as a HWP chain ($45^\circ + 0^\circ + 90^\circ$).

S6.2 Step-by-step transformation for logical ‘1’. Consider an input slot in state 1 ($|H\rangle$). Before the 45° HWP, the polarisation vector can be decomposed into two orthogonal projections along axes rotated by 45° , denoted here as components 1_1 and 1_2 .

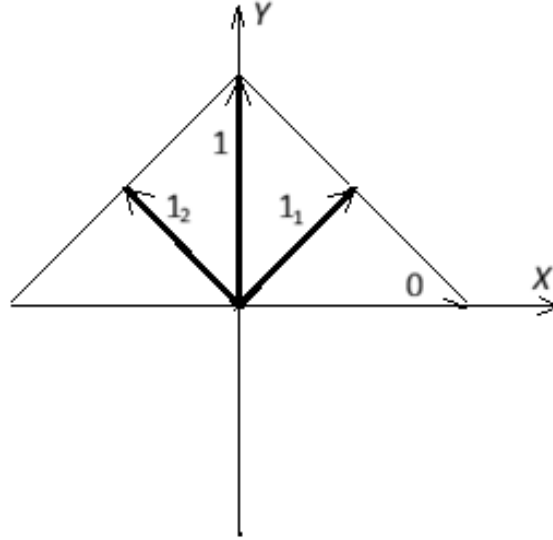


Fig. S6-2. Decomposition of the input polarisation ‘1’ into orthogonal components 1_1 and 1_2 in the 45° -rotated basis.

Passing through the HWP at 45° flips one of these components while leaving the orthogonal component unchanged. The vector recombines into the orthogonal polarisation state ($0 \equiv |V\rangle$), while the oscillation phase is shifted by π relative to the incident wave (phase lag of 180°).

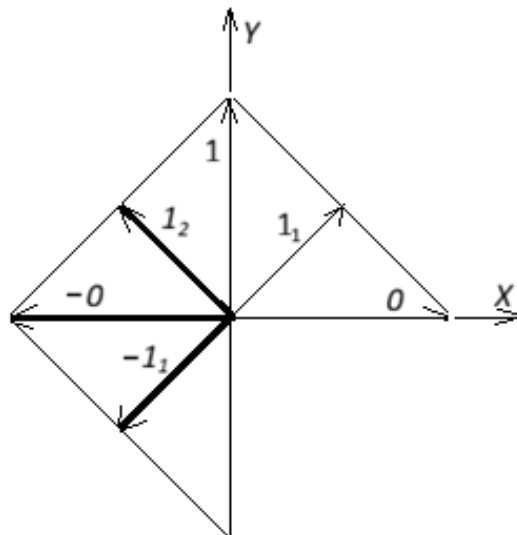


Fig. S6-3. Action of a 45° HWP: one projection changes sign, producing a net polarisation swap ($1 \rightarrow 0$).

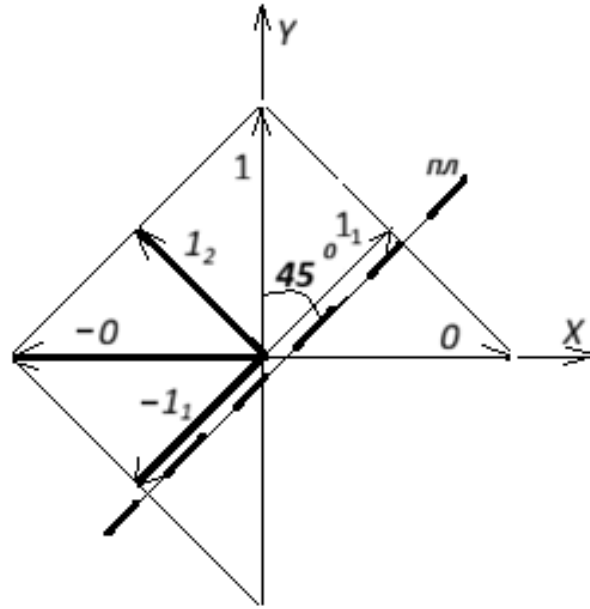


Fig. S6-4. Result after the first (45°) plate: polarisation is inverted ($1 \rightarrow 0$) while the field phase is shifted by π (180°) relative to the input reference.

S6.3 Symmetry for logical ‘0’ and phase restoration. The same reasoning applies to an input slot in state 0 ($|V\rangle$): the 45° plate swaps $0 \rightarrow 1$ and may introduce the same π phase shift.

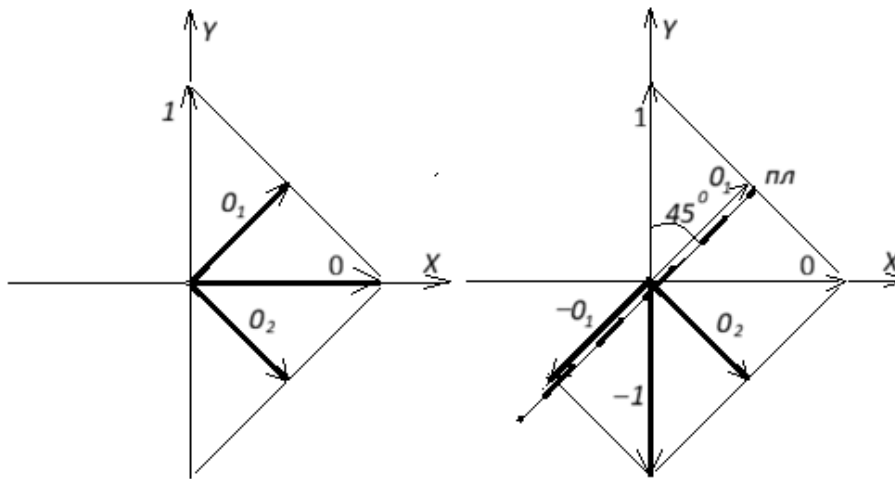


Fig. S6-5. Analogous swap for an input polarisation ‘0’: the 45° HWP produces $0 \rightarrow 1$ with the corresponding phase behaviour.

After the swap, the two additional HWPs (0° and 90°) serve to restore the baseline phase reference for both logical states. In particular, the 0° plate leaves one component in-phase

while delaying the orthogonal component by π ; the 90° plate performs the complementary correction. Thus, the final output is polarisation-inverted slot-by-slot, with a common and known phase reference for both 0 and 1, and with unchanged amplitude (intensity level).

Practical remarks. If the NOT is used only with intensity discrimination, the intermediate π phase is irrelevant and the phase-normalisation plates are optional. If the NOT output is coherently recombined or interferometrically analysed, the phase-normalisation stage should be kept and verified via a phase-sensitive diagnostic (e.g., fringe visibility maximisation).